Experiment 9

Vibration Modes of a String: Standing Waves

9.1 Objectives

- Observe resonant vibration modes on a string, i.e. the conditions for the creation of standing wave patterns.
- Determine how resonant frequencies are related to the number of nodes, tension of the string, length of the string, and density of the string.
- Determine the velocity, c, of transverse waves in the string.

9.2 Introduction

Everything you can see is due to waves. A wave is defined as an oscillation through space. The reason we can see things is because photons oscillate through space in the form of a wave and enter our eyes. This sends a signal to our brain and thus we can see. Today we will investigate waves on a much larger scale. When you apply an oscillation to the end of a string under tension, it begins to form waves with given frequencies. The **resonant frequency** is the frequency at which the wave oscillates freely without constructive interference adding to its amplitude.

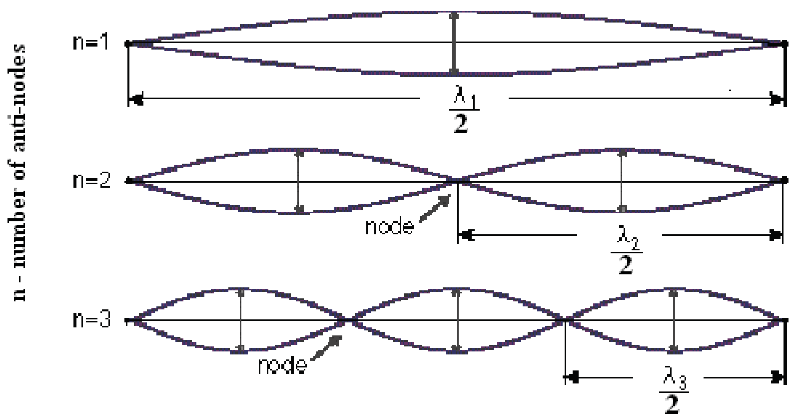

Figure 9.1: Three lowest characteristic frequencies of a string (with $n = 1, 2$ and 3 maxima).

9.3 Key Concepts

You can find a summary on-line at Hyperphysics.[1] Look for keywords: standing waves on a string, resonance, transverse waves

9.4 Theory

A wave in a string can be characterized by its **wavelength**, λ, just like a sound wave or a light wave. For a string that is fixed on both ends, a **standing wave** can develop *if an integer number (n) of half wavelengths* ($\frac{\lambda_n}{2}$) *fit into the length (L) of the string:*

$$n\left(\frac{\lambda_n}{2}\right) = L \tag{9.1}$$

Here n **refers to the number of maxima (also called antinodes)** in the wave pattern as demonstrated in Fig. 9.1. Rearranging Eq. 9.1 one can calculate the wavelength λ for a given number of antinodes n:

$$\lambda_n = \frac{2L}{n} \tag{9.2}$$

[1] http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

The **resonant frequency**[2]**, f_n,** depends on the speed of the wave, c, and its wavelength, λ_n, as:

$$f_n = \frac{c}{\lambda_n} \tag{9.3}$$

Substituting Eq. 9.2 into Eq. 9.3 gives:

$$f_n = \left(\frac{c}{2L}\right) n \tag{9.4}$$

Compare Eq. 9.4 to the equation of a line:

$$y = mx + b$$

By making a plot of f_n vs. n, the slope (m) of the line is related to the speed of the wave c. **Make sure you determine the formula that relates the slope m to c before coming to lab.** (Hint: When plotting f_n vs. n, f_n is the y variable, n is the x variable and the y-intercept is zero, i.e. $b = 0$.)

If a force acts on a string with a resonant frequency, the amplitude of the vibration will grow very large. This is a common behavior in many physical systems. An example of such behavior is pushing a child on a swing. A swing oscillates with a **characteristic frequency**. If someone exerts a push on the child with that frequency, after several cycles the amplitude of the swing becomes large, even if the pushes are gentle. If pushes are given with a different frequency, some of the pushes will be out of phase; meaning that the child will be pushed against his motion and the amplitude will not have a chance to grow. A string has **many** characteristic frequencies and the string's amplitude will grow whenever the driving force has **any** of these characteristic frequencies. If a string is set to vibrate at one of these characteristic frequencies a standing wave is set up on the string. When a standing wave is present, nodes and antinodes will be visible on the string. A node is a location on the string where the string does not move. On the other hand, an antinode is a location that undergoes a vibration with very large amplitude. Figure 9.1 shows the lowest three characteristic frequencies for a given string under constant tension.

[2] The pitch of musical instruments is determined by the resonant frequency, whether it is a string instrument, a wind instrument or a percussion instrument. Since instruments are not driven at a fixed frequency, the vibrations are composed of a mixture of several harmonic frequencies.

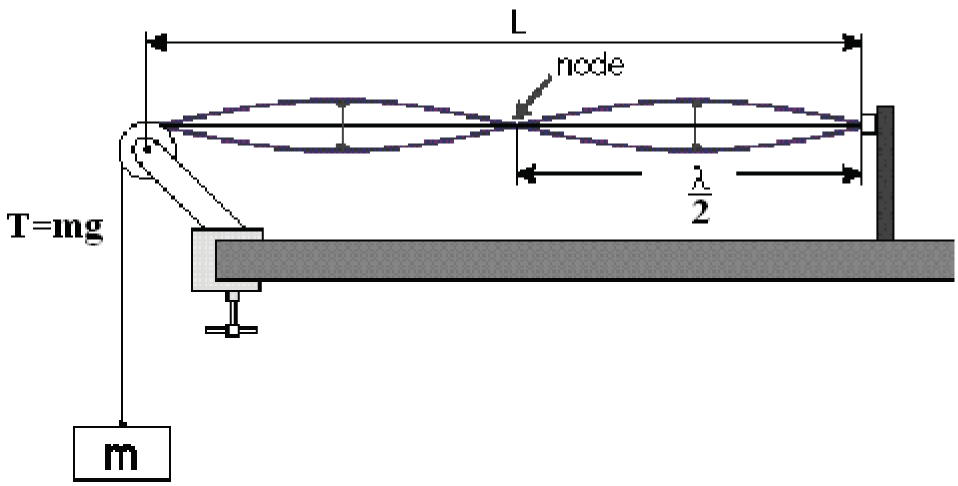

Figure 9.2: Diagram of how the string looks when driven at the second lowest resonant frequency. This configuration has two anti-nodes (points of maximum oscillation).

The speed, c, of a transverse wave in a string depends on the string's density[3], ρ, and the tension, T. Using a setup similar to Fig. 9.2 the string is put under tension by running it over a pulley and hanging a mass on the end. The tension, T, is then given by

$$T = mg \tag{9.5}$$

Knowing the amount of tension T and the string's density ρ, the speed c of the transverse wave can be found from:

$$c = \sqrt{\frac{T}{\rho}} \tag{9.6}$$

[3]Mass density (or mass per unit length) is frequently represented by the Greek letter rho (ρ).

9.5 In today's lab

Today you will investigate the speed c of a wave traveling through a string under tension using 3 methods. First you will measure the tension and density of the string in order to calculate c using Eq. 9.6. Then you will find the resonant frequencies of the string for $n = 1$–11 antinodes. You will use this information to calculate c in two additional ways: First by calculating the wavelength λ_n using Eq. 9.2 and then c from Eq. 9.3 for each resonant frequency and averaging them. Second by plotting f_n vs. n and using the slope of the best fit line with Eq. 9.4. You will check the consistency of your 3 values for c with one another.

9.6 Equipment

- Variable frequency oscillator
- Pulley and weight system
- String
- Meter stick
- Mass scale

9.7 Procedure

1. Measure the total length, l_0, of the string when it is not under tension and record it in your data sheet (in the table at the bottom). Choose a reasonable uncertainty on the length, δl_0, of the unstretched string and record it. Take into account both the uncertainty in reading the meter stick and the uncertainty associated with aligning the unstretched string with the meter stick (the string needs to be in a straight line and at the same time not stretched).
2. Measure the mass m of your hanger with weights and record it in your data sheet. This should have a value around 250 grams.
3. Attach the mass system to the end of your string and hang it over the pulley. Measure the length L of the string from the pulley to the

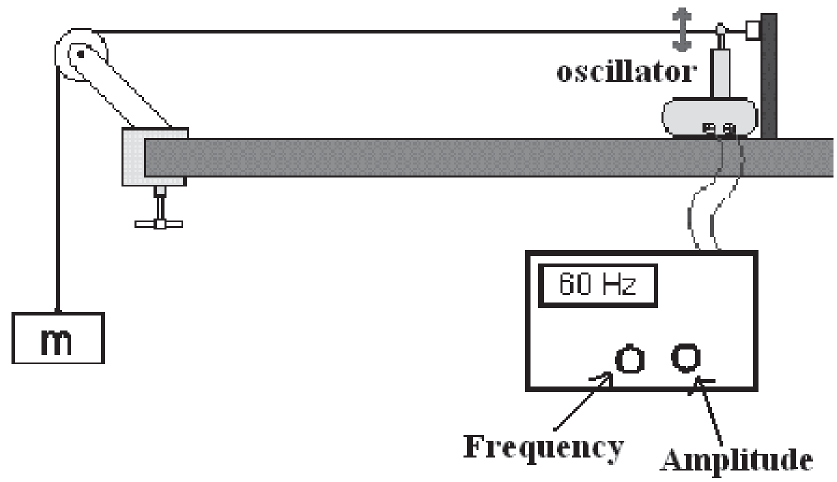

Figure 9.3: Diagram showing the experimental apparatus.

holder (see Fig. 9.2) and record it in your data sheet. This is not the length of the entire string, just the portion that will have standing waves on it. L should be approximately 150 cm. Choose a reasonable uncertainty, δL, for how well you were able to measure L. Take into account how accurately you can read the meter stick when holding it next to the string and where you think the node will be when the string touches the pulley.

4. Calculate the tension, T, in the string. Use $g = 980$ cm/sec^2.

5. Measure the **total** length of the **stretched** string, l_s, with your mass system hanging on the end and record it in your data sheet. In this case, the string changes directions as it passes over the pulley as shown in Fig. 9.3. Choose an uncertainty for δl_s that includes both the uncertainty associated with reading the meter stick and the uncertainty of aligning the meter stick with the string.

6. Calculate the density of the stretched string, ρ_s, using the density of the unstretched string, ρ_0, and your measured lengths for the stretched, l_s, and unstretched, l_0, string. The mass of the string is the same whether it is stretched or not ($m_s = m_0$). As mass is equal to density

times length ($m = \rho\ l$) the density of the stretched string, ρ_s, can be found using $\rho_s l_s = \rho_0 l_0$. The density of the unstretched string, ρ_0, you are using is $\rho_0 = 0.0375$ g/cm. Have Excel calculate the stretched string density and its uncertainty. The uncertainty $\delta\rho_s$ in the stretched string density is given by:

$$\delta\rho_s = \rho_s \left(\frac{\delta l_s}{l_s} + \frac{\delta l_0}{l_0} \right) \tag{9.7}$$

7. Use Eq. 9.6 to find the wave speed c and record it in your data sheet. Use the density of the stretched string, ρ_s, for this calculation. The uncertainty in the speed, δc, is given by:

$$\delta c = c \left(\frac{1}{2} \frac{\delta\rho_s}{\rho_s} \right) \tag{9.8}$$

   Because the uncertainties in the string's mass and its tension are both small compared to the uncertainty in ρ_s, it is not included in Eq. 9.8. Put these values in the bottom most Excel table as your first measurement of c.

8. Position the oscillator near the fixed end of the string and adjust the oscillator to "`Hz 1-100`" as shown in Fig. 9.3.

9. Starting around 100 Hz, find the first frequency mode where you have 11 antinodes in the string. Be sure to finely adjust the frequency until you get the largest possible amplitudes (peaks). Record this frequency in your data sheet.

10. Gradually decrease the frequency until you find the next lowest integer of antinodes ($n = 10$ in this case) and record the frequency in your data sheet.

11. Repeat the previous two steps for $n = 9$ to $n = 1$ antinodes.

12. Please don't leave the weights hanging on the string after completing your measurements.

13. Calculate the wavelength, λ_n, and speed of the wave, $c_n = \lambda_n f_n$, for each resonant frequency in Excel.

14. Have Excel calculate the mean (average) of your c values. You may use the formula "`=AVERAGE(E23:E33)`" for this calculation.

15. Calculate the standard deviation, s, and standard deviation of the mean, s_m, for your resonant frequencies. For the standard deviation s, use the formula "`=STDEV(E23:E33)`". For the uncertainty, δc, use the standard deviation of the mean, $s_m = s/\sqrt{N}$, where N is number of trials (measurements). This is your second measurement of c.

16. Make a plot of f_n vs. n in KaleidaGraph and include a best fit line. You do not need error bars on this plot.

17. Before printing your Excel spreadsheet answer questions #1 and 2 and fill in the third value of c.

18. Finish answering the questions #3 - 5.

9.8 Checklist

1. Excel spreadsheets (both data and formula views)
2. Plot of f_n vs. n
3. Answers to questions

9.9 Questions

1. What is the slope of your graph and its uncertainty?

2. From the slope and its uncertainty, calculate the speed of the wave and its uncertainty. Show your work. **Hint:** $\delta c = c\left(\frac{\delta\text{slope}}{\text{slope}} + \frac{\delta L}{L}\right)$.

3. Is the speed of the wave measured from your graph consistent with the mean value of your eleven $c = \lambda_n f_n$ calculations? Show your work.

4. Is the speed of the wave measured from your graph consistent with the value you obtained using the string's tension and density (Eq. 9.6)? Show your work.

5. Take 2 pens (or pencils) and use them to pinch the string first at a node and then at an antinode. Describe what happened to the standing wave when the string was pinched at those two places.

	B	C	D	E	F	G	H
1		***Vibration Modes of a String***					
2							
3		Formulae & Constants					
4		$f_n = c/\lambda_n = cn/2L$					
5		$c = \sqrt{(T/\rho_s)}$					
6		$T = mg$					
7							
8							
9		g =		[cm/s^2]	δ(L)		
10	Length	L =		[cm]			
11	Mass	m=		[grams]			
12	Tension	T=		[g cm/s^2]			
13							
14	The gray fields contain measured or import quantities.						
15	The yellow fields contain calculated quantities						
16							
17							
18	**Resonant frequencies of the string**						
19							
20							
21	n	f_n	λ_n	speed (c)			
22		[Hz]	[cm]	[cm /s]			
23						Mean c	
24							[cm /s]
25							
26						S_c	
27							[cm /s]
28							
29						S_m_c	
30							[cm /s]
31							
32							
33							
34							
35					error		
36	density of unstretched string:			ρ_0 =		NA	[g/cm]
37	length of unstretched string:			l_0 =			[cm]
38	length of stretched string:			l_s =			[cm]
39	density stretched $= \rho_0(l_0/l_s)$:			ρ_s=			[g/cm]
40	wave speed calculated from formula 4:						[cm/s]
41	wave speed average (for n=1 to n=11):				0	0	[cm/s]
42	wave speed measured (from slope):						[cm/s]
43							