"πN" scattering (no isospin)

Square the matrix element

In[1]:=
```
Remove["Global`*"]
(* invariants; Q and A *)
Qdp3 = SI * (p1dp3 + p2dp3) - UI * (p1dp3 - p4dp3)
Qdp1 = SI * (p1dp1 + p2dp1) - UI * (p1dp1 - p4dp1)
QdQ = SI^2 * s + UI^2 * u - 2 * SI * UI * (p1dp1 - p1dp4 + p2dp1 - p2dp4)
A = SI * M - UI * M
```

Remove::rmnsm : There are no symbols matching "Global`*". >>

Out[2]= $(p1dp3 + p2dp3)\ SI - (p1dp3 - p4dp3)\ UI$

Out[3]= $(p1dp1 + p2dp1)\ SI - (p1dp1 - p4dp1)\ UI$

Out[4]= $s\ SI^2 - 2\ (p1dp1 - p1dp4 + p2dp1 - p2dp4)\ SI\ UI + u\ UI^2$

Out[5]= $M\ SI - M\ UI$

In[6]:=
```
(* replacements *)
replacements1 = {p1dp1 → M^2, p2dp2 → m^2, p3dp3 → M^2, p4dp4 → m^2}
replacements2 = {
  p2dp1 → p1dp2,
  p3dp1 → p1dp3, p3dp2 → p2dp3,
  p4dp1 → p1dp4, p4dp2 → p2dp4, p4dp3 → p3dp4}
replacements3 = {
  p1dp2 → (s - M^2 - m^2) / 2,
  p1dp3 → (M^2 + M^2 - t) / 2,
  p1dp4 → (M^2 + m^2 - u) / 2,
  p2dp3 → (m^2 + M^2 - u) / 2,
  p2dp4 → (m^2 + m^2 - t) / 2,
  p3dp4 → (s - M^2 - m^2) / 2}
replacements4 = {t → 2 * M^2 + 2 * m^2 - s - u}
```

Out[6]= $\{p1dp1 \to M^2,\ p2dp2 \to m^2,\ p3dp3 \to M^2,\ p4dp4 \to m^2\}$

Out[7]= $\{p2dp1 \to p1dp2,\ p3dp1 \to p1dp3,\ p3dp2 \to p2dp3,\ p4dp1 \to p1dp4,\ p4dp2 \to p2dp4,\ p4dp3 \to p3dp4\}$

Out[8]= $\left\{p1dp2 \to \frac{1}{2}\left(-m^2 - M^2 + s\right),\ p1dp3 \to \frac{1}{2}\left(2\,M^2 - t\right),\ p1dp4 \to \frac{1}{2}\left(m^2 + M^2 - u\right),\ p2dp3 \to \frac{1}{2}\left(m^2 + M^2 - u\right),\ p2dp4 \to \frac{1}{2}\left(2\,m^2 - t\right),\ p3dp4 \to \frac{1}{2}\left(-m^2 - M^2 + s\right)\right\}$

Out[9]= $\{t \to 2\,m^2 + 2\,M^2 - s - u\}$

In[10]:=
```
MsqA0 = 2 * Qdp3 * Qdp1 - QdQ * p1dp3 + QdQ * M^2;
MsqA0 = MsqA0 /. replacements1;
MsqA0 = MsqA0 /. replacements2;
MsqA0 = MsqA0 /. replacements3;
MsqA0 = MsqA0 /. replacements4;
Expand[MsqA0]
```

Out[15]=
$$\frac{m^4\,SI^2}{2} - m^2\,M^2\,SI^2 + \frac{M^4\,SI^2}{2} + 2\,M^2\,s\,SI^2 - \frac{1}{2}\,s\,SI^2\,u + m^4\,SI\,UI + 2\,m^2\,M^2\,SI\,UI - 3\,M^4\,SI\,UI - s\,SI\,u\,UI + \frac{m^4\,UI^2}{2} - m^2\,M^2\,UI^2 + \frac{M^4\,UI^2}{2} + 2\,M^2\,u\,UI^2 - \frac{1}{2}\,s\,u\,UI^2$$

In[16]:=
```
MsqA2 = A^2 * p1dp3 + A^2 * M^2
MsqA2 = MsqA2 /. replacements1
MsqA2 = MsqA2 /. replacements2
MsqA2 = MsqA2 /. replacements3
MsqA2 = MsqA2 /. replacements4
Expand[MsqA2]
```

Out[16]=
$$M^2\,(M\,SI - M\,UI)^2 + p1dp3\,(M\,SI - M\,UI)^2$$

Out[17]=
$$M^2\,(M\,SI - M\,UI)^2 + p1dp3\,(M\,SI - M\,UI)^2$$

Out[18]=
$$M^2\,(M\,SI - M\,UI)^2 + p1dp3\,(M\,SI - M\,UI)^2$$

Out[19]=
$$M^2\,(M\,SI - M\,UI)^2 + \frac{1}{2}\left(2\,M^2 - t\right)(M\,SI - M\,UI)^2$$

Out[20]=
$$M^2\,(M\,SI - M\,UI)^2 + \frac{1}{2}\left(-2\,m^2 + s + u\right)(M\,SI - M\,UI)^2$$

Out[21]=
$$-m^2\,M^2\,SI^2 + M^4\,SI^2 + \frac{1}{2}\,M^2\,s\,SI^2 + \frac{1}{2}\,M^2\,SI^2\,u + 2\,m^2\,M^2\,SI\,UI - 2\,M^4\,SI\,UI - M^2\,s\,SI\,UI - M^2\,SI\,u\,UI - m^2\,M^2\,UI^2 + M^4\,UI^2 + \frac{1}{2}\,M^2\,s\,UI^2 + \frac{1}{2}\,M^2\,u\,UI^2$$

In[22]:=
```
MsqA1 = A * M * 2 * Qdp1 + A * M * 2 * Qdp3;
MsqA1 = MsqA1 /. replacements1;
MsqA1 = MsqA1 /. replacements2;
MsqA1 = MsqA1 /. replacements3;
MsqA1 = MsqA1 /. replacements4;
Expand[MsqA1]
```

Out[27]=
$$-2\,m^2\,M^2\,SI^2 + 2\,M^4\,SI^2 + 2\,M^2\,s\,SI^2 + 4\,m^2\,M^2\,SI\,UI - 4\,M^4\,SI\,UI - 2\,M^2\,s\,SI\,UI - 2\,M^2\,SI\,u\,UI - 2\,m^2\,M^2\,UI^2 + 2\,M^4\,UI^2 + 2\,M^2\,u\,UI^2$$

In[28]:=
```
Msq = Expand[MsqA2 + MsqA0 + MsqA1]
```

Out[28]=
$$\frac{m^4\,SI^2}{2} - 4\,m^2\,M^2\,SI^2 + \frac{7\,M^4\,SI^2}{2} + \frac{9}{2}\,M^2\,s\,SI^2 + \frac{1}{2}\,M^2\,SI^2\,u - \frac{1}{2}\,s\,SI^2\,u + m^4\,SI\,UI + 8\,m^2\,M^2\,SI\,UI - 9\,M^4\,SI\,UI - 3\,M^2\,s\,SI\,UI - 3\,M^2\,SI\,u\,UI - s\,SI\,u\,UI + \frac{m^4\,UI^2}{2} - 4\,m^2\,M^2\,UI^2 + \frac{7\,M^4\,UI^2}{2} + \frac{1}{2}\,M^2\,s\,UI^2 + \frac{9}{2}\,M^2\,u\,UI^2 - \frac{1}{2}\,s\,u\,UI^2$$

factorize M^2 ; the cross section

In[29]:=
```
Kss = Expand[(Msq /. UI → 0) / SI^2]
Kuu = Expand[(Msq /. SI → 0) / UI^2]
Ksu = Expand[(Msq /. {UI^2 -> 0, SI^2 → 0}) / (SI * UI)]
```

Out[29]= $\frac{m^4}{2} - 4\,m^2\,M^2 + \frac{7\,M^4}{2} + \frac{9\,M^2\,s}{2} + \frac{M^2\,u}{2} - \frac{s\,u}{2}$

Out[30]= $\frac{m^4}{2} - 4\,m^2\,M^2 + \frac{7\,M^4}{2} + \frac{M^2\,s}{2} + \frac{9\,M^2\,u}{2} - \frac{s\,u}{2}$

Out[31]= $m^4 + 8\,m^2\,M^2 - 9\,M^4 - 3\,M^2\,s - 3\,M^2\,u - s\,u$

In[32]:=
```
factor1 = g^4 / (2 * M^2) * (4 * M^2) / s;
dσ[S_, U_] := factor1 *
    (Kss / (s - M^2)^2 +
      Kuu / (u - M^2)^2 +
      Ksu / (s - M^2) / (u - M^2)) /. {s → S, u → U}
```

In[34]:=
```
Clear[s, u]
dσ[s, u]
```

Out[35]= $\frac{1}{s} 2\,g^4 \left(\frac{m^4 + 8\,m^2\,M^2 - 9\,M^4 - 3\,M^2\,s - 3\,M^2\,u - s\,u}{\left(-M^2 + s\right)\left(-M^2 + u\right)} + \frac{\frac{m^4}{2} - 4\,m^2\,M^2 + \frac{7\,M^4}{2} + \frac{9\,M^2\,s}{2} + \frac{M^2\,u}{2} - \frac{s\,u}{2}}{\left(-M^2 + s\right)^2} + \frac{\frac{m^4}{2} - 4\,m^2\,M^2 + \frac{7\,M^4}{2} + \frac{M^2\,s}{2} + \frac{9\,M^2\,u}{2} - \frac{s\,u}{2}}{\left(-M^2 + u\right)^2} \right)$

Numerical calculations

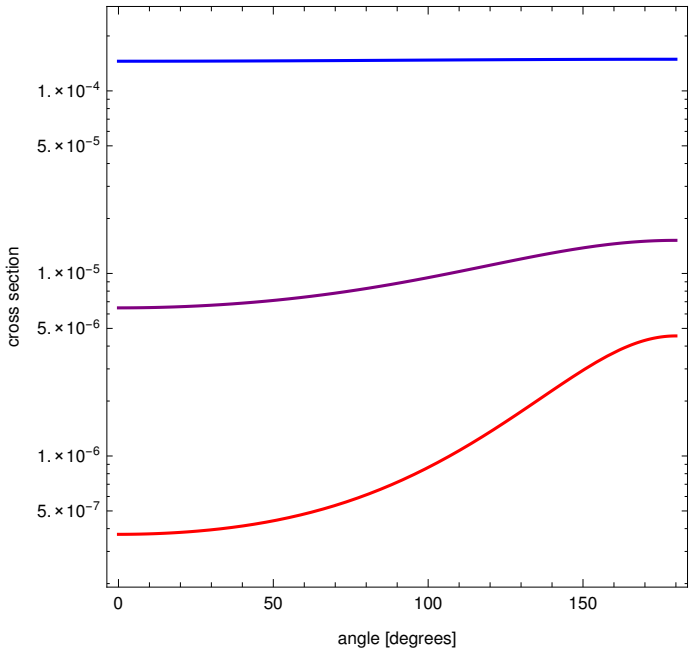
In[36]:=
```
(* Numerical calculations *)
Nreplacements = {g → 1.0, m → 140.0, M → 940.0}
f[p_, θ_] := dσ[
    (Sqrt[p^2 + M^2] + Sqrt[p^2 + m^2])^2,
    (Sqrt[p^2 + M^2] - Sqrt[p^2 + M^2])^2 - 2 * p^2 * (1 + Cos[θ])] /. Nreplacements
f[100, Pi/2]
```

Out[36]= {g → 1., m → 140., M → 940.}

Out[38]= 0.000147172

In[39]:=
```
LogPlot[{f[100, ang * Pi / 180],
  f[500, ang * Pi / 180],
  f[1000, ang * Pi / 180]},
 {ang, 0, 180},
 PlotStyle → {Blue, Purple, Red},
 Frame → True, AspectRatio → 1,
 FrameLabel → {"angle [degrees]", "cross section"}]
```

Out[39]=



Extreme relativistic limit (m → 0 and M → 0)

In[40]:=
```
test = Msq;
test = test /. {m → 0, M → 0, SI → 1 / s, UI → 1 / u};
test = test /. {s → 4 * p^2, u → -2 p^2 (1 + Cos[θ])};
λ[th_] := test /. {θ → th};
LogPlot[λ[th], {th, 0, Pi}]
```

Out[44]=