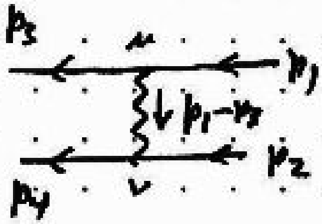
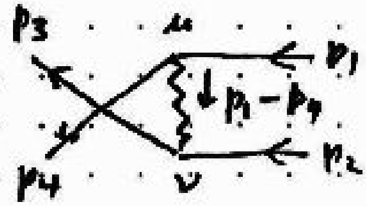
Moller scattering

$\mathcal{M} =$  $+$  $\quad D_F^{\mu\nu}(q) = \frac{-g^{\mu\nu}}{q^2}$

$$\mathcal{M}_t = e^2\, \bar{u}_3 \gamma_\mu u_1\, \bar{u}_4 \gamma^\mu u_2 / t \qquad t = (p_1 - p_3)^2$$

$$\mathcal{M}_u = -e^2\, \bar{u}_3 \gamma_\nu u_2\, \bar{u}_4 \gamma^\nu u_1 / u \qquad u = (p_1 - p_4)^2$$

$$|\mathcal{M}|^2 = |\mathcal{M}_t|^2 + |\mathcal{M}_u|^2 + \mathcal{M}_t \mathcal{M}_u^* + \mathcal{M}_t^* \mathcal{M}_u$$

$$|\mathcal{M}_t|^2 = \frac{e^4}{t^2}\, \bar{u}_3 \gamma_\mu u_1\, \bar{u}_4 \gamma^\mu u_2\, \bar{u}_1 \gamma_\nu u_3\, \bar{u}_2 \gamma^\nu u_4$$

$$|\mathcal{M}_t|^2 = \frac{e^4}{t^2} \left[\bar{u}_3 \gamma_\mu u_1\, \bar{u}_1 \gamma_\nu u_3\right] \left[\bar{u}_4 \gamma^\mu u_2 \bar{u}_2 \gamma^\nu u_4\right]$$

$$= \frac{e^4}{t^2} \left[\mathrm{Tr}\, \gamma_\mu u_1 \bar{u}_1 \gamma_\nu u_3 \bar{u}_3\right] \left[\mathrm{Tr}\, \gamma^\mu u_2 \bar{u}_2 \gamma^\nu u_4 \bar{u}_4\right]$$

$$\overline{|\mathcal{M}_t|^2} = \frac{e^4}{4t^2} \left[\mathrm{Tr}\, \gamma_\mu \frac{\not{p}_1 + m}{2m} \gamma_\nu \frac{\not{p}_3 + m}{2m}\right]$$
$$\left[\mathrm{Tr}\, \gamma^\mu \frac{\not{p}_2 + m}{2m} \gamma^\nu \frac{\not{p}_4 + m}{2m}\right]$$

$$= \frac{e^4}{4t^2} \left[p_{1\mu} p_{3\nu} - g_{\mu\nu}\, p_1 \cdot p_3 + p_{3\mu} p_{1\nu} + m^2 g_{\mu\nu}\right] \frac{4}{(2m)^2}$$
$$\left[p_2^\mu p_4^\nu - g^{\mu\nu}\, p_2 \cdot p_4 + p_4^\mu p_2 + m^2 g^{\mu\nu}\right] \frac{4}{(2m)^2}$$

$$\overline{|\mathcal{M}_t|^2} = \frac{e^4}{4t^2}\frac{16}{(2m)^4}\left[p_{1\mu}p_{3\nu} + p_{3\mu}p_{1\nu} + g_{\mu\nu}(m^2 - p_1\cdot p_3)\right]$$

$$\left[p_2^\mu p_4^\nu + p_4^\mu p_2^\nu + g^{\mu\nu}(m^2 - p_2\cdot p_4)\right]$$

$$= \frac{e^4}{4t^2}\frac{16}{(2m)^4}\Big\{ 2\, p_1\cdot p_2\; p_3\cdot p_4 + 2\, p_1\cdot p_4\; p_2\cdot p_3$$

$$+ 2\, p_1\cdot p_3\,(m^2 - p_2\cdot p_4) + 2\, p_2\cdot p_4\,(m^2 - p_1\cdot p_3)$$

$$+ 4\,(m^2 - p_1\cdot p_3)(m^2 - p_2\cdot p_4)\Big\}$$

$$s = (p_1+p_2)^2 = 2m^2 + 2p_1\cdot p_2 = (p_3+p_4)^2 = 2m^2 + 2p_3\cdot p_4$$

$$t = (p_1-p_3)^2 = 2m^2 - 2p_1\cdot p_3 = (p_4-p_2)^2 = 2m^2 - 2p_4\cdot p_2$$

$$u = (p_1-p_4)^2 = 2m^2 - 2p_1\cdot p_4 = (p_3-p_2)^2 = 2m^2 - 2p_3\cdot p_2$$

$$= \frac{e^4}{4t^2}\frac{16}{(2m)^4}\left\{ \frac{1}{2}(s-2m^2)^2 + \frac{1}{2}(2m^2-u)^2 + (2m^2-t)\,t + t^2\right\}$$

$$\overline{|\mathcal{M}_t|^2} = \frac{e^4}{4t^2}\frac{16}{(2m)^4}\left\{ \frac{1}{2}(s-2m^2)^2 + \frac{1}{2}(2m^2-u)^2 + 2m^2 t\right\}$$

Similarly,

$$\overline{|\mathcal{M}_u|^2} = \frac{e^4}{4u^2}\frac{16}{(2m)^4}\left\{ \frac{1}{2}(s-2m^2)^2 + \frac{1}{2}(2m^2-t)^2 + 2m^2 u\right\}$$

Because ee are identical, the transition probability is invariant under $t \longleftrightarrow u$.

$$\mathcal{M}_1 = \frac{e^2}{t}\ \bar{u}_3 \gamma_\mu u_1\ \bar{u}_4 \gamma^\mu u_2$$

$$\mathcal{M}_2 = -\frac{e^2}{u}\ \bar{u}_3 \gamma_\nu u_2\ \bar{u}_4 \gamma^\nu u_1$$

$$\mathcal{M}_2^* = \frac{-e^2}{u}\ \bar{u}_2 \gamma_\nu u_3\ \bar{u}_1 \gamma^\nu u_4$$

$$\mathcal{M}_1 \mathcal{M}_2^* = \frac{-e^4}{tu}\ \bar{u}_3 \gamma_\mu u_1\ \bar{u}_1 \gamma^\nu u_4\ \bar{u}_4 \gamma^\mu u_2\ \bar{u}_2 \gamma_\nu u_3$$

$$= \frac{-e^4}{tu} \mathrm{Tr}\left[\gamma^\mu u_1 \bar{u}_1 \gamma^\nu u_4 \bar{u}_4 \gamma_\mu u_2 \bar{u}_2 \gamma_\nu u_3 \bar{u}_3\right]$$

$$\overline{\mathcal{M}_1 \mathcal{M}_2^*} = \frac{-e^4}{4tu} \frac{1}{(2m)^4} \mathrm{Tr}\Big[\gamma^\mu (\not{p}_1 + m)$$

$$\gamma^\nu (\not{p}_4 + m) \gamma_\mu (\not{p}_2 + m) \gamma_\nu \ (\not{p}_3 + m)\Big]$$

$$= \frac{-e^4}{4tu} \frac{1}{(2m)^4} \mathrm{Tr}\left[\gamma^\mu (\not{p}_1 + m)\ X_\mu\ (\not{p}_3 + m)\right]$$

$$X_\mu = \gamma^\nu (\not{p}_4 + m) \gamma_\mu (\not{p}_2 + m) \gamma_\nu$$

$$= \gamma^\nu (\not{p}_4 \gamma_\mu \not{p}_2 + m \gamma_\mu \not{p}_2 + m \not{p}_4 \gamma_\mu + m^2 \gamma_\mu) \gamma^\mu$$

$$= -2 \not{p}_2 \gamma_\mu \not{p}_4 + 4 m\, p_{2\mu} + 4m\, p_{4\mu} + -2m^2 \gamma_\mu$$

$$\underline{F} = \mathrm{Tr}\left[\gamma^\mu (\not{p}_1 + m)\ X_\mu\ (\not{p}_3 + m)\right]$$

$$= \mathrm{Tr}\Big[\gamma^\mu (\not{p}_1 + m)(-2 \not{p}_2 \gamma_\mu \not{p}_4)(\not{p}_3 + m)$$

$$+ 4m (\not{p}_2 + \not{p}_4)(\not{p}_1 + m)(\not{p}_3 + m)$$

$$- 2m^2\ \gamma^\mu (\not{p}_1 + m) \gamma_\mu (\not{p}_3 + m)\Big]$$

$$\Sigma = -8\, p_1\cdot p_2\; 4 p_3\cdot p_4 + 4m^2\, 4\, p_2\cdot p_4$$

$$+ 4m^2\, 4\,(p_2\cdot p_1 + p_2\cdot p_3 + p_4\cdot p_1 + p_4\cdot p_3)$$

$$+ 4m^2\, 4\, p_1\cdot p_3 - 8m^4 \cdot 4$$

$$= -32(p_1\cdot p_2)^2 + 16m^2(2p_1\cdot p_3 + 2 p_1\cdot p_2 + 2 p_1\cdot p_4) - 32m^4$$

$$= -32(p_1\cdot p_2)^2 + 16m^2(\cancel{2m^2} + 4p_1\cdot p_2) \cancel{-32m^4}$$

$$= -32\,(p_1\cdot p_2)\,[\, p_1\cdot p_2 - 2m^2]$$

$2p_1\cdot p_2 = s - 2m^2$

$$= -8\,(s-2m^2)\,(s-6m^2)$$

$$\overline{\mathcal{M}_1 \mathcal{M}_2^*} = \frac{-e^4}{4tu}\,\frac{1}{(2m)^4}\,(-8)(s-2m^2)\,(s-6m^2)$$

$$\overline{(\mathcal{M}_1 \mathcal{M}_2^* + \mathcal{M}_1^* \mathcal{M}_2)} = \frac{e^4}{4tu}\,\frac{16}{(2m)^4}\,(s-2m^2)(s-6m^2)$$

$$\overline{|\mathcal{M}|^2} = \frac{e^4\, 16}{4(2m)^4}\Big\{ \frac{1}{t^2}\Big[\tfrac{1}{2}(s-2m^2)^2 + \tfrac{1}{2}(2m^2-u)^2 + 2m^2 t\Big]$$

$$+ \frac{1}{u^2}\Big[\tfrac{1}{2}(s-2m^2)^2 + \tfrac{1}{2}(2m^2-t)^2 + 2m^2 u\Big]$$

$$+ \frac{1}{tu}\,(s-2m^2)\,(s-6m^2)\Big\}$$

The center of mass cross section is

$$\left(\frac{d\sigma}{d\Omega_3}\right)_{CM} = \frac{1}{64\pi^2 s}\,(2m)^4\,\overline{|\mathcal{M}|^2} \qquad \text{(Eq. 8.18)}$$

in units ~~wet~~ $\hbar = 1$
and $c = 1$

$$\frac{e^2}{4\pi\hbar c} = \frac{1}{137}\;; \quad (\hbar c)^2 = (0.197\ \text{GeV-fm})^2$$

$$= (1.97)^2 \times \text{GeV}^2 \times 10^{-4}\ \text{barns}$$