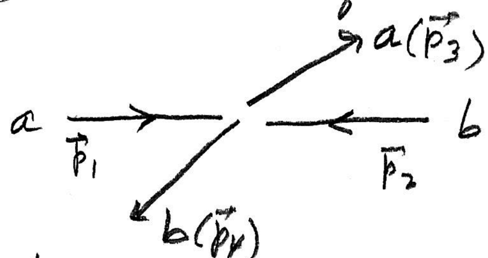
Homework Assignment 2 (12/12)

Problem 4

$$T\,\psi_x\bar{\psi}_y - N\,\psi_x\bar{\psi}_y = iS_F(x-y)$$ 2 points

Problem 5

(A) In the center of mass frame:



$$\vec{p}_1+\vec{p}_2=0 \Rightarrow |\vec{p}_2|=|\vec{p}_1|$$
$$\vec{p}_3+\vec{p}_4=0 \Rightarrow |\vec{p}_3|=|\vec{p}_4|$$

also

$$E_1+E_2 = E_3+E_4$$

$$\sqrt{p_1^2+m_a^2}+\sqrt{p_2^2+m_b^2} = \sqrt{p_3^2+m_a^2}+\sqrt{p_4^2+m_b^2}$$

$$\sqrt{p_1^2+m_a^2}+\sqrt{p_1^2+m_b^2} = \sqrt{p_3^2+m_a^2}+\sqrt{p_3^2+m_b^2}$$

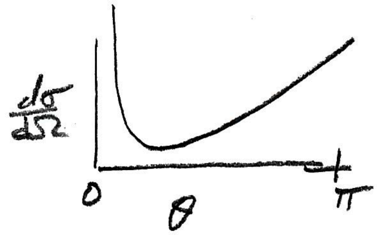
which implies $|\vec{p}_1|=|\vec{p}_3|$. 2 points

(B) $$\left(\frac{d\sigma}{d\Omega_3}\right)_{COM} = \frac{1}{64\pi^2(E_1+E_2)^2}\frac{|\vec{p}_3|}{|\vec{p}_1|}\left(\prod_\ell (2m_\ell)\right)|\mathcal{M}|^2 \quad (8.18)$$

In the center of mass frame, $s=(E_1+E_2)^2$ and $|\vec{p}_3|=|\vec{p}_1|$. Thus 2 points

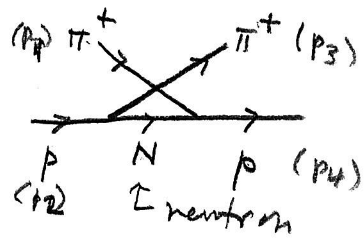
$$\left(\frac{d\sigma}{d\Omega_3}\right)_{COM} = \frac{1}{64\pi^2 s}\left(\prod_\ell (2m_\ell)\right)|\mathcal{M}|^2$$

Problem 6



for $\pi^+ p$ scattering

(A) The Feynman diagram is



The neutron propagator $\propto \dfrac{1}{(p_2-p_3)^2 - M_n^2} = \dfrac{1}{u - M_n^2}$

so the cross section $\propto \dfrac{1}{(u-M^2)^2}$. (1)

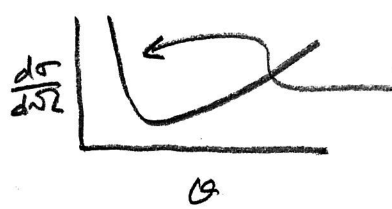
(B) $u - M^2 = (p_2 - p_3)^2 - M_n^2 = M_p^2 + m_\pi^2 - 2p_2 \cdot p_3 - M_n^2$

$$\approx -2\, p_2 \cdot p_3 = -2\, E_2 E_3 + 2\,\vec{p}_2 \cdot \vec{p}_3$$

$= -2M\sqrt{p_3^2 + m_\pi^2}$ in the lab frame of reference.

This quantity decreases as $\theta \to \pi$ so the cross section increases as $\theta \to \pi$. (1)

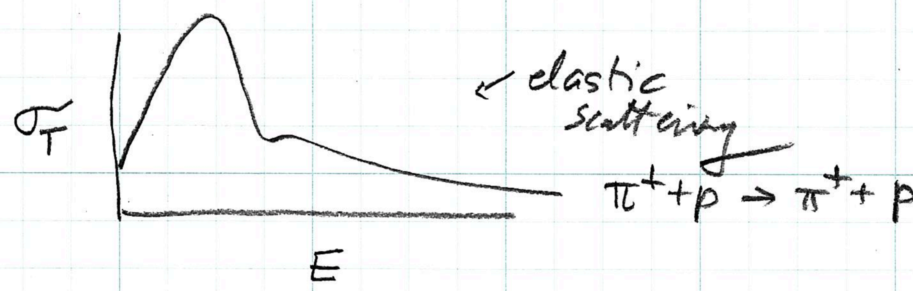
(C)



This part is Coulomb scattering which increases as $\theta \to 0$. (1)

3 points

Problem 7



The large peak in σ is the Δ^{++} resonance

1 point

Problem 8 Mandl & Shaw problem 5.1

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad \text{where} \quad F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$$

$$\mathcal{L}' = \mathcal{L} - \frac{1}{2}(\partial_\mu A^\mu)(\partial_\nu A^\nu)$$

$$= -\frac{1}{2}(\partial_\nu A_\mu)(\partial^\nu A^\mu) + \frac{1}{2}(\partial_\nu A_\mu)(\partial^\mu A^\nu) - \frac{1}{2}(\partial_\mu A^\mu)(\partial_\nu A^\nu)$$

$$= -\frac{1}{2}(\partial_\nu A_\mu)(\partial^\nu A^\mu) + \frac{1}{2}\partial_\nu\left[A_\mu(\partial^\mu A^\nu)\right] \underbrace{- \frac{1}{2} A_\mu(\partial^\mu \partial_\nu A^\nu) - \frac{1}{2}(\partial_\mu A^\mu)(\partial_\nu A^\nu)}_{= -\frac{1}{2}\partial_\mu(A^\mu \partial_\nu A^\nu)}$$

$$= -\frac{1}{2}(\partial_\nu A_\mu)(\partial^\nu A^\mu) + \frac{1}{2}\partial_\rho\left\{A_\mu(\partial^\mu A^\rho) - A^\rho(\partial_\nu A^\nu)\right\}$$

$$= -\frac{1}{2}(\partial_\nu A_\mu)(\partial^\nu A^\mu) + \frac{1}{2}\partial_\rho \Lambda^\rho$$

2 points

which is equivalent to $-\frac{1}{2}(\partial_\nu A_\mu)(\partial^\nu A^\mu)$ by Mandl & Shaw problem 2.1.