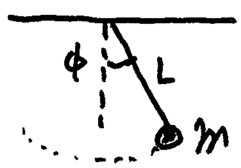
(1)

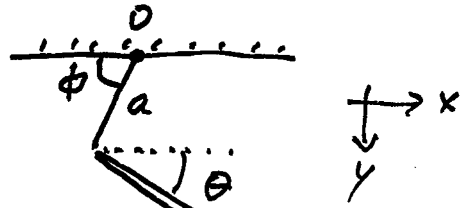


$T = \frac{1}{2} m L^2 \dot{\phi}^2$ and $V = -mgL\cos\phi$

$\mathcal{L} = T - V = \frac{1}{2} m L^2 \dot{\phi}^2 + mgL\cos\phi$ (3)

$\frac{\partial \mathcal{L}}{\partial \phi} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}}\right) \Rightarrow \ddot{\phi} = -\frac{g}{L}\sin\phi$ (3) [6]

(2) (A)



(3)

(B) $T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) + \frac{1}{2} I \dot{\theta}^2$

where

$x = -a\cos\phi + \frac{1}{2} L\cos\theta$

$y = a\sin\phi + \frac{1}{2} L\sin\theta$

$V = -mgy$

$\mathcal{L} = T - V = \frac{1}{2} m \left[a^2 \dot{\phi}^2 + \frac{1}{4} L^2 \dot{\theta}^2 + aL\dot{\theta}\dot{\phi}\cos(\theta + \phi)\right]$ [6]

$+ \frac{1}{2} I \dot{\theta}^2 + mg\left[a\sin\phi + \frac{1}{2} L\sin\theta\right]$ (3)

(3) (A) $\mathcal{L} = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - V(x, y, z)$ (3) [6]

(B) $\frac{\partial \mathcal{L}}{\partial x} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) \Rightarrow m\ddot{x} = -\frac{\partial V}{\partial x}$ (3)

& y and z similarly

(4) $J = \int_0^{10} x\sqrt{1 - y'^2}\, dx \Rightarrow f(y, y', x) = x\sqrt{1 - y'^2}$ (2)

Euler's equation $\frac{\partial f}{\partial y} = \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)$.

$\frac{\partial f}{\partial y} = 0 \Rightarrow \frac{\partial f}{\partial y'} = C$, (2) a constant. ; $\frac{\partial f}{\partial y'} = \frac{-xy'}{\sqrt{1 - y'^2}} = C$

$\therefore \quad y'^2 = \frac{C^2}{x^2}(1 - y'^2)$ so $y' = \frac{C}{\sqrt{C^2 + x^2}}$. [6]

$y(x) = \int_0^x \frac{C\, d\hat{x}}{\sqrt{C^2 + \hat{x}^2}} = C\,\mathrm{ArcSinh}(x/C)$. (2)