

## Homework Assignment #3

Problem 9 = Mesh #5.2

$$\textcircled{A} \quad [a_3(\vec{k}) - a_0(\vec{k}), a_3^\dagger(\vec{k}') - a_0^\dagger(\vec{k}')] \\ = [a_3, a_3^\dagger] + [a_0, a_0^\dagger] = 1 - 1 = 0$$

$$\textcircled{B} \quad \text{Let } \alpha_i^\dagger = a_3^\dagger(\vec{k}_i) - a_0^\dagger(\vec{k}_i).$$

$$\text{Then } [a_3(\vec{k}) - a_0(\vec{k}), \alpha_i^\dagger |0\rangle = \alpha_i^\dagger [a_3(\vec{k}) - a_0(\vec{k})] |0\rangle \\ = 0 \quad \text{because } a_3|0\rangle = 0 \text{ and } a_0|0\rangle = 0.$$

So  $\prod_i (\alpha_i^\dagger)^{n_i} |0\rangle$  is a physical state  
(i.e.,  $(a_3 - a_0)|\psi\rangle = 0$ )

and any linear combination  $|\Psi_{SL}\rangle$  is also an allowed physical state.

© A state with  $n$  transverse photons is

$$[a_r^\dagger(\vec{k})]^n |\Psi_{SL}\rangle \quad (r=1 \text{ or } 2)$$

3 points

Problem 10 = M & S. # 5.3

Assume  $a_0 |\Psi_T\rangle = 0$  and  $a_3 |\Psi_T\rangle = 0$ .

Let  $|\Psi_T'\rangle = \{1 + a [a_3^\dagger(\omega) - a_0^\dagger(\omega)]\} |\Psi_T\rangle$

Now calculate  $\langle \Psi_T' | A^\mu(x) | \Psi_T' \rangle$

$$\begin{aligned} &= \langle \Psi_T | A^\mu(x) | \Psi_T \rangle + \langle \Psi_T | A^\mu(x) a (a_3^\dagger - a_0^\dagger) | \Psi_T \rangle \\ &\quad + \langle \Psi_T | a^\dagger (a_3 - a_0) A^\mu(x) | \Psi_T \rangle \\ &\quad + |a|^2 \langle \Psi_T | (a_3 - a_0) A^\mu(x) (a_3^\dagger - a_0^\dagger) | \Psi_T \rangle \end{aligned}$$

$$= \langle \Psi_T | A^\mu(x) | \Psi_T \rangle + 2\text{Re } a \langle \Psi_T | A^\mu(x) (a_3^\dagger - a_0^\dagger) | \Psi_T \rangle$$

$$= \sum_{\vec{k}} \frac{1}{k^4} \sqrt{\frac{1}{2\omega\omega^2}} \epsilon_r^\mu a_r(\omega) e^{-ik \cdot x}$$

$$\langle \Psi_T | a_3 a_3^\dagger | \Psi_T \rangle = 1, \quad \langle \Psi_T | a_0 a_0^\dagger | \Psi_T \rangle = -1$$

$$= \langle \Psi_T | A^\mu(x) | \Psi_T \rangle + 2\text{Re } a \sqrt{\frac{1}{2\omega\omega^2}} (\epsilon_3^\mu + \epsilon_0^\mu) e^{-ik \cdot x}$$

$$\epsilon_3^\mu = (0, \vec{k}/|\vec{k}|)^\mu \text{ and } \epsilon_0^\mu = (1, \vec{0})^\mu = (k^0/\omega, \vec{0})^\mu$$

$$= \langle \Psi_T | A^\mu(x) | \Psi_T \rangle + 2\text{Re } a \sqrt{\frac{1}{2\omega\omega^2}} \frac{1}{\omega} (+i\partial_0, -i\partial_i)^\mu e^{-ik \cdot x}$$

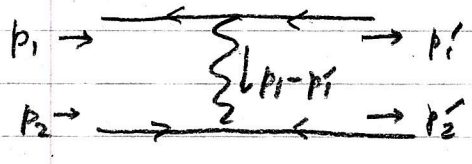
$$= \langle \Psi_T | A^\mu(x) + \sqrt{\frac{2}{\omega^2\omega^3}} \partial^\mu \text{Re} [i a e^{-ik \cdot x}]$$

2 points

Problem 11 = M & S. #7.1

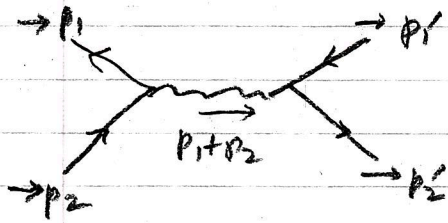
The process  $e^+(p_1) + e^-(p_2) \rightarrow e^+(p_1') + e^-(p_2')$

Spinors:  $\bar{v}_1$   $u_2$  and  $v_1'$   $\bar{u}_2'$



A Feynman diagram for s-channel annihilation. Two incoming fermion lines from the left meet at a vertex. The top line is labeled  $p_1$  and the bottom line is labeled  $p_2$ . They meet at a central vertex where a wavy line (representing a photon) is exchanged. From this vertex, two outgoing fermion lines go to the right. The top line is labeled  $p_1'$  and the bottom line is labeled  $p_2'$ . A curly bracket on the left side of the wavy line is labeled  $\{p_1 - p_1'$ .

$$= e^2 (\bar{u}_2' \gamma^\mu u_2) (\bar{v}_1 \gamma_\mu v_1') \frac{1}{(p_1 - p_1')^2}$$



$$= e^2 (\bar{u}_2' \gamma^\mu v_1') (\bar{v}_1 \gamma_\mu u_2) \frac{1}{(p_1 + p_2)^2}$$

2 points

Mott cross section for 3.16, 10, 31.6 MeV

