PHY411 Homework Set 14

1. [10 pts] Kittel-Kroemer, problem 14-1.

2. [5 pts] In class we discussed particle diffusion and found that the particle flux density is related to the concentration gradient according to the transport equation

$$\boldsymbol{J}_n = -D\,\boldsymbol{\nabla} n\,.$$

   We assumed then that the system was in a steady state, but the relation turns out to be also valid when the concentration and flux density are functions of time

   (a) When particles are conserved, then a change in particle concentration in a region must result from particles crossing the boundary of the region. Show that

$$\boldsymbol{\nabla}\cdot\boldsymbol{J}_n + \frac{\partial n}{\partial t} = 0\,.$$

   The Gauss' theorem may be useful here. The derived equation is called the continuity equation.

   (b) By combining the continuity equation with the transport equation above, obtain a partial differential equation for n not containing $\boldsymbol{J}$. For simplicity assume D independent of position. The resulting equation is called the diffusion equation.

3. [5 pts] Kittel-Kroemer, problem 14-3. The relation can be used for assessing charge of the carriers.

4. [10 pts] Kittel-Kroemer, problem 14-4. For (a) consult Ch. 7 of the textbook. For (b) and (c) use, respectively, results (31) and (94) from Ch. 14. Assume that electrons active in transport move at Fermi velocity.

5. [5 pts] Kittel-Kroemer, problem 14-6. Consider the situation of a stationary flow dominated by dissipation. For a cylinder of liquid of radius r and length L, the momentum delivered per unit time due to the pressure difference, on the two sides of the cylinder, is completely balanced by the loss of momentum at the boundary of the cylinder, to the external liquid. Use this to obtain a differential equation for $v(r)$. Solve that equation and find the flowing volume from $\dot{V} = \int_0^a \mathrm{d}r\, v(r)\, 2\pi r$.