Chapter 5. Photons: Covariant Theory
5.1. The classical fields ✔
5.2. Covariant quantization ✔
5.3. The photon propagator ✔

Chapter 6. The S-Matrix Expansion
6.1. Natural Dimensions and Units ✔
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Chapter 7. Feynman Diagrams and Rules in QED
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7.3. Feynman rules for QED ✔
7.4. Leptons (SKIPPED)

Chapter 8. QED Processes in Lowest Order
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8.2. Spin sums ✔
8.3. Photon polarization sums
8.4-7. Examples
8.8-9. Bremsstrahlung

# *QED 2 to 2 processes in leading order*

## § Charge = −2e

. $e + e \rightarrow e + e$ (Møller cross section)

## § Charge = −e

. $\gamma + e \rightarrow \gamma + e$ (Compton scattering; Klein-Nishina cross section)

## § Charge = 0

. $e + \bar{e} \rightarrow e + \bar{e}$ (Bhabha cross section)

. $e + \bar{e} \rightarrow \gamma + \gamma$ (pair annihilation)

. $\gamma + \gamma \rightarrow e + \bar{e}$ (pair creation)

. $\gamma + \gamma \rightarrow \gamma + \gamma$ (light by light scattering)

## § Charge = +e

. charge conjugation of −e

## § Charge = +2e

. charge conjugation of −2e

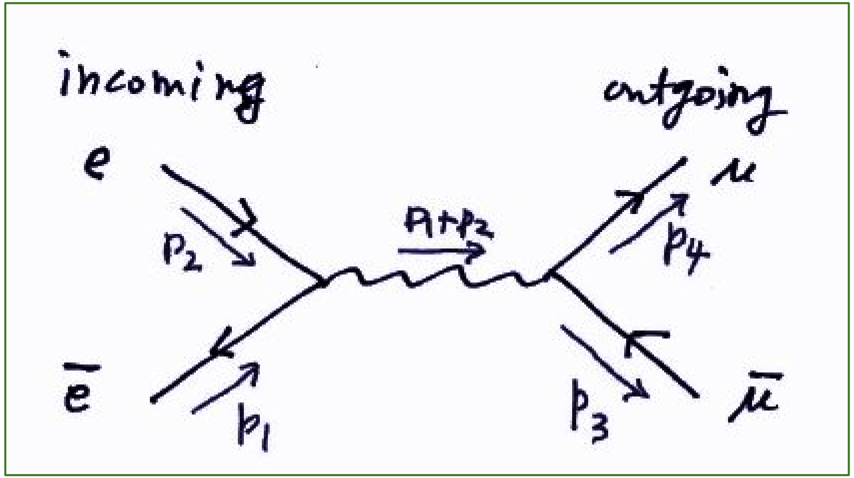
*SECTION 8.4*
*LEPTON PAIR PRODUCTION IN e+e- COLLISIONS*

$$e^+(p_1) + e^-(p_2) \rightarrow \mu^+(p_3) + \mu^-(p_4)$$

$$p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu$$

Masses : m m → M M

There is only one Feynman diagram,




Thus the transition matrix element is

$$\mathcal{M} = i e^2 \left(\bar{u}_4 \gamma_\alpha v_3\right)_M \frac{1}{(p_1+p_2)^2} \left(\bar{v}_1 \gamma^\alpha u_2\right)_m$$

$$\overline{|\mathcal{M}|^2} = \frac{1}{4} \sum_{\lambda_1} \sum_{\lambda_2} \sum_{\lambda_3} \sum_{\lambda_4} |\mathcal{M}|^2$$

$$= \frac{e^4}{4s^2} E^{\alpha\beta} M_{\alpha\beta}$$

$$E^{\alpha\beta} = \sum_{\lambda_1} \sum_{\lambda_2} \bar{v}_1 \gamma^\alpha u_2 \, \bar{u}_2 \gamma^\beta v_1$$

$$= \sum_{\lambda_1} \sum_{\lambda_2} \mathrm{Tr}\left[ v_1 \bar{v}_1 \gamma^\alpha u_2 \bar{u}_2 \gamma^\beta \right]$$

$$E^{\alpha\beta} = \frac{1}{(2m)^2} \mathrm{Tr}\, (\not{p}_1 - m)\, \gamma^\alpha (\not{p}_2 + m)\, \gamma^\beta$$

$$= \frac{1}{(2m)^2} \Big\{ 4 p_1^\alpha p_2^\beta - 4\, p_1 \cdot p_2\, g^{\alpha\beta} + 4 p_1^\beta p_2^\alpha - m^2\, 4 g^{\alpha\beta} \Big\}$$

$$= \frac{4}{(2m)^2} \Big\{ p_1^\alpha p_2^\beta + p_2^\alpha p_1^\beta - g^{\alpha\beta}(p_1 \cdot p_2 + m^2) \Big\}$$

$$M_{\alpha\beta} = \sum_{\lambda_3} \sum_{\lambda_4} \overline{u_4}\, \gamma_\alpha v_3 \overline{v_3}\, \gamma_\beta u_4$$

$$= \frac{4}{(2M)^2} \Big\{ p_{3\alpha} p_{4\beta} + p_{4\alpha} p_{3\beta} - g_{\alpha\beta}( p_3 \cdot p_4 + M^2) \Big]$$

$$E^{\alpha\beta} M_{\alpha\beta} = \frac{1}{m^2 M^2} \Big[ (p_1 p_2 + p_2 p_1)^{\alpha\beta} (p_3 p_4 + p_4 p_3)_{\alpha\beta} - 2 p_3 \cdot p_4 (p_1 \cdot p_2 + m^2) - 2 p_1 \cdot p_2 (p_3 \cdot p_4 + M^2) + 4 (p_1 \cdot p_2 + m^2)(p_3 \cdot p_4 + M^2) \Big]$$

Mandelstam variables

$$s = (p_1 + p_2)^2 = 2m^2 + 2 p_1 \cdot p_2$$
$$= (p_3 + p_4)^2 = 2M^2 + 2 p_3 \cdot p_4$$

$$t = (p_1 - p_3)^2 = m^2 + M^2 - 2 p_1 \cdot p_3$$
$$= (p_4 - p_2)^2 = M^2 + m^2 - 2 p_2 \cdot p_4$$

$$u = (p_1 - p_4)^2 = m^2 + M^2 - 2 p_1 \cdot p_4$$
$$= (p_3 - p_2)^2 = M^2 + m^2 - 2 p_2 \cdot p_3$$

$$E^{\alpha\beta}M_{\alpha\beta} = \frac{1}{m^2M^2}\Big[\, 2\,p_1\cdot p_3\, p_2\cdot p_4 + 2\,p_1\cdot p_4\, p_2\cdot p_3 - 2\,\tfrac{1}{2}(s-2M^2)\,\tfrac{s}{2} - 2\,\tfrac{1}{2}(s-2m^2)\,\tfrac{s}{2} + 4\,\tfrac{s}{2}\cdot\tfrac{s}{2}\,\Big]$$

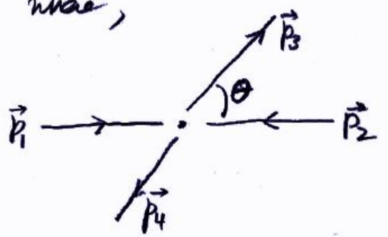
$$= \frac{1}{m^2M^2}\Big[\,\tfrac{1}{2}(m^2+M^2-t)^2 + \tfrac{1}{2}(m^2+M^2-u)^2 + s(m^2+M^2)\,\Big]$$

$$\overline{|\mathcal{M}|^2} = \frac{e^4}{4s^2}\,\frac{1}{m^2M^2}\Big[\quad " \quad\Big]$$

The CoM differential cross section

$$\left(\frac{d\sigma}{d\Omega_3}\right)_{CoM} = \frac{1}{64\pi^2(E_1+E_2)^2}\,\frac{|\vec{p}_3|}{|\vec{p}_1|}\Big[\prod_l (2m_l)\Big]\,\overline{|\mathcal{M}|^2}$$

where,




$$\vec{p}_1 + \vec{p}_2 = 0$$
$$E_1 = E_2 = \sqrt{p_1^2 + m^2}$$
$$\vec{p}_3 + \vec{p}_4 = 0$$
$$E_3 = E_4 = \sqrt{p_3^2 + m^2}$$

$$s = (p_1+p_2)^2 = (E_1+E_2)^2 = 4E_1^2 = 4p_1^2 + 4m^2$$
$$= (p_3+p_4)^2 = (E_3+E_4)^2 = 4E_3^2 = 4p_3^2 + 4M^2$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{CoM} = \frac{1}{64\pi^2 s}\left(\frac{s/4 - M^2}{s/4 - m^2}\right)^{1/2} 16m^2M^2\,|\mathcal{M}|^2$$

$$= \frac{e^4}{4\pi^2 s}\left(\frac{s-4M^2}{s-4m^2}\right)^{1/2}\frac{1}{4s^2}\Big[\,\tfrac{1}{2}(m^2+M^2-t)^2 + \tfrac{1}{2}(m^2+M^2-u)^2 + s(m^2+M^2)\,\Big]$$

Now use Mathematica.

$$\left(\frac{d\sigma}{d\Omega}\right)_{CoM} = \frac{1}{64\pi^2 s}\left(\frac{s/4 - M^2}{s/4 - m^2}\right)^{1/2} 16 m^2 M^2 |\mathcal{M}|^2$$

$$= \frac{e^4}{4\pi^2 s}\left(\frac{s-4M^2}{s-4m^2}\right)^{1/2} \frac{1}{4s^2}$$

$$\left[\tfrac{1}{2}(m^2+M^2-t)^2 + \tfrac{1}{2}(m^2+M^2-u)^2 + s(m^2+M^2)\right]$$

**$e+\ e- \longrightarrow \mu+\ \mu-$**

```
In[169]:= Remove["Global`*"]

In[302]:= Φ = (1 / 2) * (m^2 + M^2 - t) ^2 + (1 / 2) * (m^2 + M^2 - u) ^2 + s * (m^2 + M^2)
          (* CoM replacements *)
          R1 = {s → 4 * E1^2};
          R2 = {t → m^2 + M^2 - 2 * E1^2 + 2 * p1 * p3 * Cos[θ]};
          R3 = {u → m^2 + M^2 - 2 * E1^2 - 2 * p1 * p3 * Cos[θ]};
```

Out[302]= $\left(m^2+M^2\right) s + \frac{1}{2}\left(m^2+M^2-t\right)^2 + \frac{1}{2}\left(m^2+M^2-u\right)^2$

```
In[306]:= W = Φ /. Join[R1, R2, R3];
          W = Expand[W / 4 / s^3 /. R1];
          W = Expand[W /. {p1^2 → E1^2 - m^2, p3^2 → E1^2 - M^2}]
```

Out[308]= $\frac{1}{64\,\mathrm{E1}^2} + \frac{m^2}{64\,\mathrm{E1}^4} + \frac{M^2}{64\,\mathrm{E1}^4} + \frac{\mathrm{Cos}[\theta]^2}{64\,\mathrm{E1}^2} - \frac{m^2\,\mathrm{Cos}[\theta]^2}{64\,\mathrm{E1}^4} - \frac{M^2\,\mathrm{Cos}[\theta]^2}{64\,\mathrm{E1}^4} + \frac{m^2 M^2\,\mathrm{Cos}[\theta]^2}{64\,\mathrm{E1}^6}$

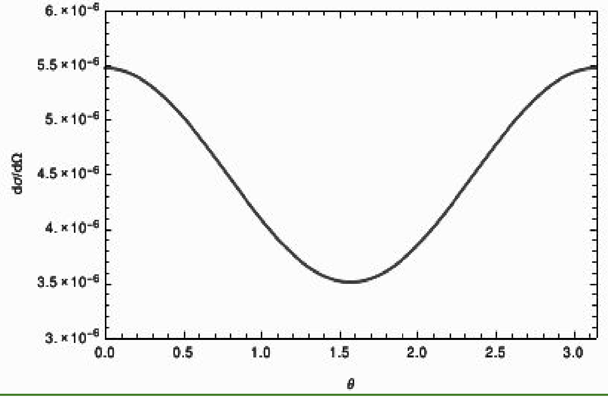
```
In[321]:= const = (4 * Pi * alpha) ^2 / (4 * Pi ^2) * (hbc) ^2;
          constV = const /. {alpha -> 1 / 137., hbc -> 197.} ;
          CrSection = constV * W * Sqrt[(E1 ^ 2 - M ^ 2) / (E1 ^ 2 - m ^ 2)]
```

Out[323]=

$$8.27087 \sqrt{\frac{\mathrm{E1}^2-\mathrm{M}^2}{\mathrm{E1}^2-\mathrm{m}^2}} \left(\frac{1}{64\,\mathrm{E1}^2}+\frac{\mathrm{m}^2}{64\,\mathrm{E1}^4}+\frac{\mathrm{M}^2}{64\,\mathrm{E1}^4}+\frac{\mathrm{Cos}[\theta]^2}{64\,\mathrm{E1}^2}-\frac{\mathrm{m}^2\,\mathrm{Cos}[\theta]^2}{64\,\mathrm{E1}^4}-\frac{\mathrm{M}^2\,\mathrm{Cos}[\theta]^2}{64\,\mathrm{E1}^4}+\frac{\mathrm{m}^2\,\mathrm{M}^2\,\mathrm{Cos}[\theta]^2}{64\,\mathrm{E1}^6}\right)$$

```
In[325]:= Plot[
           CrSection /. {m → 0.511, M → 106, E1 → 200},
           {θ, 0, Pi}, PlotRange → {{0, Pi}, {3*^-6, 6*^-6}},
           Frame → True, FrameLabel → {"θ", "dσ/dΩ"}]
```

Out[325]=

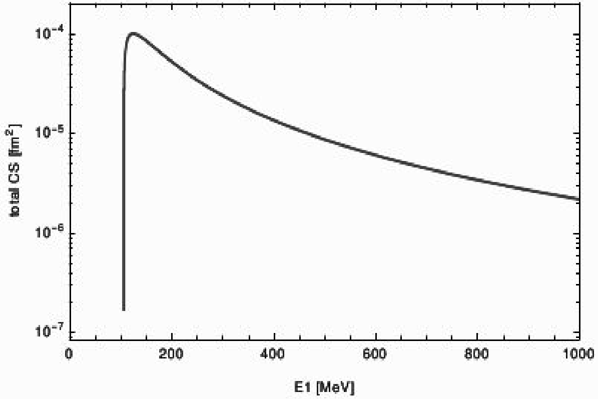


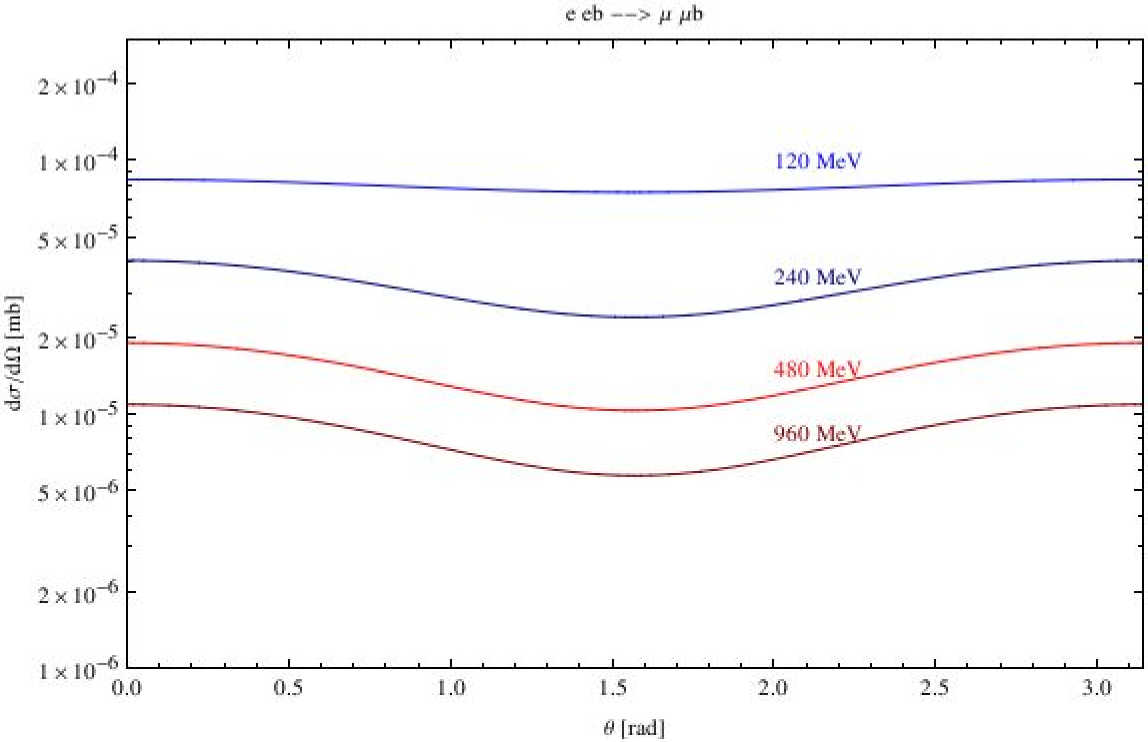

```
In[334]:= totalCS = Integrate[2 * Pi * Sin[θ] * CrSection, {θ, 0, Pi}];
          TCS[En_] := totalCS /. {m -> 0.511, M → 106, E1 → En}
          TCS[Ee]
          LogPlot[TCS[Ee], {Ee, 106, 1000},
           PlotRange → {{0, 1000}, {0, 1.2*^-4}},
           Frame → True,
           FrameLabel → {"E1 [MeV]", "total CS [fm²]"}]
```

Out[336]=

$$\frac{0.811991 \sqrt{\frac{-11236+\mathrm{Ee}^2}{-0.261121+\mathrm{Ee}^2}} \left(1955.97+14981.7\,\mathrm{Ee}^2+2.66667\,\mathrm{Ee}^4\right)}{\mathrm{Ee}^6}$$

Out[337]=






In the high energy limit, i.e.,

$\sqrt{s} >> M$

. $\sigma \sim \text{const} / s$

The constant is $\propto e_e^2\, e_\mu^2$ ,

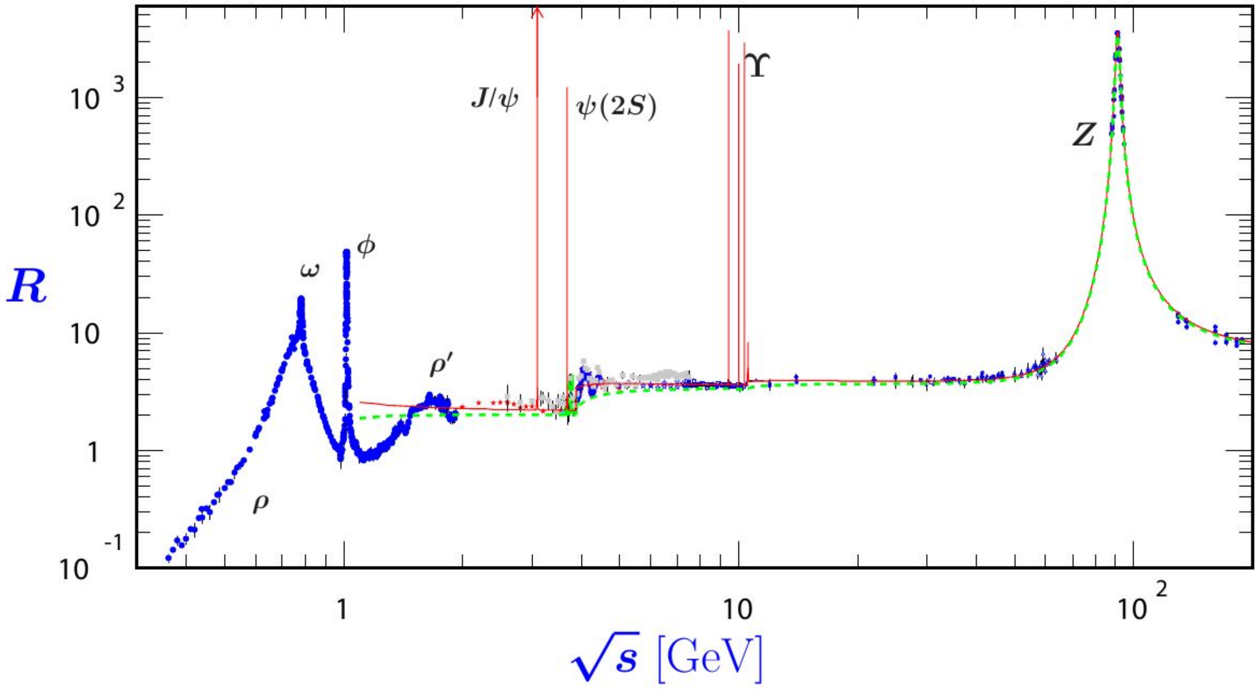
and independent of M .

# Electron - positron annihilation to hadrons

These were important experiments in the history of high-energy physics.
From the Particle Data Group, the figure shows the ratio
R = σ ( e ebar → hadrons ) / σ ( e ebar → μ μbar) .

***The underlying process in hadron production is e + ebar → q + qbar.***

Neglecting QCD interactions we would just have R = constant (green = naive quark model; red = QCD 3rd order).



R depends on √{s} because of 2 effects:

- thresholds
- resonances

Between thresholds we do indeed have

R ≈ constant.

Naively estimate the constant above the b-quark threshold,

$$R = \Sigma e_q^2 / e^2$$
$$= ( 1/9 + 4/9 + 1/9 + 4/9 + 1/9) \times 3$$
$$= 11 /3 = 3.67 .$$

This is a great success of QCD.

Historically, measurements of R provided strong evidence for the quark model and QCD.