

Geometrical Optics

Introduction

In geometrical optics, refraction is described by Snell's Law. Refraction refers to the bending of light as it passes from one medium to another. Snell's Law will be studied in this lab. It states that

$$n_i \sin \theta_i = n_r \sin \theta_r \quad (1)$$

where n is the index of refraction of the incident or refracted material, and θ is the angle of the incident or refracted light ray measured from the normal to the surface. When the incident index of refraction is greater than the refracted index of refraction, there is a critical angle beyond which refraction can no longer take place and the beam of light is totally internally reflected. This critical angle is given by

$$\theta_c = \sin^{-1} \left(\frac{n_r}{n_i} \right) \quad (2)$$

The index of refraction of a material will be measured using a semi-circular lens and equations 1 and 2.

When light passes through a rectangular piece of material of width, t , its lateral displacement, d , is given by

$$d = t \frac{\sin(\theta_i - \theta_r)}{\cos(\theta_r)} \quad (3)$$

Equation 3 presumes that the light emerging from the rectangular plate is parallel to the incident path. This is the "optical micrometer" case. If there is some angle of deviation from the incident path, it can be found using

$$\theta_{dev} = \frac{d_{far} - d_{near}}{L} \quad (4)$$

where L is the distance between the two different locations where the lateral displacement is measured.

In the first part we will use refraction measurements to find the index of refraction n , then compare it to the values obtained by subsequent refraction measurements and the critical angle method. We will then use Eq 3 to predict displacement vs angle of the optical micrometer, and measure the deviation angle for this configuration, which should ideally be 0.

Procedure

A 632.8 nm laser mounted on an optical track at lab table number 2 will be used for all experiments. The laser was aligned on the track and the beam was projected onto a plate perpendicular to the track at the other end of the track. A piece of graph paper was used on this plate to mark the positions of laser. Between the laser and the plate was a rotating protractor. The two types of lenses can be mounted on this protractor. The protractor was aligned so that the beam passes directly over its pivot point. The distance between the pivot point and the face of the plate was measured with a meter or 2-meter stick.

Initially the semi-circular lens was mounted so that the face of the lens faces the laser (we called it the "D" configuration, because the lens looked like a D when the laser was on our left); see page 22 of the lab book. The lens was positioned so that the beam reflects back towards the laser, and the mounting apparatus was rotated so that at this position, the protractor reads 0 degrees. The initial beam position was marked with pen or pencil on the piece of graph paper. We then calculated the angle at which the laser exited the lens by xxxxxx. Then the protractor was rotated clockwise and counter clockwise in 5 degree increments, and the beam positions are also marked on the graph paper. The displacement of these marks was measured with respect to the initial position using calipers. The intensity of the reflected and refracted beams were also noted qualitatively as the angle changes (page 23 of lab book).

The same procedure was followed where the lens was now oriented so that the round part of the lens was facing the laser (the "C" configuration, see page 24 of lab book). The critical angle of the lens was also measured while the beam was in this position. This was done by rotating the protractor until there was no more refraction at the flat surface, and the beam was totally internally reflected. This was done both clockwise and counterclockwise, and the angles in the two directions were very similar.

Next the rectangular bar-shaped lens was placed on the protractor. The laser was again oriented so that the reflected beam returns to the laser. Again, the displacement was measured on the piece of graph paper, but this time, the protractor was turned in 10 degree increments. This is done twice. Once while the plate was close to the protractor, and once while it was far away. The distance between these measurements, L , was measured, as well as the thickness of the block, D .

Questions start as a new page. Do not just slide them into the procedure.

Q1) It's important that the beam passes over the center of the lens because....

...

Q4) As stated below, our results are consistent with the expected values with no optical elements in place.

Measurements, Calculations, and Results

All data and calculations can be found on the attached spreadsheets.

For the measurement of n in the “D” configuration, we found we had to subtract angles xxx and yyy to get an angle corresponding to θ_r in formula 1); see sketch on p 23 of lab book for the definitions of the angles. We also translated between – and plus angles in the spreadsheet using $180 -$ the angle at the indicator for the xxx angle case.

In the “C” configuration, the angles are defined in the sketch below, and θ_i corresponds to xxx xxxxx. The critical angle used the same angle definitions, just setting the angle zzz to 90 degrees, and using eq 2.

The optical micrometer part was analyzed using the index of refraction from the “D” configuration, and eq 3, where the displacements x_{near} were calculated by subtracting off the position X_o when the lens was removed. We repeated this again in the near position, where we moved the table as seen on page 24 of the lab book.

1/6/2014		Geometrical Optics Summary Table									
		f	df	df/f	expect	dexpect	D	D(fract)	dD	t value	OK?
Index of refraction	units	no units	no units		no units	no units					
(1) Flat toward laser		1.568	0.06900	4.40%							
(2) Curved toward laser		1.543	0.017	1.10%	1.5680	0.00069	-0.03	-1.594%	0.0170	-1.47	Y
critical angle	units	radian	radian		radian	radian					
		1.606	0.016	1.01%	1.5680	0.00069	0.04	2.404%	0.0162	2.33	N
displaced/predicted	units	no units	no units		no units	no units					
		1.0791	0.0209	1.94%	1.00	0	0.08	7.9%	0.02	3.78	N
dev angle	units	radian	radian		radian	radian					
		0.003	0.0017	56.7%	0	0	0.00300		0.00170	1.76	Y

(list all major results discussed)

You could just put the table here in the page order but as a separate piece of paper.

The results are shown in the spreadsheet Summary Table on the previous page.

With the D-shaped lens mounted so that the flat part of the lens faces the laser, the index of refraction was found to be 1.568 ± 0.069 , a measurement with 4% uncertainty. With the curved part facing the laser, the index of refraction was found to be 1.543 with 1% uncertainty; it was slightly surprising that this measurement was more accurate. Comparing these two measurements gave a t value of -1.47, indicating the measurements were statistically compatible. Using the critical angle measurement, the index of refraction was found to be 1.606 ± 0.016 , which gave a t value of 2.33 compared to the initial measurement, and therefore incompatible. When using the bar lens with the plate located far away from the pivot point, the measured d divided by the calculated d using $n = 1.568$ had a mean of 1.0791 ± 0.0209 (sdm). Since a ratio of 1.0 was expected between results and the predictions, this was also not compatible with expectations. Finally, the angle of deviation of the beam was calculated for several incident angles. One expects 0 for the deviation angle, and we measured 3 milliradians; with the standard deviation of the mean of 1.7 milliradians, which was compatible with expectations.

Conclusion

The two measurements of the index of refraction using the D-lens were compatible. This was actually sort of surprising, since we could see the surface was a little wavy, and the values from the first measurement had significantly larger uncertainties than the second measurement. But the critical angle measurement of the index of refraction was not compatible with the first n measured. We had a lot of discussions about how to define the critical angle (see the crossed out page in the lab notebook), and it was not so easy to say just when it met our criterion. And we might have picked a criterion that gave a systematically biased version of the critical condition, biasing the index of refraction measurement. The equation predicting the optical micrometer displacement worked well. The deviation angle was pretty small, but its fractional error was large; in the end it was compatible with the expected value of zero. Overall, these three measurements appear to be compatible with what ray optics predicts, and indeed we saw the transmitted beam fading as we got closer to total internal reflection.

There were several possible sources of error in this lab. There were random errors associated with the measurements of angles, and lengths. There were also many possible systematic errors. The laser beam may not have been aligned on the track. The plate at the end of the track or the paper attached to it may not have been perpendicular to the laser. The laser may not have passed directly through the pivot point of the protractor device. Perhaps the most obvious source of error was the fact that the lenses were of low quality, and all of the laser images were fuzzy, making it quite difficult to locate the true position of the beam on the graph paper.

Extra material at the end of the report as follows:

Any computer plots number the pages so they are easy to refer to

Then calculation spreadsheets number the pages so they are easy to refer to

Finally, photocopied pages from your lab notebook

Uncertainty calculations example

(Here you describe how did you derive the uncertainties of the key results used to draw your conclusions, e.g. the column df in the summary table.)

For the measurements of index of refraction, we just repeated the measurements several times. We used the mean value, then calculated the standard deviation of the mean
by $\text{stdev} / \sqrt{N_{\text{measurements}}}$

For the test of the displacement equation, we calculated the ratio measured/predicted for each displacement trial and again reported mean and standard deviation of mean as the value and error.

For the deviation angle, we tried doing the same thing but all the trials gave the same value of deviation angle. So in the end we derived $d(\text{angle})$ from

$$\text{Dev} = (x_f - x_n) / L = \text{diff} / L$$

$d(\text{diff}) = d(x_f) (+) d(x_n) = \sqrt{2} dx = .14 \text{ mm}$ (or, instead of (+) use \oplus from symbol font)

then $d\text{Dev} / \text{Dev} = d\text{diff}/\text{diff} (+) dL/L = .14\text{mm} / .35\text{mm} (+) 2\text{mm} / 2\text{m} = .57$