PHY 955 Assignment #5

Problem 17 The decay process $e^- + e^+ \to \gamma$ is not possible because it is not possible to conserve both energy and momentum.

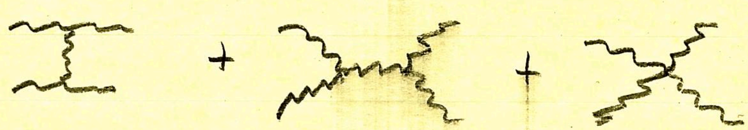
Assume $\vec{p}_1 + \vec{p}_2 = \vec{k}$

Then $E_1 + E_2 = \sqrt{p_1^2 + m^2} + \sqrt{p_2^2 + m^2}$ and $\omega = |\vec{p}_1 + \vec{p}_2|$

$$(E_1+E_2)^2 - \omega^2 = p_1^2 + m^2 + p_2^2 + m^2 + 2E_1E_2 - (p_1^2 + p_2^2 + 2p_1p_2\cos\theta)$$

$$= 2m^2 + 2E_1E_2 - 2p_1p_2\cos\theta \quad > 0.$$

Problem 18 gluon gluon scattering

(A) 

(B) $$\frac{d\sigma}{d\Omega} = \frac{9\alpha_s^2}{8s}\left(3 - \frac{ut}{s^2} - \frac{su}{t^2} - \frac{st}{u^2}\right)$$ P.D.G. (48.12)

(C) Plot $d\sigma/d\Omega$ versus t for $s = (100\,\mathrm{GeV})^2$.

Note : $s + t + u = \Sigma m^2 = 0$

Problem 19 $\quad \frac{d\Gamma}{dE_e} = \frac{G^2}{12\pi^3}\sqrt{E_e^2 - m_e^2}\left[3E_3(M^2+m^2) - 4ME_e^2 - 2Mm^2\right]$

(A)

Neglect the electron mass $\Rightarrow$

$$\frac{d\Gamma}{dE_e} = \frac{G^2}{12\pi^3} E_e\left(3E_e M^2 - 4ME_e^2\right)$$

$$\Gamma = \int_0^{M/2} \frac{d\Gamma}{dE}\,dE = \frac{G^2}{12\pi^3}\left[M^2\left(\frac{M}{2}\right)^3 - M\left(\frac{M}{2}\right)^4\right]$$

$$= \frac{G^2 M^5}{192\pi^3}$$

$\Gamma = \frac{1}{\tau}$ so $G^2 = \frac{192\pi^3}{\tau M^5}$ $\qquad \frac{G}{(\hbar c)^3} = \left[\frac{192\pi^3\hbar}{\tau (Mc^2)^5}\right]^{1/2}$

$$= 1.166\times 10^{-5}\ \text{GeV}^{-2}$$

(B) For the standard model,

$$\alpha = \frac{e^2}{4\pi}\ ;\quad G_F = \frac{\sqrt{2}\,g_W^2}{8M_W^2}\ ;\quad \sin\theta_W = \frac{e}{g_W}$$

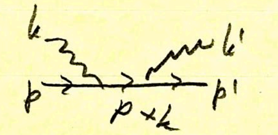
$$\therefore\ \sin\theta_W = \frac{\sqrt{4\pi\alpha}}{\sqrt{8M_W^2 G_F/\sqrt{2}}} = 0.464$$

$$(\sin^2\theta_W = 0.215)$$

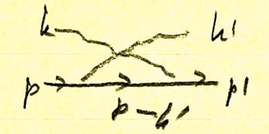
Problem 20 Mandl & Shaw #8.7 5.3

Compton scattering

$$\mathcal{M}_a = -ie^2\, \bar u(p')\, \not\epsilon' (\not p + \not k + m) \not\epsilon\, u(p)/2p\cdot k$$



$$\mathcal{M}_b = +ie^2\, \bar u(p')\, \not\epsilon (\not p - \not k' + m) \not\epsilon'\, u(p)/2p\cdot k'$$



Now consider this gauge transformation:

$\epsilon^\mu \to \epsilon^\mu + \lambda k^\mu$ and $\epsilon'^\mu \Rightarrow \epsilon'^\mu + \lambda k'^\mu$

Calculate the changes in $\mathcal{M}_a$ and $\mathcal{M}_b$.

The terms $\propto \lambda$

$$\delta_1 m_a = -ie^2\lambda\, \bar u(p') \{ \not k' \underbrace{(\not p + \not k + m)}_{\not p' + \cancel{\not k'}} \not\epsilon + \not\epsilon' (\not p + \cancel{\not k} + m) \not k \} u/2p\cdot k$$

$$= -ie^2\lambda\, \bar u' \{ [2 \underbrace{k'\cdot p'}_{k\cdot p} - (\not p' - m)\not k'] \not\epsilon + \not\epsilon' [2p\cdot k - \not k(\not p - m)] u/2p\cdot k$$

$$= -ie^2\lambda\, \bar u' (\not\epsilon + \not\epsilon') u$$

$$\delta_1 m_b = +ie^2\lambda\, \bar u' \{ \not k \underbrace{(\not p - \not k' + m)}_{\not p' - \cancel{\not k}} \not\epsilon' + \not\epsilon (\not p - \cancel{\not k'} + m) \not k' \} u/2p\cdot k'$$

$$= +ie^2\lambda\, \bar u' \{ \underbrace{2k\cdot p'}_{=2k'\cdot p} \not\epsilon' + \not\epsilon\, 2p\cdot k' \} u/2p\cdot k'$$

$$= +ie^2\lambda\, \bar u' (\not\epsilon' + \not\epsilon) u \qquad \underline{\delta_1 m_a + \delta_1 m_b = 0}$$

The terms $\propto \lambda^2$

$$\delta_2 m_a = -ie^2\lambda^2\, \bar u' (\not k' (\not p + \cancel{\not k} + m) \not k) u/2p\cdot k$$

$$= -ie^2\lambda^2\, \bar u' (\not k' [2p\cdot k - \not k(\not p - m)]) u/2p\cdot k$$

$$= -ie^2\lambda^2\, \bar u' \not k' u$$

$$\delta_2 m_b = +ie^2\lambda^2\, \bar u' \not k (\not p - \cancel{\not k'} + m) \not k' u/2p\cdot k'$$

$$= ie^2\lambda^2\, \bar u' \not k [2p\cdot k' - \not k'(\not p - m)] u/2p\cdot k'$$

$$= ie^2\lambda^2\, \bar u' \not k u = ie^2\lambda^2\, \bar u' [\not k' + \cancel{\not p'} - \cancel{\not p}] u = ie^2\lambda^2\, \bar u' \not k' u$$