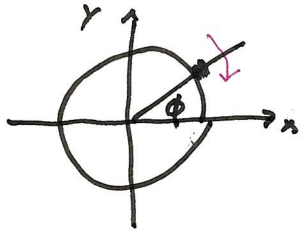
Solutions for Exam 1

(1)



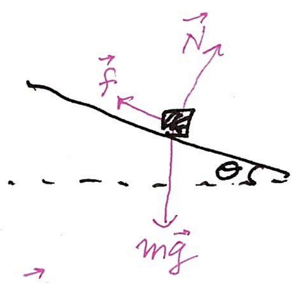
(a) $\vec{r} = \hat{i} R\cos\phi + \hat{j} R\sin\phi$; $\phi = \frac{\pi}{2} - \omega t$

$$\vec{v} = \dot{\vec{r}} = -R\omega(-\hat{i}\sin\phi + \hat{j}\cos\phi)$$

$$\vec{a} = \dot{\vec{v}} = R\omega^2(-\hat{i}\cos\phi - \hat{j}\sin\phi)$$

(b) $\vec{F} = +m\vec{a} = -mR\omega^2\,\hat{i}$ at $\phi = 0$

(2) (a)

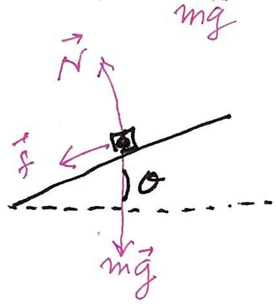


At terminal velocity, $\vec{F} = 0$.

$\therefore\ mg\sin\theta - cv_T^2 = 0$

$$\boxed{v_T = \sqrt{\frac{mg}{c}\sin\theta}}$$

(b)



Going uphill, $m\dot{\vec{v}} = -mg\sin\theta - cv^2$ (tangent component)

$$m\frac{dv}{dt} = -c(v_T^2 + v^2)$$

$$\int_{v_0}^{v} \frac{dv'}{v_T^2 + v'^2} = -\frac{c}{m}\int_0^t dt' = -\frac{ct}{m}$$

$v_0 = v_T$

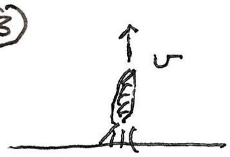
$$= -\int_{v}^{v_T} \frac{dv'}{v_T^2 + v'^2}$$

$$= -\frac{1}{v_T}\int_{v/v_T}^{1} \frac{du}{1+u^2}$$

$$= -\frac{1}{v_T}\frac{\pi}{4} \quad \text{when } v = 0,$$

$$\underline{t_{\text{max height}} = \frac{m}{cv_T}\frac{\pi}{4}}$$

(3)



(a) $m\frac{dv}{dt} = Kv_{ex} - mg$ and $\frac{dm}{dt} = -K$

$$v_{ex} = \frac{\text{thrust}}{K} = \frac{2.5\times10^5\ \text{N}}{(11.5\times10^3\ \text{kg})/(65\ \text{s})} = 1413\ \frac{\text{m}}{\text{s}}$$

(b) $m\,dv = -v_{ex}\,dm - g\,m\,dt$

$$\Delta v = -v_{ex}\,\Delta(\ln m) - g\,\Delta t$$

$$\Delta v = -v_{ex}\ln\frac{m_b}{m_0} - gt_b = 2932\ \text{m/s}$$