Chapter 5. Photons: Covariant Theory
5.1. The classical fields
5.2. Covariant quantization
5.3. The photon propagator

Chapter 6. The S-Matrix Expansion
6.1. Natural Dimensions and Units ✔
6.2. The S-matrix expansion ✔
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Chapter 7. Feynman Diagrams and Rules in QED
7.1. Feynman diagrams in configuration space
7.2. Feynman diagrams in momentum space
7.3. Feynman rules for QED
7.4. Leptons

Chapter 8. QED Processes in Lowest Order
8.1. The cross section
8.2. Spin sums
8.3. Photon polarization sums
8.4-7. Examples
8.8-9. Bremsstrahlung

Example. " "Pion nucleon scattering" "
... but ignoring isospin.
(a purely academic exercise)

Fields: $\psi_\alpha(\boldsymbol{x})$ and $\varphi(\boldsymbol{x})$

Lagrangian density :

$$\mathcal{L} = \mathcal{L}_{free\,\varphi} + \mathcal{L}_{free\,\psi} + \mathcal{L}_{interaction}$$

$$\mathcal{L}_{free\,\varphi} = you\ know$$

$$\mathcal{L}_{free\,\psi} = you\ know$$

$$\mathcal{L}_{interaction} = g\,\overline{\psi}\psi\varphi$$

Note that $\mathcal{L}$ is relativistically invariant.

INTERACTING FIELDS

In Chapter 7 we will derive the methods for calculating transition matrix elements (Feynman diagrams) and reaction cross sections.

But today we'll do an example, to illustrate:

- computational techniques
- theoretical issues

Here is the example

$$\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_\phi + \mathcal{L}_{int}$$

$$\mathcal{L}_\psi = \bar{\psi}\,(i\gamma - M)\,\psi$$

$$\mathcal{L}_\phi = \tfrac{1}{2}\,\partial^\mu\phi\,\partial_\mu\phi - \tfrac{m^2}{2}\,\phi$$

$$\mathcal{L}_{int} = g\,\phi\,\bar{\psi}\psi$$

No isospin : ψ, M, "N"

ϕ, m, "π"

Recall the transition amplitude

$$\langle f |\, T \exp(-i) \int_{-T}^{T} H_I \, dt \,| i \rangle$$

$$= \text{probability amplitude} = S_{fi}$$

= the probability amplitude for evolution from initial state |i> at time –T to final state |f> at time +T; take the limit T → infinity.

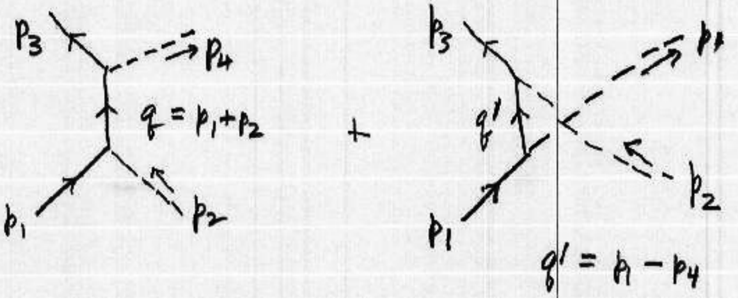
The interaction Hamiltonian is

$$H_I = -\int \mathcal{L}_{int} \, d^3x$$

For the lowest order calculation, the relevant term in the perturbation expansion is L_{int}^{2} ;

there are two Feynman diagrams .

$$\langle f |\, T \; g\,\phi(x)\,\bar\psi\psi(x) \, d^4x \;\; g\,\phi(y)\,\bar\psi\psi(y) \, d^4y \,| i \rangle$$

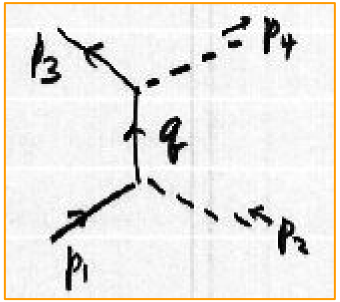


The scattering amplitude

$$\langle 0|\; a_{p_4}\, c_{p_3}\; \overset{T}{\wedge}\; g\,\phi(x)\,\bar{\psi}\psi(x)\; \overset{d^4x}{\wedge}\; g\,\phi(y)\,\bar{\psi}\psi(y)\; \overset{d^4y}{\wedge}\; a^{+}_{p_2}\, c^{+}_{p_1}\;|0\rangle$$

$$g^2\, e^{ip_4\cdot x}\, e^{ip_3\cdot x}\, \bar{u}_3\, \underbrace{S_F(x-y)}_{\downarrow\; e^{-iq\cdot(x-y)}\,\hat{S}_F(q)}\, e^{-ip_2\cdot y}\, e^{-ip_1\cdot y}\, u_1$$

$\times$(other factors)



$$\delta^4(p_3+p_4-q)\;\delta^4(p_1+p_2-q)\;d^4q$$

$$S = g^2 \delta^4(p_3 + p_4 - p_1 - p_2) \quad \times(\text{other factors})$$

$$\times \bar{u}_3 \left\{ \frac{\not{q} + M}{q^2 - M^2} - \frac{\not{q}' + M}{q'^2 - M^2} \right\} u_1$$

$$q = p_1 + p_2 \qquad q' = p_1 - p_4$$

$$q^2 = s \qquad (q')^2 = u$$

Mandelstam variables

$$\mathcal{M} = g^2 \bar{u}_3 (\not{Q} + A) u_1 \quad \text{where} \quad \not{Q} = \frac{\not{q}}{s - M^2} - \frac{\not{q}'}{u - M^2}$$

$$A = \frac{M}{s - M^2} - \frac{M}{u - M^2}$$

The cross section for polarized scattering

$$d\sigma = (2\pi)^4 \delta^4(P_f - P_i) \prod_i \sqrt{2m_i}$$

$$\frac{1}{4E_1E_2 v_{rel}} \prod_f \frac{d^3p_f}{(2\pi)^3 2E_f} |\mathcal{M}|^2 ;$$

for unpolarized scattering,

$$d\sigma_u = \frac{1}{2}\sum_{S_1} \sum_{S_3} d\sigma$$

$$|\mathcal{M}|^2 = \mathcal{M}^* \mathcal{M}$$

$$= g^4 \; u_1^\dagger (\not{Q}^\dagger + A)(\gamma^0)^\dagger u_3 \bar{u}_3 (\not{Q} + A) u_1 \qquad u_3^\dagger\gamma^0$$

$$\gamma^0 \not{Q}^\dagger \gamma^0 = \gamma^0 (\gamma^{0\dagger} Q^0 - \gamma^{i\dagger} Q^i) \gamma^0$$

$$= \gamma^0 (\gamma^0 Q^0 + \gamma^i Q^i) \gamma^0$$

$$= \gamma^0 Q^0 - \gamma^i Q^i = \not{Q}$$

$$|\mathcal{M}|^2 = g^4 \; \bar{u}_1 (\not{Q} + A) u_3 \bar{u}_3 (\not{Q} + A) u_1$$

$$= g^4 \; \mathrm{Trace}\left[(\not{Q} + A) \; u_3 \bar{u}_3 \; (\not{Q} + A) \; u_1 \bar{u}_1 \right]$$

$$\sum_{S_3} u_3 \bar{u}_3 = \Lambda^+(p_3) = \frac{\not{p}_3 + M}{2M}$$

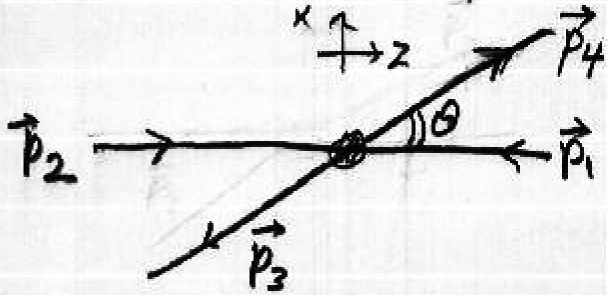
$$\sum_{S_1} u_1 \bar{u}_1 = \frac{\not{p}_1 + M}{2M}$$

$$\overline{|\mathcal{M}|^2} = g^4 \, \frac{1}{2} \, \mathrm{Trace}\left[(\not{Q} + A) \frac{\not{p}_3 + M}{2M} (\not{Q} + A) \frac{\not{p}_1 + M}{2M} \right]$$

For the CoM frame of reference, we can use

Eq. (8.18) [Homework Problem 5]

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{1}{64\pi^2 s}\left(\prod_f 2m_f\right)|\mathcal{M}|^2$$



$$p_1^\mu = (E_1, 0, 0, -p)$$

$$p_2^\mu = (E_2, 0, 0, p)$$

$$p_3^\mu = (E_3, -p\sin\theta, 0, -p\cos\theta)$$

$$p_4^\mu = (E_4, p\sin\theta, 0, p\cos\theta)$$

(5) TRACE IDENTITIES (Appendix A)

$$\overline{|\mathcal{M}|^2} = g^4 \frac{1}{2} \mathrm{Trace}\left[(\slashed{Q}+A)\frac{\slashed{p}_3+M}{2M}(\slashed{Q}+A)\frac{\slashed{p}_1+M}{2M} \right]$$

$$\begin{aligned} = \frac{g^4}{8M^2}\mathrm{Tr}\Big[& \slashed{Q}(\slashed{p}_3+M)\slashed{Q}(\slashed{p}_1+M) \\ & + \slashed{Q}(\slashed{p}_3+M)A(\slashed{p}_1+M) \\ & + A(\slashed{p}_3+M)\slashed{Q}(\slashed{p}_1+M) \\ & + A^2(\slashed{p}_3+M)(\slashed{p}_1+M)\Big] \end{aligned}$$

$\downarrow$ Trace identities

$$\begin{aligned} = \frac{g^4}{8M^2}\Big[& 4\,Q\cdot p_3\,Q\cdot p_1 - 4Q^2\,p_1\cdot p_3 + 4\,Q\cdot p_1\,Q\cdot p_3 \\ & + M^2\,4\,Q^2 \\ & + AM\,4\,Q\cdot p_1 + AM\,4\,Q\cdot p_3 \\ & + AM\,4\,Q\cdot p_1 + AM\,4\,Q\cdot p_3 \\ & + A^2\,4\,p_1\cdot p_3 + A^2\,4M^2\Big] \end{aligned}$$

(5) MANDELSTAM VARIABLES (see the handout)

$$\begin{aligned} \overline{|\mathcal{M}|^2} = \frac{g^4}{2M^2}\Big[& 2\,Q\cdot p_1\,Q\cdot p_3 - Q^2\,p_1\cdot p_3 + Q^2M^2 \\ & + AM(2\,Q\cdot p_1 + 2\,Q\cdot p_3) \\ & + A^2(p_1\cdot p_3 + M^2)\Big] \end{aligned}$$

Recall,

$$Q^\mu = \frac{p_1^\mu + p_2^\mu}{s-M^2} - \frac{p_1^\mu - p_4^\mu}{u-M^2} \quad \text{and} \quad A = \frac{M}{s-M^2} - \frac{M}{u-M^2}$$

$$p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu$$

$$s = (p_1+p_2)^2 = M^2 + m^2 + 2p_1\cdot p_2$$

$$t = (p_1-p_3)^2 = M^2 + M^2 - 2p_1\cdot p_3$$

$$u = (p_1-p_4)^2 = M^2 + m^2 - 2p_1\cdot p_4 \quad \text{etc}$$

Now we need Mathematica.

Some data on *low energy* π^+ - proton scattering (Homework Problem 6)

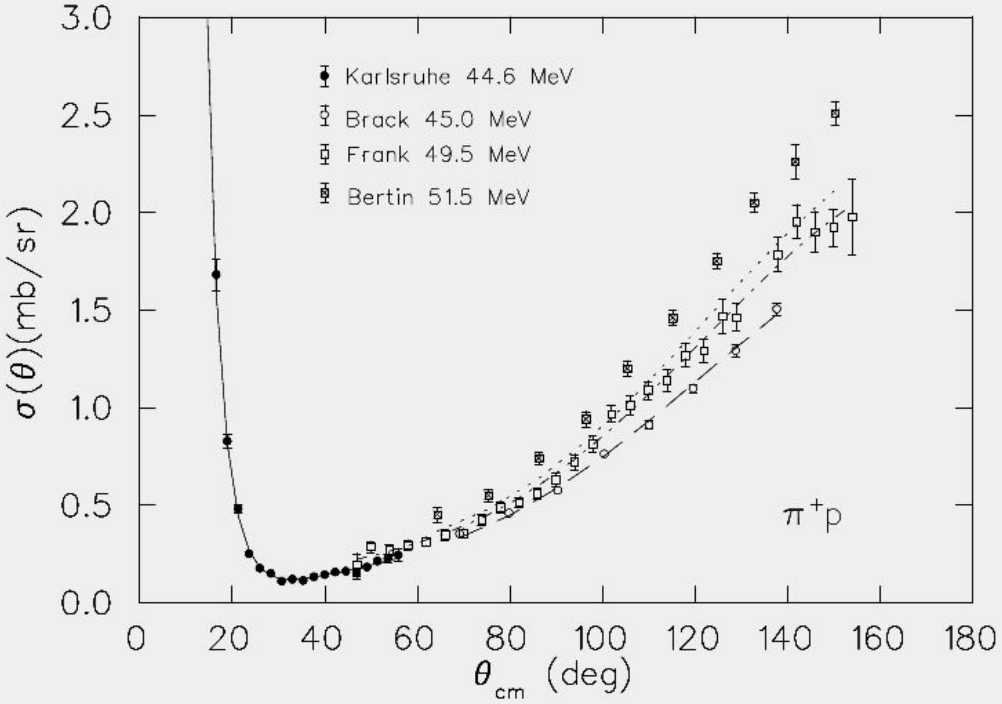
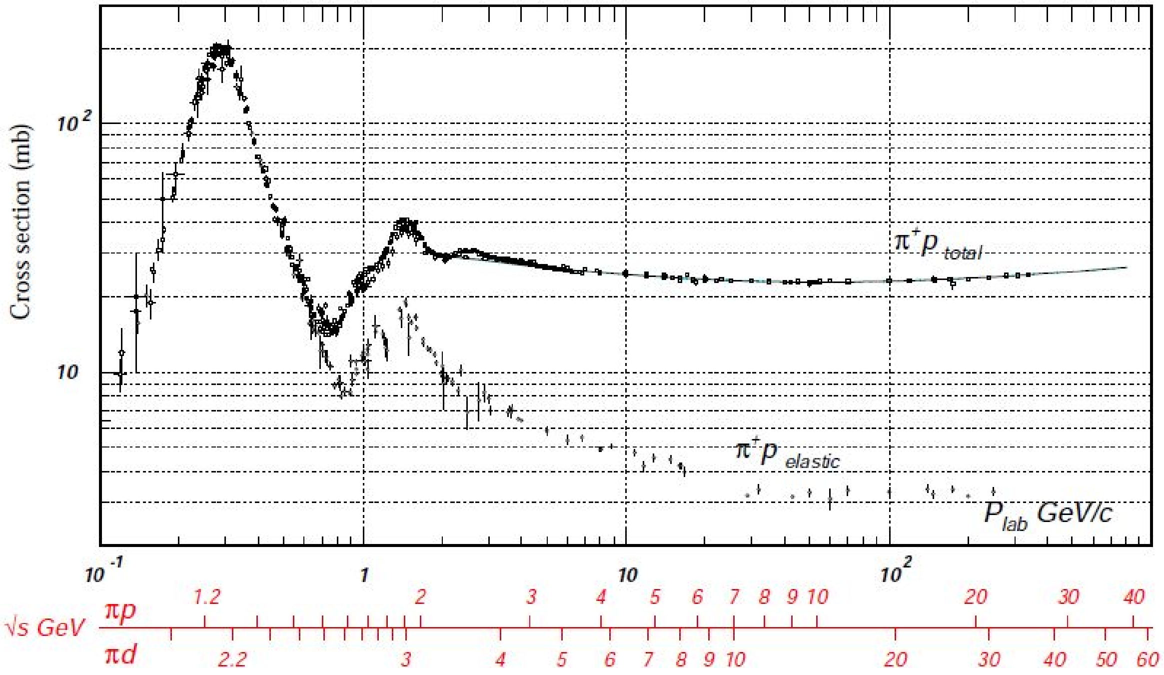


FIG. 1. Cross sections from π^+ proton scattering around 50 MeV. The Bertin data would agree with the prediction given by the dotted curve if they were consistent with the other data sets. The solid, long dash and dash-dot curves come from our fit and correspond to the Karlsruhe, Brack and Frank data sets.

π+ - proton scattering from the Particle Data Group



The cross section has a peak at $\sqrt{s}$ ~ 1200 MeV .

Why?