Experiment 3

Analysis of a Freefalling Body

3.1   Objectives

- Verify how the distance of a freely-falling body varies with time.

- Investigate whether the velocity of a freely-falling body increases linearly with time.

- Calculate a value for g, the acceleration due to gravity.

3.2   Introduction

Everyday, you experience gravity. This happens because the Earth is so massive it pulls us down and keeps us on the ground. But what happens when we drop something? We notice that as this thing falls to the Earth, it moves faster and faster until it hits the ground. From this we can tell that gravity is **accelerating** the object the entire time the object is in free fall. Today, we will measure how much gravity actually accelerates an object by using the Behr Free Fall apparatus and your mathematical skills.

3.3   Key Concepts

As always, you can find a summary on-line at Hyperphysics.[1] Look for keywords: gravity, velocity, and acceleration

---

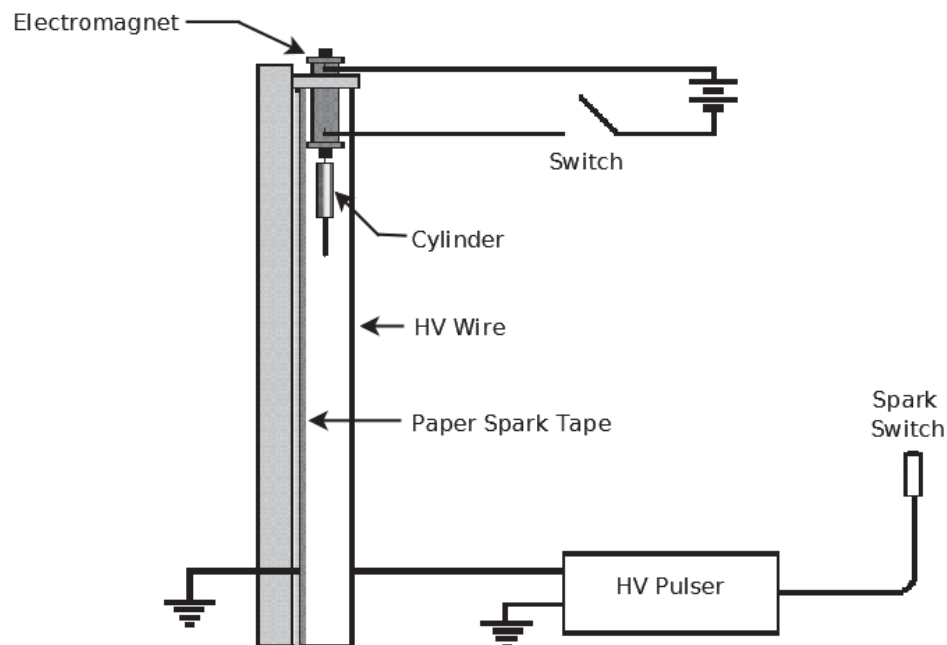[1]http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

Figure 3.1: Schematic of the Behr Free Fall apparatus.

3.4   Apparatus

A Behr Free Fall Apparatus and Spark Timing System will be used in this experiment. A schematic representation of the apparatus is shown above in Fig. 3.1 and a photograph of the apparatus is shown in Fig. 3.2.
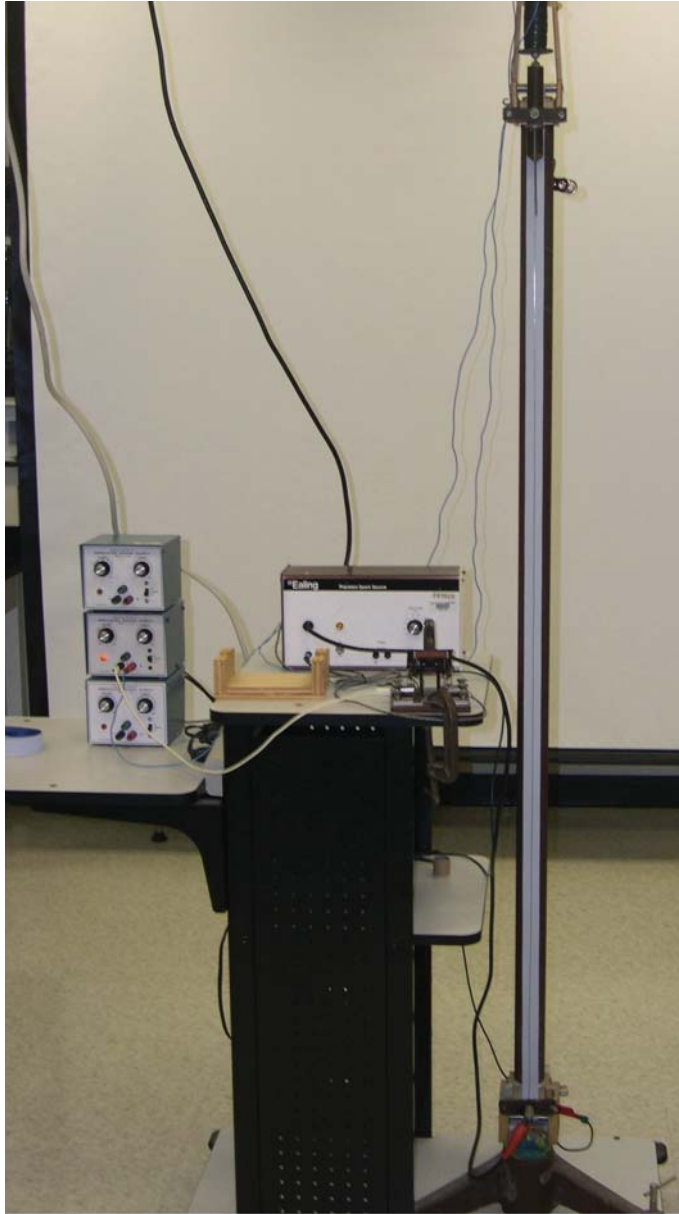
Last updated August 20, 2015

Figure 3.2: Photograph of the Behr Free Fall apparatus.

3.5   Theory

In this experiment a cylinder is dropped and a record of its free fall is made. Before the measurement, the cylinder is suspended at the top of the stand with the help of an electromagnet. When the electromagnet is turned off, the cylinder is released and starts to fall. Simultaneously, the spark timer starts to send high-voltage pulses through two wires which are stretched along the cylinder's path. At the time of each pulse a spark goes through the wires and the cylinder, leaving a mark on the special paper tape that lies between the cylinder and one of the wires. The **time interval** between two adjacent sparks is constant and is denoted by the Greek letter tau "τ". For this experiment, $\tau = 1/60$ of a second. Measuring the distances between any two marks, Δy, and knowing the time interval between the corresponding sparks, Δt, it is possible to calculate the **average** velocity during this interval using the formula

$$v = \frac{\Delta y}{\Delta t} \tag{3.1}$$

If Δt is small enough, we can assume that the velocity at any instant within this interval is approximately equal to this average velocity. In the case where acceleration is constant, the **instantaneous velocity** at the middle of the time interval Δt is exactly equal to the average velocity of the object during the time interval Δt.

In general, for the motion of a body with a <u>constant</u> acceleration a, the velocity v is given by the equation

$$v = v_0 + at \tag{3.2}$$

where v_0 is the velocity of the cylinder at $t = 0$. Since in our case the body is falling freely,

$$a = -g \tag{3.3}$$

where g is the magnitude of the acceleration due to gravity. The negative sign in front of g is to indicate that the direction of the acceleration is in the negative direction (i.e. downward). Therefore it follows from Eq. 3.2 that for a freely-falling body

$$v = v_0 - gt \tag{3.4}$$

Last updated August 20, 2015

By comparing Equation 3.4 to the equation for a line you can determine what quantities to plot on the x and y axes to obtain a graph whose slope corresponds to the acceleration due to gravity, g. (If you need a reminder of how to match up the equations refer back to the "Introduction to Computer Tools and Uncertainties" lab and look at Step 1 in the section called "Plotting a best-fit line".) The value of g obtained from the slope of your graph can then be compared with the known value $g = 9.806 \pm 0.002$ m/s^2. If you want to know why this value isn't just 9.81 m/s^2 please see the footnote.[2]

In addition to the velocity, you can also determine the position of the cylinder, y, as a function of time using the standard equation for an object that is undergoing constant acceleration. For a freely falling body, where at time $t = 0$ the object has height y_0 and velocity in the vertical direction v_0, then this equation looks like:

$$y = y_0 + v_0 t - \frac{1}{2} g t^2 \tag{3.5}$$

3.6 In today's lab

At the start of today's lab, your instructor will demonstrate the operation of the Behr Free Fall apparatus. In the interest of saving time, you will be supplied with a pre-made shock tape, instead of making one yourself. On this tape, you will measure the distance between the "dots" and using the given time interval, τ, calculate the velocity of cylinder. You will make 2 plots, obtain the value for g and answer the questions at the end of the lab.

3.7 Equipment

- Behr Free Fall Apparatus

- Shock tape

- Meter stick

---

[2]The value of g depends on your latitude on Earth. For example, $g = 9.832$ m/s^2 at the poles, $g = 9.806$ m/s^2 at 45 degrees and $g = 9.780$ m/s^2 at the equator. The value that is usually quoted in textbooks, $g = 9.81$ m/s^2, is obtained by taking the average over all latitudes, $g = 9.80665$ m/s^2, and then rounding up. East Lansing is located at 42 degrees latitude so g should be close to $g = 9.806$ m/s^2. For this lab when comparing your value for g to the known value use $g = 9.806 \pm 0.002$ m/s^2.

3.8   Procedure

1. Secure both ends of the shock tape to the desk using masking tape, making sure that the shock tape is as flat as possible.

2. The "bottom" of the tape is defined as the end of the tape with the largest, bold black dot which is where the cylinder stopped and multiple sparks hit the tape. Also note that the dots should be getting farther apart as you approach the bottom of the tape. Starting from the **third** dot from the bottom, label each successive point from #25 to #1 in **descending** order. It is ok if you have unused marks left over at the top of the tape. Point #25 (at the bottom) is defined to be where $y = 0$ and point #1 (at the top) is defined to be when $t = 0$.

3. Using your meter stick, measure each point's distance (in cm) from $y = 0$ (point #25) and write it down on the shock tape.

4. Input your measured distances into the spreadsheet. Be careful and make sure you are putting the correct distance for each numbered point into the correct cell. Also input a reasonable uncertainty on your distance measurements, δy, in the spreadsheet. When choosing the size of your uncertainty consider things like how well can you measure the distance between the 2 dots which each have a width to them. For example, do you measure to the middle of the dot, the bottom edge of the dot, etc. Also realize that there is an uncertainty on each end of the measurement as you are measuring from one dot to another dot.

5. Input the correct times for each point using the given value of the time interval, τ, which is $1/60$ sec between dots.

6. Calculate the instantaneous velocity v_i for each point y_i. As discussed in the theory section, for constant acceleration the average velocity over the time interval Δt is equal to the instantaneous velocity at the middle of the time interval. So the instantaneous velocity v_i at position y_i is equivalent to calculating the average velocity during the time interval of t_{i-1} to t_{i+1} for each of your points using $v = \frac{\Delta y}{\Delta t}$. In this equation Δy for each point i is the difference between the position of the mark FOLLOWING and the position of the mark PRECEDING the point for which you are trying to calculate a velocity, which is written as $\Delta y = y_{i+1} - y_{i-1}$. Likewise, Δt is the time interval between

the FOLLOWING mark and the PRECEDING mark, which is written as $\Delta t = t_{i+1} - t_{i-1}$. For example, the instantaneous velocity at point #2 is:

$$v_2 = \frac{y_3 - y_1}{t_3 - t_1}.$$ (3.6)

In this lab the time interval between each point is always the same, τ, so all Δt values will also be the same. Notice that Δt is defined to be across two time intervals (e.g. in Eq. 3.6 it is from point #1 to point #3) so $\Delta t = 2\tau$.

7. Now calculate the uncertainty in your velocity at each point using the equation $\delta v_i = \frac{\delta y}{\tau}$. For this lab it is assumed that there is no uncertainty in the value of the time interval. See Section 3.9 for a more complete description of the error calculation.

8. Transfer your data columns for "Time", "y", and "v" into Kaleida-Graph.

9. Graph I - Make a graph of y vs. t. You do NOT need a best fit line or error bars for this plot.

10. Graph II - Make a graph of v vs. t. Be sure to include a best-fit line and error bars on v. (If you don't remember how to get the best fit line or include error bars on your plot refer back to the first lab, Introduction to Computer Tools and Uncertainties.)

3.9 Error Calculation

When measuring the position, y, between points you will assign an uncertainty based on how accurately you can measure that distance. This error is called δy and will be used to determine all the other uncertainties in this lab. In order to determine the uncertainty in the velocity, $\delta(v_y)$, you need to know the uncertainty in Δy and Δt. For this lab it is assumed that there is no uncertainty in the time interval so $\delta(\Delta t) = 0$. The error in Δy, at each point i is the same and is given by:

$$\delta(\Delta y) = 2\delta y$$ (3.7)

The error in the velocity at each point i is then given by:

$$\delta(v_y) = v_y \frac{\delta(\Delta y)}{\Delta y} = v_y \frac{2\delta y}{\Delta y} = \frac{(\Delta y)2\delta y}{2\tau(\Delta y)} = \frac{\delta y}{\tau} \tag{3.8}$$

3.10 Checklist

1. Your Excel spreadsheet (both lab partners turn in a copy).

2. Formula view of the spreadsheet (both lab partners turn in a copy).

3. Graph I — Plot of the height vs. time with hand-written observation at the bottom.

4. Graph II - Plot of velocity vs. time with hand-written observation at the bottom.

5. Answers to questions.

6. One member of each group should turn in your spark tape record of the free-fall. Make sure to put your name and your lab partner's name on the tape.

3.11 Questions

1. From Graph II - What is the value of the y-intercept (determined from the best-fit line)? What does it mean (i.e. what physical quantity does it correspond to)?

2. From Graph II - What is your value of g in cm/s^2? Is it compatible with the accepted value of $g = 980.6 \pm 0.2$ cm/s^2? If not, give possible reasons why your result is inconsistent. (Never say it is because of "human error" or a "mistake", instead give a concrete example of what may have gone wrong.)

3. Using Equation 3.4 and your results from the best-fit line on Graph II, calculate the time at which $v = 0$ cm/s. Show your work and explain the result.

4. If you wanted Graph I to have a linear relationship, what would you plot instead of position vs. time: y^2 vs. t, y^2 vs. t^2, or y vs. t^2? Assume the initial position (y_0) and initial velocity (v_0) of the cylinder are exactly zero. Justify your response.

	A	B	C	D	E	F	G	H	I	J	K
1			**Free Fall with Behr Apparatus Part I**								
2											
3								*value*	*units*		
4	Time Interval between sparks τ:							0.0167	[seconds]		
5											
6	$\Delta y_i = y_{i+1} - y_{i-1}$										
7	Error in -y- : $\delta(y)$							xxx	[cm]		
8	Error in -Δy- : $\delta(\Delta y) = 2\,\delta(y)$							xxx	[cm]		
9	Error in Vy: $\delta(Vy) = \delta y/\tau$							xxx	[cm/sec]		
10											
11											
12								Recall:			
13	Dot	Time	y_i	Δy_i	Vy_i	δVy		Blue = measured values			
14	i	[sec]	[cm]	[cm]	[cm/sec]	[cm/sec]		Yellow = calculated values			
15	1										
16	2										
17	3										
18	4										
19	5										
20	6										
21	7										
22	8										
23	9										
24	10										
25	11										
26	12										
27	13										
28	14										
29	15										
30	16										
31	17										
32	18										
33	19										
34	20										
35	21										
36	22										
37	23										
38	24										
39	25										
40											