Experiment 5

Inelastic Collisions

5.1 Objectives

- Measure the momentum and kinetic energy of two objects before and after a perfectly inelastic one-dimensional collision.
- Observe that the concept of **conservation of momentum** is independent of **conservation of kinetic energy**, that is, the total momentum remains constant in an inelastic collision but the kinetic energy does not.
- Calculate the percentage of KE which will be lost (converted to other forms of energy) in a perfectly inelastic collision between an initially stationary mass and an initially moving mass.

5.2 Introduction

One of the most important concepts in the world of physics is the concept of conservation. We are able to predict the behavior of a system through the **conservation of energy** (energy is neither created nor destroyed). An interesting fact is that while **total energy is ALWAYS conserved**, kinetic energy might not be as it can be converted to other forms of energy, such as potential energy or heat. Like total energy, **momentum is ALWAYS conserved**. In this experiment and the following week's experiment, you will demonstrate that momentum is always conserved while kinetic energy may or may not by studying inelastic and elastic collisions.

5.3 Key Concepts

You can find a summary on-line at Hyperphysics.[1] Look for keywords: elastic collision and inelastic collision.

5.4 Theory

This experiment and the following will deal with two different types of one-dimensional collisions: inelastic and elastic. Below is a discussion of the principles and equations that will be used in analyzing both kinds of collisions.

For a single particle, **momentum** is defined as the product of the mass and the velocity of the particle:

$$\vec{p} = m\vec{v} \tag{5.1}$$

Momentum is a **vector** quantity[2] which means that **direction** is a necessary part of the data. For example, in the one-dimensional case the momentum could have a direction in either the $+x$ direction or the $-x$ direction. For a system of more than one particle, the **total momentum** is the **vector sum** of the individual momenta:

$$\vec{p} = \vec{p_1} + \vec{p_2} + ... = m_1\vec{v_1} + m_2\vec{v_2} + ... \tag{5.2}$$

So you add the momenta of all the particles together making sure to take into account the direction each particle is moving.

One of the most fundamental laws of physics is that the **total momentum, $\vec{p}$,** of any system of particles is **conserved**, or constant, as long as the net external force on the system is zero. Assume we have two particles with masses m_1 and m_2 and initial velocities $\vec{v}_{1i}$ and $\vec{v}_{2i}$ which collide with each other without any external force acting on them. Suppose their velocities after the collision are $\vec{v}_{1f}$ and $\vec{v}_{2f}$. **Conservation of momentum** then states that the total momentum before the collision ($\vec{p}_{initial} = \vec{p_i}$) is equal to the total momentum after the collision ($\vec{p}_{final} = \vec{p_f}$):

$$\vec{p_i} = \vec{p_f} \quad \text{where} \quad \vec{p_i} = m_1\vec{v}_{1i} + m_2\vec{v}_{2i} \quad \text{and} \quad \vec{p_f} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \tag{5.3}$$

[1] http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

[2] Vector quantities will be denoted by having an arrow over them.

In any given system, the **total energy** is generally the sum of several different forms of energy. **Kinetic energy, KE,** is the form associated with motion and for a single particle is written as:

$$KE = \frac{mv^2}{2} \tag{5.4}$$

In contrast to momentum, kinetic energy is **NOT** a vector.[3] For a system of many particles the **total kinetic energy** is simply the sum of the individual kinetic energies of each particle:

$$KE = KE_1 + KE_2 + ... \tag{5.5}$$

Another fundamental law of physics is that the **total energy** of a system is **always conserved**. However within a given system one form of energy may be converted to another. For example, in the Free Fall lab the potential energy of the falling cylinder was converted into kinetic energy. *Kinetic energy* ***alone*** *is often not conserved.*

There are two basic kinds of collisions, elastic and inelastic. You will study inelastic collisions this week and elastic collisions next week. The following discussion will be pertinent for both weeks.

In an **elastic collision**, two or more bodies come together, collide, and then move apart again with **no loss in kinetic energy**. An example would be two identical "superballs," colliding and then rebounding off each other with the same speeds they had before the collision. Since there is no loss in kinetic energy the initial kinetic energy ($KE_{initial} = KE_i$) must equal the final kinetic energy ($KE_{final} = KE_f$).

$$KE_i = KE_f \quad \text{so} \quad \frac{m_1 v_{1i}^2}{2} + \frac{m_2 v_{2i}^2}{2} = \frac{m_1 v_{1f}^2}{2} + \frac{m_2 v_{2f}^2}{2} \tag{5.6}$$

In an **inelastic collision**, the bodies collide and come apart again, but ***some*** *kinetic energy is lost.* That is, some of the kinetic energy is converted to another form of energy. An example would be the collision between a baseball and a bat where some of the kinetic energy is used to deform the ball and converted into heat. If the bodies collide and stick together, the collision is called **perfectly inelastic**. In this case, ***much*** *of the kinetic*

[3]Notice there is no arrow over the velocity variable in the equation for kinetic energy.

energy is lost in the collision. That is, much of the kinetic energy is converted to other forms of energy.

In the following two experiments you will be dealing with a perfectly inelastic collision in which much of the kinetic energy is lost, and with a nearly elastic collision in which kinetic energy is conserved. Remember, in both of these kinds of collisions total momentum should always be conserved.

Today you will be dealing with a perfectly inelastic collision of two carts on an air track. Cart #2 will be sitting at rest so $v_{2i} = 0$, while Cart #1 is given a slight push (v_{1i}) in order to initiate a collision. Let's consider what the kinetic energy should be in the initial state before the carts have hit each other. Using Equations 5.4 and 5.5, the initial kinetic energy KE_i is:

$$KE_i = \frac{m_1 v_{1i}^2}{2} + \frac{m_2 v_{2i}^2}{2} = \frac{m_1 v_{1i}^2}{2} \tag{5.7}$$

Whereas, the final kinetic energy, after the carts have hit and stuck together, is given by:

$$KE_f = \frac{(m_1 + m_2) v_f^2}{2} \tag{5.8}$$

Notice that because the carts are now stuck together the mass is their total mass $(m_1 + m_2)$ and they have a common velocity, $v_f = v_{1f} = v_{2f}$.

In today's lab you will be comparing your measured value for the final kinetic energy with a predicted value. In order to get the predicted value we need to derive an equation that relates the final kinetic energy to the initial kinetic energy of a perfectly inelastic collision. Remember for an inelastic collision, kinetic energy is NOT conserved but momentum IS. Using conservation of momentum ($\vec{p_i} = \vec{p_f}$) and the fact that Cart #2 is initially at rest gives:

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1i} = (m_1 + m_2) \vec{v}_f \tag{5.9}$$

Using Eqs. 5.7, 5.8, and 5.9, we arrive at an equation for KE_f in terms of KE_i. (You are asked to show the complete derivation of this formula in the questions section.)

$$\boxed{KE_f = \left(\frac{m_1}{m_1 + m_2} \right) KE_i} \tag{5.10}$$

This is the prediction for the final kinetic energy of a perfectly inelastic collision.

5.5 In today's lab

Today you will look at perfectly inelastic collisions and see how momentum is conserved but kinetic energy is not. You will vary the amount of mass on the two colliding carts and see how that changes the kinetic energy lost. You will show there is a significant energy loss in perfectly inelastic collisions and try to figure out where this energy goes.

5.6 Equipment

- Air Track
- Air Supply
- Two carts one with needle and one with clay (carts are sometimes called gliders)
- Photogate Circuit
- 4 - 50g masses

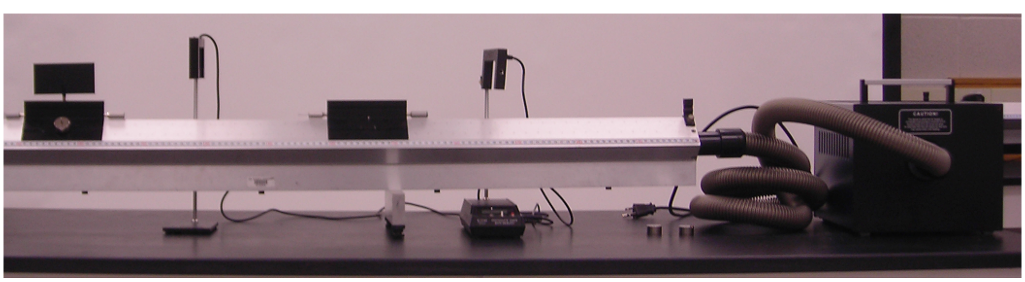

Figure 5.1: A photo of the lab setup.

5.7 Procedure

Do not move the carts on the air track when the air is not turned on. It will scratch the track and ruin the "frictionless" environment we need to get accurate data.

1. Start by making sure that the air track is level. Your instructor will demonstrate how at the beginning of class.

2. We will define Cart #1 as the cart with the fin and Cart #2 as the cart without. For each trial, you will always push Cart #1 and have Cart #2 stationary ($v_{2i} = 0$ cm/s) in the middle between the photogates. See Figure 6.1.

3. Set up the photogates such that there is sufficient room for the collision to happen in between the gates AND enough room after the second photogate that both carts can pass all the way through the second photogate. Make sure the fin on Cart #1 passes all the way through the first photogate before colliding with the second cart and all the way through the second photogate before the carts hit the end of the track.

4. Set the photogates to `GATE` mode and make sure the memory switch is set to ON.

5. Measure the mass of your carts. Make sure that the pegs are in the **bottom holes** on the carts. There should be a peg with a needle on Cart #1 and a counter balance peg on the opposite side. Cart #2 should have a peg filled with putty and a counter balance peg on the other side. Cart #1 should have a fin on top. Measure the mass of each of the carts including the pegs and fin but **without** the 50 g masses and record it in the spreadsheet in the cells for the "Mass of empty cart" 1 and 2.

6. Measure the length of the fin on Cart #1 and record this in Excel. Be sure to pick a reasonable uncertainty for the fin length as well.

7. Input the uncertainty for the times measured by the photogate into Excel. You will use an uncertainty of 0.0005 seconds.

8. Put all four 50 gram masses on Cart #2 such that they are evenly distributed (2 masses on each side).

9. Input the mass of Cart #1 in the column labeled m_1 and the total mass of both carts **including the mass disks** into the column labeled $m_3 = (m_1 + m_2)$.

10. Place Cart #2 in between the photogates and have one partner hold it stationary until just before the collision takes place. Place Cart #1 "outside" of the first photogate.

11. Making sure that your photogates are reset to 0, give a brief but firm shove to Cart #1 such that it collides and sticks together with Cart #2. Allow the two carts to pass through the second photogate completely before stopping them. **Do not allow the carts to bounce off the end of the track and pass back through the photogates again as this will mess up your times.**

12. Record the time for Cart #1 to pass through the first photogate (t_i) in Excel, then press the `READ` switch and record (t_{mem}) as well.

13. Calculate the time for the combined cart system to pass through the second photogate using the formula $t_f = t_{\text{mem}} - t$ and input that into Excel.

14. All other formulas have already been programmed into Excel for you. You must provide sample calculations (written out by hand on a separate piece of paper) for the cells shown in blue in the spreadsheet: δv_{1i}, P_{1i}, P_f, δP_{1i}, KE_{1i} and δKE_{1i}. Note that the initial velocity v_{1i} of Cart #1 is calculated using the formula L/t_i and that the final velocity of the combined cart system v_f is calculated using the formula L/t_f. The formulas needed for the uncertainties are given below.

15. You will do a total of 6 trials with the following distribution of the 50 gram masses on the carts:

    - Trials 1 & 2: 0 mass disks on Cart #1, 4 mass disks on Cart #2
    - Trials 3 & 4: 2 mass disks on Cart #1, 2 mass disks on Cart #2
    - Trials 5 & 6: 2 mass disks on Cart #1, 0 mass disks on Cart #2

Make sure to keep the masses distributed symmetrically on the carts and to record the correct masses for m_1 and m_3 (including the mass disks) into the spreadsheet.

16. The absolute value of the percent difference between the initial and final momentum should be **less than** 5% indicating that momentum was conserved. (The spreadsheet does this calculation for you in the column called "change" in the momentum summary table.) If your values are not <5%, rerun the trials. If you are having problems getting good values check the following things:

    - Make sure to compress the putty in the peg after every collision. If you hear a metallic clink when the carts collide you aren't getting an inelastic collision so push the clay back in.
    - Make sure you've entered the correct mass values into the spreadsheet in the "Velocity Data" table for columns m_1 and m_3. Notice that the second mass value is the total mass of the two carts together and is labeled as $m_3 = m_1 + m_2$. Make sure you include the mass of the disks on each cart correctly.
    - Make sure you're giving Cart #1 a firm (not too hard) shove but NOT still pushing the cart when it goes through the first photogate.
    - Make sure the fin on Cart #1 is completely through the first photogate before the collision occurs.
    - Make sure the carts don't go back through the second photogate.
    - Make sure the carts are balanced.
    - Make sure the fin is parallel with the cart (not tilted).

    Do not struggle to get values of <5% for too long. If there is fewer than 30 minutes left in lab then keep values which have a percent change of <10%.

17. Be sure to include hand calculations for the light blue boxes in Excel.

5.8 Uncertainties and Percent Change

In today's experiment we have already input all of the equations into Excel for you out of the interest of brevity, but it is important to understand the uncertainties for the values you used in this experiment.

The uncertainty for velocity is:

$$\delta v = v\left(\frac{\delta L}{L} + \frac{\delta t}{t}\right)$$

The uncertainty for momentum is:

$$\delta P = P\frac{\delta v}{v}$$

And the uncertainty for kinetic energy is:

$$\delta KE = 2KE\frac{\delta v}{v}$$

The uncertainties for the differences for the momenta and kinetic energies are then:

$$\delta P_{\text{diff}} = \delta P_f + \delta P_i \quad \text{and} \quad \delta KE_{\text{diff}} = \delta KE_f + \delta KE_i$$

Today you are asked to look at the change in the initial and final values of momentum and kinetic energy. It is often convenient to express how much a value changed in terms of a percent difference. This is often calculated by taking the difference between the final and initial values, dividing by the initial value and then multiplying by 100 to get a percentage. For example, the percent change in momentum given in the spreadsheet is calculated by:

$$\frac{P_f - P_{1i}}{P_{1i}} * 100 \tag{5.11}$$

Small values of percent change mean the measured final momentum was very close to the measured initial momentum, which is expected as momentum should be conserved. What about the percent changes for kinetic energy?

5.9 Checklist

1. Excel sheets (both data view and formula view)
2. Answer to questions.
3. Hand calculations for the 6 highlighted cells.
4. There are no graphs or plots this week.

5.10 Questions

1. For which of your trials was momentum conserved? The scientific way to address this question is to ask if the difference between the initial and final momenta, $P_{\text{diff}} \pm \delta P_{\text{diff}}$, is compatible with zero. Show a consistency check for at least one trial. If momentum was not conserved for any of your trials, describe possible sources of error.

2. Was kinetic energy conserved for any of your trials? Show work for at least one trial. Where did the kinetic energy go?

3. How much kinetic energy was lost? How did the amount of kinetic energy lost change as the distribution of the masses on the carts changed?

4. Compare one of your measured KE_f trials with the predicted value $KE_{f\ calc}$ for a perfectly inelastic collision. (Use your measured values for the masses and KE_i in Equation 5.10 to calculate $KE_{f\ calc}$.) Are they compatible? Was your collision a perfectly inelastic collision? The uncertainty on the predicted value of $KE_{f\ calc}$ is given by:

$$\left(\delta KE_{f\ calc} = KE_{f\ calc}\frac{\delta KE_i}{KE_i}\right)$$

5. Combine Equations 5.7, 5.8 and 5.9 to obtain the expression in Eq. 5.10. Show work. Hint: Solve Eq. 5.9 for v_f, then substitute this into Eq. 5.8.