Name GRADING KEY

Homework Assignment #2 ; due Friday, September 15

Cover sheet: Staple this page in front of your solutions.

INSTRUCTIONS: Write the requested ***answers*** to the problems (without calculations) on this page; and write your ***solutions*** to the problems (your work, written clearly; no scratch paper) on your own paper.

[6] Problem 1.35.* *Answer: The distance where the golf ball hits the ground is ...*

$xF = 2 (v_0^2 /g) \sin\theta \cos\theta$

1 point

[7] Problem 1.38.* *Answer: The distance from the puck to O, when the puck returns to floor level, is ...*

$xF = 2 v_{0x} v_{0y} / (g \sin\theta)$

1 point

[8] Problem 1.39.** *(There is no answer to report here.)* 2 points

*[9] Problem 1.44.** *(There is no answer to report here.)* 1 point

*[10] Problem 1.51.***[computer]*

(Refer to the Mathematica sample program to get started.)

Hand in the Mathematica program and the plots.

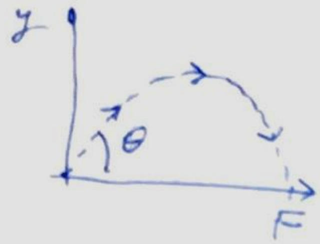
Answer here : Comment on your two graphs ...

THE PERIOD OF OSCILLATION OF THE EXACT SOLUTION IS GREATER THAN THAT OF THE HARMONIC APPROXIMATION.

5 points

HOMEWORK ASSIGNMENT 2

[6] PROBLEM 1.35 — A GOLF BALL ...



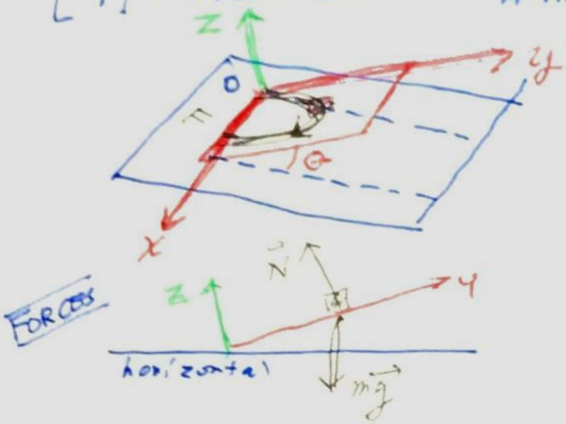
$x(t) = v_0 \cos\theta \; t$

$y(t) = v_0 \sin\theta \; t - \frac{1}{2} g t^2$

At the final point (F) $y = 0$.

- Final time $t_F = \frac{2 v_0}{g} \sin\theta$
- Final distance $x_F = v_0 \cos\theta \; t_F = \frac{2 v_0^2}{g} \sin\theta \cos\theta$

[7] PROBLEM 1.38 — A MASS SLIDES ON A RAMP ...



Initial position $(x_0, y_0, z_0) = (0, 0, 0)$

Initial velocity $= (v_{0x}, v_{0y}, v_{0z}) = (v_{0x}, v_{0y}, 0)$

Force $\vec{F} = m\vec{g} + \vec{N}$

$= -mg \sin\theta \; \hat{e}_y + (N - mg\cos\theta)\, \hat{e}_z$

<u>Equations</u> of <u>motion</u> $\ddot{x} = 0$ so $x = v_{0x} t$

$\ddot{y} = -g \sin\theta$ so $y = v_{0y} t - \frac{1}{2} g \sin\theta \; t^2$

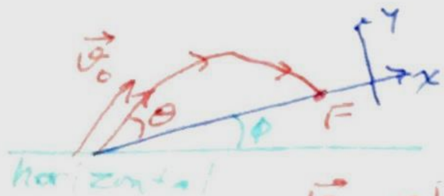
(We don't need the equation $\ddot{z} = F_z$; that determines N because $z(t) = 0$.)

- Time to travel to F $= t_F = \frac{2 v_{0y}}{g \sin\theta}$

  $y = 0$

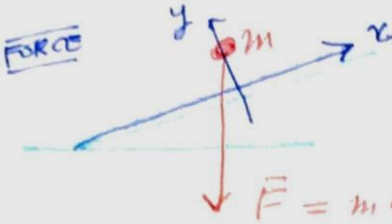
- Distance from O to F $= x_F = v_{0x} t_F = \frac{2 v_{0x} v_{0y}}{g \sin\theta}$

[8] PROBLEM 1.38 THROW A BALL UP A SLOPE...



$$\vec{v}_0 = v_0\left[\hat{e}_x \cos\theta + \hat{e}_y \sin\theta\right]$$

FORCE



$$\vec{F} = m\vec{g} = mg\left[-\hat{e}_x \sin\phi - \hat{e}_y \cos\phi\right]$$

The equations of motion, and solutions, are

$$\ddot{x} = -g\sin\phi \quad \Rightarrow \quad x(t) = v_0\cos\theta\, t - \tfrac{1}{2} g \sin\phi\, t^2$$

$$\ddot{y} = -g\cos\phi \quad \Rightarrow \quad y(t) = v_0\sin\theta\, t - \tfrac{1}{2} g\cos\phi\, t^2$$

- Calculate the range, R

At F, $y_F = 0$ and $x_F = R$. Therefore $t_F = \dfrac{2v_0\sin\theta}{g\cos\phi}$

and $R = v_0\cos\theta \dfrac{2v_0\sin\theta}{g\cos\phi} - \dfrac{1}{2} g\sin\phi\left(\dfrac{2v_0\sin\theta}{g\cos\phi}\right)^2$

After some algebra, $R = \dfrac{2v_0^2}{g}\,\dfrac{\sin\theta\cos(\theta+\phi)}{\cos^2\phi}$

- Calculate the maximum range as a function of θ

$$\frac{dR}{d\theta} = 0 \quad ; \quad \frac{dR}{d\theta} = \frac{2v_0^2}{g\cos^2\phi}\cos(2\theta+\phi)$$

So, $2\theta + \phi = \pi/2$.

Then
$$R_{max} = \frac{2v_0^2}{g\cos^2\phi}\sin\left(\frac{\pi}{4} - \frac{\phi}{2}\right)\cos\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$$

$$= \frac{2v_0^2}{g\cos^2\phi}\left\{\frac{1}{\sqrt{2}}\left[\cos\tfrac{\phi}{2} - \sin\tfrac{\phi}{2}\right]\frac{1}{\sqrt{2}}\left[\cos\tfrac{\phi}{2} - \sin\tfrac{\phi}{2}\right]\right\}$$

$$= \frac{v_0^2}{g\cos^2\phi}\left\{1 - 2\cos\tfrac{\phi}{2}\sin\tfrac{\phi}{2}\right\} = \frac{v_0^2}{g\cos^2\phi}(1 - \sin\phi)$$

$$R_{max} = \frac{v_0^2}{g\left[1 + \sin\phi\right]}$$

[9] PROBLEM 1.44 — HARMONIC OSCILLATIONS...

Let $\Phi(t) = A \sin\omega t + B \cos\omega t$

Derivatives

$$\dot{\Phi} = A\omega \cos\omega t + B(-\omega)\sin\omega t$$

$$\ddot{\Phi} = A(-\omega^2)\sin\omega t + B(-\omega^2)\cos\omega t$$

Thus

$$\ddot{\Phi} = -\omega^2 (A\sin\omega t + B\cos\omega t) = -\omega^2 \Phi.$$

The function Φ depends on two constants, A and B;
so it is the general solution of the differential equation
$\ddot{\Phi} = -\omega^2 \Phi$.

[10] PROBLEM 1.51

This is a computer problem.
Hand in the program (2 points) the figures (2 points).
and write the comment on the cover page (1 point)