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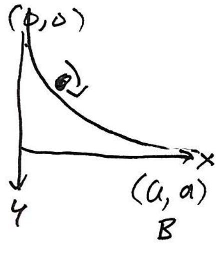
EXTRA CREDIT FOR EXAM 2 – due in class Friday November 10

To get extra credit, write solutions to the Exam 2 questions. Write the solutions in the boxes on this page, and write clearly. *NEATNESS COUNTS !*

QUESTION 1A. By conservation of energy, $\frac12 mv^2 + \frac12 I\omega^2 - mgy = 0$. Also, for rolling motion, $v = R\omega$. Now calculate v_B = the speed at B.



$$\frac12 mv^2 + \frac12\left(\frac25 MR^2\right)\omega^2 = mgy$$

$$\left(\frac12+\frac15\right)v^2 = gy$$

$$v_B = \sqrt{\frac{10}{7}gy}$$

QUESTION 1B. By conservation of energy, $(7/10)\,mv^2 - mgy = 0$. Now, $v^2 = \dot x^2+\dot y^2$, and x and y are related by $x^2+y^2 = a^2$. Write $\dot y = G(y)$. Then the time to roll down the track is

$t = \int_{\text{init.y}}^{\text{final y}} dy/G(y)$. Calculate t in the form #.### SQRT(a/g).

$$v^2 = \dot x^2 + \dot y^2 \quad\text{and}\quad x^2+y^2 = a^2.$$

$$\dot x = -\frac{y}{x}\dot y \Rightarrow v^2 = \frac{y^2}{x^2}\dot y^2 + \dot y^2 = \frac{a^2-x^2}{x^2}\dot y^2 + \dot y^2$$

$$\dot y^2 = v^2\frac{a^2-y^2}{a^2} = \frac{10}{7}gy\left(1-\frac{y^2}{a^2}\right) = G^2(y)$$

$$t = \int_0^a \frac{dy}{\sqrt{\frac{10}{7}gy(1-y^2/a^2)}} = \sqrt{\frac{7a}{10g}}\int_0^1\frac{du}{\sqrt{u(1-u^3)}}$$

$$= 2.19\sqrt{a/g}$$

QUESTION 2A. If the particle remains at rest, then the force on the particle is 0.

So, the relevant equation is **F** = 0.

Now, calculate r_0.

$$\vec F = -\nabla U \quad\text{and}\quad U = \frac{a}{r} + br^2$$

$$F_r = -\frac{dU}{dr} = \frac{a}{r^2} + 2br$$

$$F_r = 0 \Rightarrow r_0 = \left(\frac{a}{2b}\right)^{1/3} \text{ (equilibrium)}$$

QUESTION 2B. Consider the initial conditions $(x_0, y_0) = (r_0, 0)$ and $(v_x(0), v_y(0)) = (0, v_0)$. The conserved quantities are ***angular momentum*** and ***energy***. Write equations, using polar coordinates, for the two constants, like this:

$\dot r = f1$; $\dot\varphi = f2$; simplify the functions f1 and f2.

$$\vec L = m\vec r\times\vec v = m r_0 v_0\,\hat e_z$$

$$= mr\hat r\times(\dot r\hat r + r\dot\phi\hat\phi) = mr^2\dot\phi\,\hat e_z$$

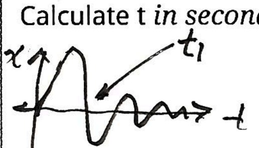
$$\dot\phi = r_0 v_0/r^2$$

$$E = \frac12 mv^2 + U(r) = \frac12 m v_0^2 + U(r_0)$$

$$\dot r^2 = -r^2\dot\phi^2 + \frac{2}{m}\left[U(r_0) - U(r)\right]$$

QUESTION 3A. This is an underdamped oscillator. Sketch a graph of x versus t. The solution, consistent with initial conditions, has the form $x(t) = \exp(-\beta t)\,A\sin(\omega t)$. You figure out A and ω.

To calculate the time when x = 0, solve $\sin(\omega t) = 0$. Calculate t *in seconds.*



Try $x = e^{pt}$

$$\ddot x + 2\beta\dot x + \omega_0^2 x = 0$$

$$p^2 + 2\beta p + \omega_0^2 = 0$$

$$p = -\beta \pm\sqrt{\beta^2-\omega_0^2} = -\beta \pm i\omega$$

$$\omega = \sqrt{\omega_0^2-\beta^2} = \frac{\sqrt3}{2}\omega_0$$

$$\omega t_1 = \pi \Rightarrow t_1 = \frac{\pi}{\omega} = \frac{2\pi}{\sqrt3}\frac{1}{\omega_0} = 3.63 \text{ sec.}$$

QUESTION 3B. Calculate the velocity at the time in part (A), x(t), and express the answer in *meters per second.*

$$v = \dot x = -\beta e^{-\beta t}A\sin\omega t + e^{-\beta t}A\omega\cos(\omega t)$$

$$v(0) = v_0 = A\omega$$

In part a, $t = t_1 = \pi/\omega \Rightarrow$

$$v(t_1) = e^{-\beta t_1}A\omega(-1) = -v_0 e^{-\pi\beta/\omega}$$

$$= -v_0 e^{-\pi\frac{\beta}{\omega_0}\frac{2}{\sqrt3}} = -v_0 e^{-\pi/\sqrt3}$$

$$= 0.054 \text{ m/s}$$