Homework Assignment #3

Problem 9 = M & Sh #5.2

(A) $[a_3(\vec{k}) - a_0(\vec{k}),\ a_3^\dagger(\vec{k}') - a_0^\dagger(\vec{k}')]$

$= [a_3, a_3^\dagger] + [a_0, a_0^\dagger] = 1 - 1 = 0$

(B) Let $\alpha_i^\dagger = a_3^\dagger(\vec{k}_i) - a_0^\dagger(\vec{k}_i)$.

Then $[a_3(\vec{k}) - a_0(\vec{k})]\ \alpha_i^\dagger|0\rangle = \alpha_i^\dagger\ [a_3(\vec{k}) - a_0(\vec{k})]|0\rangle$

$= 0$ because $a_3|0\rangle = 0$ and $a_0|0\rangle = 0$.

So $\prod_i (\alpha_i^\dagger)^{n_i}|0\rangle$ is a physical state

(i.e, $(a_3 - a_0)|s\rangle = 0$)

and any linear combination $|\Psi_{LS}\rangle$ is also an allowed physical state.

(C) A state with n transverse photons is

$$[a_r^\dagger(\vec{k})]^n\ |\Psi_{SL}\rangle \qquad (r = 1 \text{ or } 2)$$

3 points

Problem 10 = M & Sh. # 5.3

Assume $a_0 |\Psi_T\rangle = 0$ and $a_3 |\Psi_T\rangle = 0$.

Let $|\Psi_T'\rangle = \{1 + a[a_3^+(k) - a_0^+(k)]\} |\Psi_T\rangle$

Now calculate $\langle \Psi_T' | A^\mu(x) | \Psi_T' \rangle$

$$= \langle \Psi_T | A^\mu(x) | \Psi_T \rangle + \langle \Psi_T | A^\mu(x)\, a(a_3^+ - a_0^+) | \Psi_T \rangle$$
$$+ \langle \Psi_T | a^* (a_3 - a_0)\, A^\mu(x) | \Psi_T \rangle$$
$$+ |a|^2 \langle \Psi_T | (a_3 - a_0)\, A^\mu(\vec{x})\, (a_3^+ - a_0^+) | \Psi_T \rangle$$

$$= \langle \Psi_T | A^\mu(x) | \Psi_T \rangle + 2\mathrm{Re}\, a \langle \Psi_T | A^\mu(x)\, (a_3^+ - a_0^+) | \Psi_T \rangle$$

$$= \sum_{kr} \sqrt{\frac{1}{2\omega\Omega}}\, \epsilon_r^\mu\, a_r(k)\, e^{-ik\cdot x}$$

$\langle \Psi_T | a_3 a_3^+ | \Psi_T \rangle = 1$; $\langle \Psi_T | a_0 a_0^+ | \Psi_T \rangle = -1$

$$= \langle \Psi_T | A^\mu(x) | \Psi_T \rangle + 2\underline{\underline{\mathrm{Re}}}\, a \sqrt{\frac{1}{2\omega\Omega}} (\epsilon_3^\mu + \epsilon_0^\mu)\, e^{-ik\cdot x}$$

$\epsilon_3^\mu = (0, \vec{k}/|\vec{k}|)^\mu$ and $\epsilon_0^\mu = (1, \vec{0})^\mu = (k^0/\omega, \vec{0})^\mu$

$$= \langle \Psi_T | A^\mu(x) | \Psi_T \rangle + 2\mathrm{Re}\, a \sqrt{\frac{1}{2\omega\Omega}}\, \frac{1}{\omega} (+i\partial_0, -i\partial_i)^\mu e^{-ik\cdot x}$$

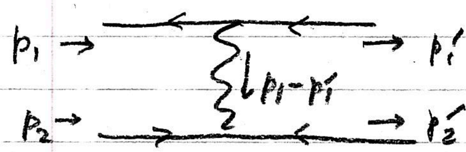
$$= \langle \Psi_T | A^\mu(x) + \sqrt{\frac{2}{\Omega\omega^3}}\, \partial^\mu\, \mathrm{Re}\left[i a e^{-ik\cdot x} \right]$$

2 points

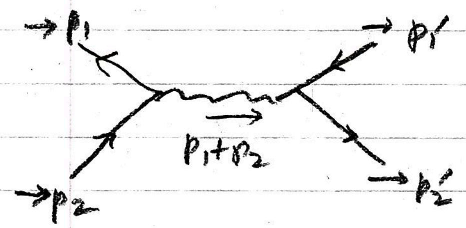
Problem 11 = M & Sh. II 7.1

The process $e^+(p_1) + e^-(p_2) \to e^+(p_1') + e^-(p_2')$

Spinors: $\bar{v}_1 \quad u_2$ and $v_1' \quad \bar{u}_2'$



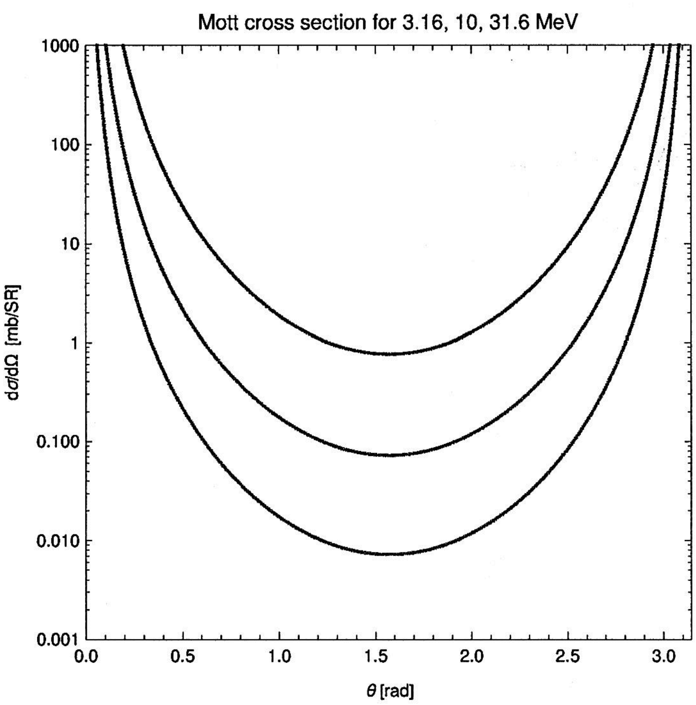
$$= e^2 \left(\bar{u}_2' \gamma^\mu u_2\right)\left(\bar{v}_1 \gamma_\mu v_1'\right) \frac{1}{(p_1 - p_1')^2}$$



$$= e^2 \left(\bar{u}_2' \gamma^\mu v_1'\right)\left(\bar{v}_1 \gamma_\mu u_2\right) \frac{1}{(p_1 + p_2)^2}$$

2 points

Out[9]=



Mott cross section for 3.16, 10, 31.6 MeV
dσ/dΩ [mb/SR]
1000
100
10
1
0.100
0.010
0.001
0.0
0.5
1.0
1.5
2.0
2.5
3.0
θ [rad]