PHY 451

Notes for Computerized Cavendish Balance

1. **Calibration**: Use the "optical-lever" method for establishing the relationship between voltage output (V_{out}) of the electronics and the angle of rotation θ of the boom containing the small masses. To obtain proper alignment between the laser and the balance, there are two Si mirrors mounted on the balance. One mirror is on the front glass cover of the balance and is used for aligning the laser beam ⊥ to this glass. The second mirror is attached to the boom and is accurately situated so its plane is parallel to the axis of the boom. Thus one can tell when the boom is close to parallel to the plane of the front glass plate. A third Si mirror is mounted on the meter stick underneath the laser so you can align the meter stick ⊥ to the laser beam.

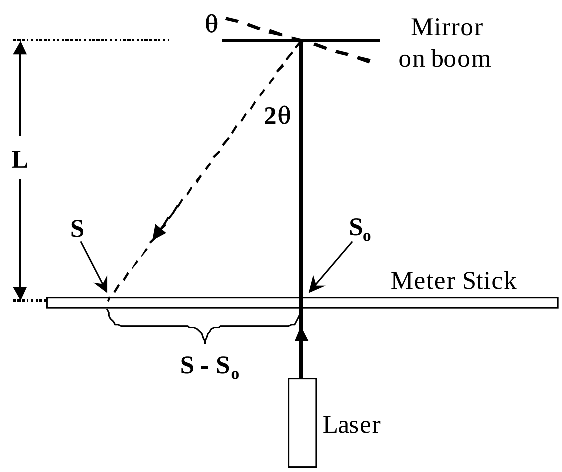


Figure 4 in the manufacturer's manual is somewhat confusing. See revised figure, above. S_o is the position of the laser beam on the meter stick after reflecting off the mirror on the glass cover of the balance. S is the position of the laser beam on the meter stick after reflecting off the mirror on the boom. Angle $\theta = f(S\text{-}S_o, L)$, where f is some trigonometric that you have to work out. A plot of V_{out} vs. θ should follow a straight line whose slope(and uncertainty) you want to calculate.

Procedure: After aligning the laser beam, so you know where S_o appears on the meter stick, measure L. Then rotate slightly the top contact of the torsion fiber via the brass lever arm on top of the balance housing until $S \approx S_o$. Because of the long period of oscillation for the balance, it will take some time to get S to be steady and $\approx S_o$. Once S is steady, make sure V_{out} is close to zero by adjusting the DC offset knob on the electronics box next to the apparatus.

Now you must set the torsional balance into oscillation in order to do the calibration. Before you start the oscillation, open the Labview program; and to avoid the digitization error, set the voltage range to ~ 0.1 V **before you tell the program to start recording data.** Try to obtain an oscillation amplitude about S_o of $|S_{max} - S_o| \approx 10$ cm on the meter stick. You induce the oscillation by rotating the brass lever arm slightly and then returning it to its original position. This will take some practice. If you induce too large an amplitude, just wait until it damps down. The Labview program provides a way of relating V_{out} to S. Over an oscillation from one extremum to the next, you watch S and push a button each time S crosses a cm mark (or every other cm mark) on the meter stick. Of course, you need to write down your beginning and ending value of S. V_{out} will be recorded by the computer each time you push the button.

2. **Determination of damping constant, resonance frequency**: Wait for the oscillations to damp out. Use geometry to estimate how far you need to move the brass lever to shift the quiescent value of S by ≤ ~5 cm on the meter stick. This will induce an oscillation amplitude of $|S_{max} - S_o| \leq$ ~10 cm. Once the oscillation has been established, use the Labview program to record the free damped harmonic oscillation (DHO) of the torsional balance. When the oscillations have damped out, return the brass lever arm to its original S_o position and record another DHO. You can fit the DHO with the equation provided in the manual to determine the frequency and damping constant. You need to write down the value of the damping constant b and the period of oscillation T.

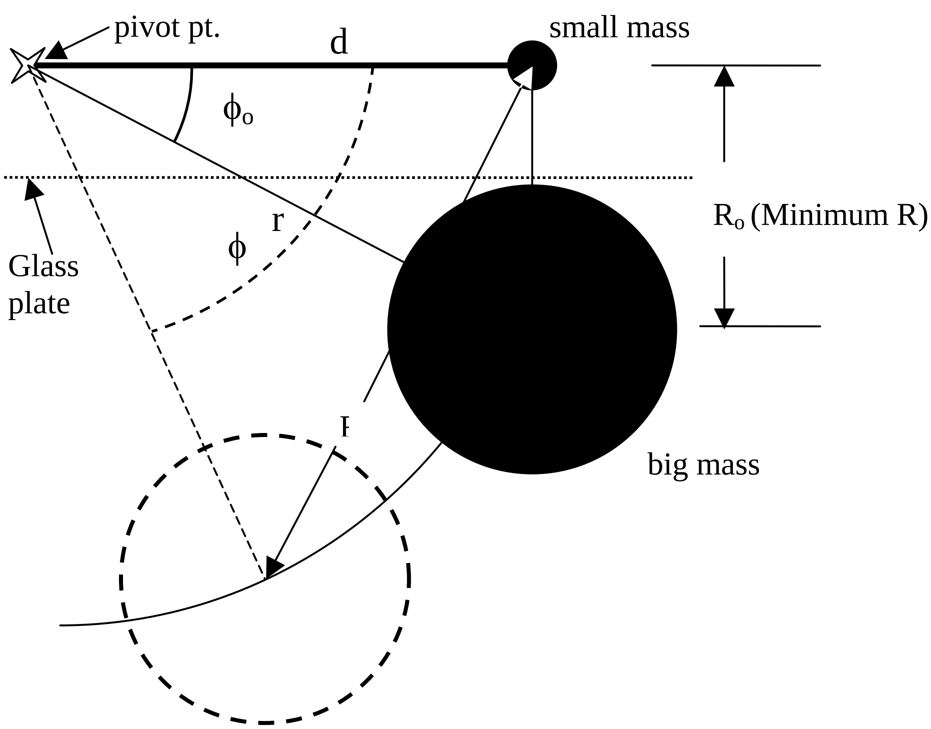
3. **A useful way to move the two large Pb balls**: To measure the gravitational attraction between the large and small balls, you move the large balls back and forth in phase with the angular motion of the torsional balance. An apparatus has been constructed for this purpose, and a "Timer" program is provided to you to help you keep track of the time. The balance and the large balls are now mechanically "decoupled." Thus when you move the large balls as close as you can to the glass plates on the front and back sides of the balance, no direct contact will occur between the balls and the glass plates, eliminating possible excitations of the torsional balance. The heavy granite table top now absorbs the shock of the large balls reaching their limits of travel. First you need to make sure that the large balls DO NOT make contact with the glass plates.
   Procedure: There are two ways to obtain data for determining G. (1) Just use the turning-point analysis outlined in the manual as the oscillations build up during your reversal of the large balls. (2) In the Appendix, we suggest a perhaps more accurate method where you keep reversing the large balls until the amplitude of boom oscillation becomes steady. Of course, you can use both methods and see which gives a better value of G. It is very important to maintain an accurate rhythm as you reverse the large balls. If you lose this rhythm, the extrema of the oscillations will exhibit an annoying "beat pattern."

4. **Calculating G**: Eq. (18) in the lab manual reads $G = \dfrac{K\theta_D R^2}{2Mmd}$, where K is the torsion constant of the tungsten fiber. Given that you have measured T (period of oscillation) and can calculate the total rotational inertia I of the boom (including the two small Pb balls), you can use Eq. (20) to eliminate K from Eq. (18). This gives

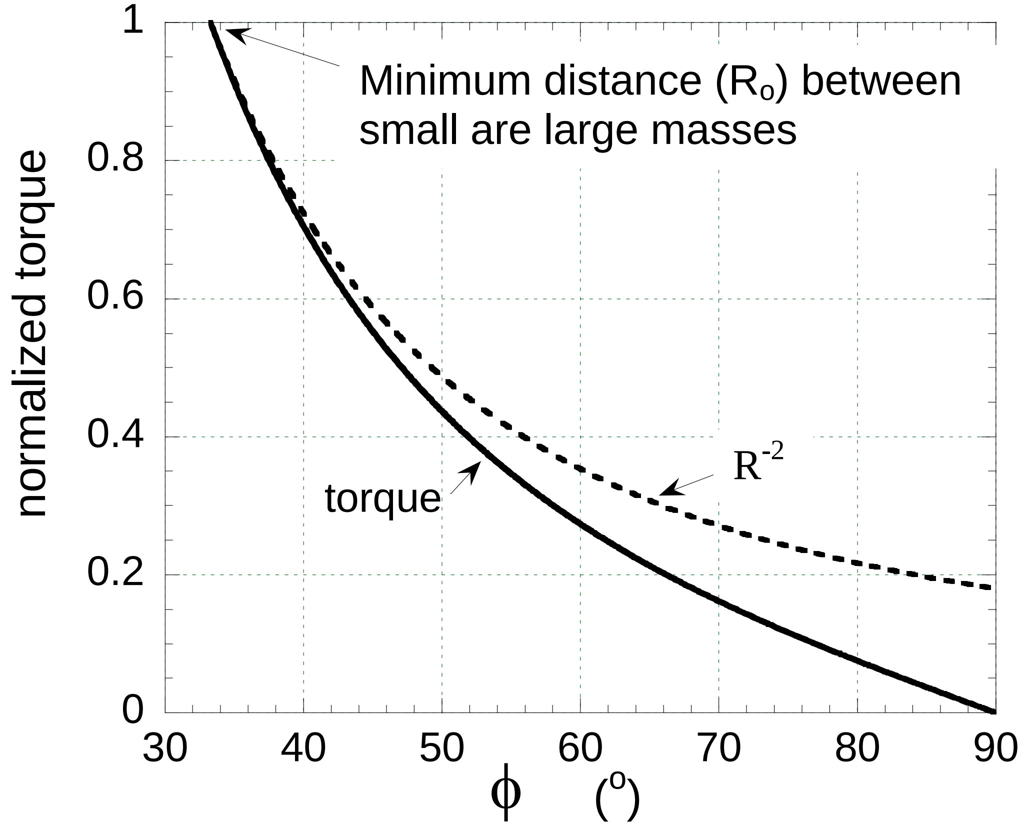
$$G = \frac{\left[\left(\dfrac{2\pi}{T}\right)^2 \theta_D\right] I R^2}{2Mmd} \text{ since } (2\pi/T)^2 >> b^2.$$

   Note that this equation has a slightly different notation from the one at the end of the Appendix.

5. **The "R^{-2}" gravitational force between two masses**: This improved apparatus, mentioned in Sect. 4, will allow you to measure this expected R^{-2} force, where R is the distance between the centers of the masses. What you will do is oscillate the big masses through smaller amplitudes so that R is larger, and you should be able to quantify the expected lower amplitude of torsional excitation of the balance. Note: ϕ is the angle of closest approach between the big and small masses during the oscillation. Angle ϕ_o ($\approx 33°$) is the smallest ϕ can be without the big mass hitting the balance housing. See drawing, below. r and d are the distances between the centers of the big and small masses and the pivot point, respectively, where $d = 6.665 \pm 0.004$ cm and $r = 8.11 \pm 0.01$ cm. To find ϕ_o you will need to measure the minimum distance (R_o) between the big and small masses.



The following graph shows a computation of the normalized total torque on the balance vs. ϕ and compares the torque with the normalized R^{-2} contribution to this torque. For $\phi < \sim 45°$, the R^{-2} contribution dominates the torque. Note, as expected, that the torque is zero when $\phi = 90°$, because both small masses are equidistant from the two big masses.



Procedure: Aluminum shims (in 5 ° and 2.5 ° increments) are provided so you can set the value of ϕ. A student has come up with following useful procedure: Choose the largest ϕ you want to have and put in the appropriate shims. When the boom is at rest, start oscillating the large balls until you build up a steady-state amplitude of oscillation θ_s. The value of θ_s is proportional to the torque. Without breaking rhythm, remove some shims (smaller ϕ) so you can oscillate the large balls over a bigger angle and keep oscillating until a new θ_s is established. Keep repeating this procedure until $\phi = \phi_o$. Of course, in order to calculate the torque curve on the above graph, you will have to work out $\tau \propto g(r, d, \phi)$ where g is some trigonometric function.

Appendix

Determining G from steady-state oscillations under driven resonance

This is a new way to determine G that may have higher accuracy. You reverse the big balls as instructed, but you keep going until you obtain several oscillations of constant amplitude. In this limit, the work done by gravity in each cycle is equal to the loss due to friction, parameterized by the damping constant b. From the decay of oscillations, you can accurately determine b and ω_1 that are needed for this analysis, where it will turn out that $b \ll \omega_1$. The only other quantity needed is θ_s, the steady-state amplitude of the driven oscillations.

For decay of oscillations, the differential equation is $\ddot{\theta} + 2b\dot{\theta} + \omega_0^2\theta = 0$, and its solution is

$$\theta(t) = \theta_0(e^{-bt})\cos(\omega_1 t + \varphi),$$

where $\omega_1^2 = \omega_0^2 - b^2$. Note for your experiment that $b \ll \omega_0$, giving $\omega_1 = \omega_0$.

For the driven case at frequency ω, the differential equation is

$$\ddot{\theta} + 2b\dot{\theta} + \omega_0^2\theta = \left(\frac{\tau}{I}\right)\cos(\omega t),$$

where τ is the torque and I is the rotational inertia. Here the steady-state solution is

$$\theta(t) = \theta_s \cos(\omega t + \varphi).$$

At resonance, $\omega = \omega_2$ where $\omega_2^2 = \omega_0^2 - 2b^2$ and $\varphi = \pi/2$, giving

$$\theta(t) = \theta_s \sin(\omega_2 t) \text{ and } \theta_s = \frac{\tau}{2b\omega_2 I}.$$

Note that since $b \ll \omega_0$, you have $\omega_2 = \omega_0 = \omega_1$.

However, we are applying a "square-wave" torque of amplitude τ_0 at frequency ω_2. The square wave has Fourier components that consist of odd harmonics of ω_2, but we just want the Fourier component at ω_2 which gives $\tau = (4/\pi)\tau_0$. Thus you have

$$\tau_0 = \left[\left(\frac{\pi}{2}\right)b\omega_2\theta_s\right]\cdot I,$$

and this looks very similar to the case of the build-up of oscillations where $\tau_0 = \left[\omega_1^2\theta_D\right]\cdot I$. Thus the quantities inside the "[ ]" should multiply to the same value for each method—something you can test.

Note also that $\tau_0 = \dfrac{2GMmd}{R^2}$.

Finally, for the "steady-state" case, one has $G = \dfrac{\left[\left(\frac{\pi}{2}\right)b\omega_2\theta_s\right]IR^2}{2Mmd}$, and

for the build-up of oscillations, $G = \dfrac{\left[\omega_1^2\theta_D\right]IR^2}{2Mmd}$.