Name_______GRADING KEY___________________________

Homework Assignment #3 due in class Wednesday, September 20

Cover sheet : Staple this page in front of your solutions.

INSTRUCTIONS : Write the requested *answers* (without calculations) on this page;
write the detailed *solutions* (your work written clearly; no scratch paper) on your own paper.

[11] Problem 2.2.* *Answer: the value of β is*

1.6×10^{-4} Ns/m^2 **1 point**

[12] Problem 2.3.* *Answer: the Reynolds number (part b) is*

0.0108 **1 point**

[13] Problem 2.10.** *Answer: the terminal speed is*

1.18×10^{-3} m/s **2 points**

[14] Problem 2.18.* *Answer: the Taylor series for ln(1+δ) is*

$\delta - \frac{1}{2}\delta^2 + \frac{1}{3}\delta^3$ **1 point**

[15] Problem 2.26.* *Answer: the time to slow to 15 m/s is*

6.33 s **1 point**

[16] The terminal velocity of a drop of water (diameter = D, mass = m) is the velocity such that F = mg − bv − cv^2 = 0. The parameter values for air at STP are

$b = (1.6 \times 10^{-4}\ Ns/m^2)\ D$ and $c = (0.25\ Ns^2/m^4)\ D^2$;

also, $m = (0.52 \times 10^3\ kg/m^3)\ D^3$.

Determine v_{ter} as a function of D. Plot an accurate graph of v_{ter} versus D, from D = 0.1 mm to 3 mm. (Use a computer to make the plot.) The result shows why water droplets in a cloud do not fall as rain. Hand in the plot.

Answer here: Explain why water droplets in a cloud do not fall as rain. ***(2 + 1 points)***

Water droplets in a cloud are very small, so their terminal velocity is less than the velocities of updrafts in the cloud. The water droplets are constantly carried upward by updrafts.

[17] Consider these equations for a baseball fly ball near the surface of the Earth:

$m x'' = -c\,(v_x^2 + v_y^2)\cos\theta$; $m y'' = -mg - c\,(v_x^2+v_y^2)\sin\theta$; $\tan\theta = v_y / v_x$.

[Initial values: $(x_0,y_0) = (1,0)$ m and $(v_{0x}, v_{0y}) = (30,30)$ m/s ; terminal speed = 40 m/s.]

Hand in an accurate plot of the trajectory, i.e., y versus x. (Use a computer.) ***(4 points)***

[redacted]

Homework Assignment 3

[11] Problem 2.2 STOKES'S LAW

Stokes's law for viscous drag: $f_{lin} = 3\pi\eta D v$

Thus $f_{lin} = bv$ where $b = 3\pi\eta D = \beta D$

where $\beta = 3\pi\eta$.

For air, $\eta = 1.7\times10^{-5}\ Ns/m^2$ so $\beta = 1.6\times10^{-4}\ Ns/m^2$

[12] Problem 2.3 REYNOLDS's NUMBER

(a) Given $f_{lin} = 3\pi\eta D v$ and $f_{quad} = \underset{(\kappa \approx 0.25)}{\kappa} \rho A v^2$.

The Reynolds's number is defined by $Re = \dfrac{D v \rho}{\eta}$.

The ratio f_{quad}/f_{lin} is

$$\frac{f_{quad}}{f_{linear}} = \frac{\kappa \rho \pi (D/2)^2 v^2}{3\pi\eta D v} = \frac{\kappa}{12} Re$$

$$\text{ratio} = \frac{Re}{48} \quad \text{for } \kappa = 0.25.$$

(b) For a steel ball bearing in glycerine, with the given parameter values,

$$Re = \frac{2mm \quad 5cm/s \quad 1.3\times10^{-3}\ kg/cm^3}{12\ Ns/m^2}$$

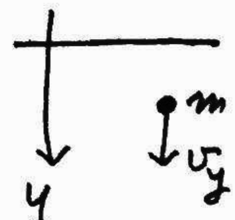
$$Re = 0.0108$$

Since Re is small, the linear resistive force is dominant.

[13] Problem 2.10 A steel ball bearing sinking in glycerine

Use linear resistance, $f_{lin} = 3\pi\eta D v$.

(a) Characteristic time and terminal velocity



$$m\frac{dv}{dt} = F_y$$

$$= mg - \underbrace{\rho V g}_{\text{BUOYANCY FORCE}} - bv$$

Solution by separation of variables

$$\frac{dv}{dt} = g - \frac{\rho V g}{m} - \frac{b}{m}v$$

$$= g\left[1 - \rho/\rho_{steel}\right] - \frac{b}{m}v$$

$$= g' - \frac{b}{m}v \quad \leftarrow \text{(already solved in the book)}$$

Thus $v_{terminal} = \frac{mg'}{b}$ and $\tau = \frac{m}{b}$.

(b) Numerical $m = 3.27\times10^{-5}$ kg

$b = 3\pi\eta D =$

$\tau = 1.44\times10^{-4}$ s; $v_{ter} = 1.18\times10^{-3}\ \frac{m}{s}$

95% of terminal speed

$$v = v_{ter}(1 - e^{-t/\tau}) = 0.95\, v_{ter}$$

$$t_{95} = 4.33\times10^{-4}\,s$$

Reynolds's number at $v = v_{ter}$

$$\frac{f_{quad}}{f_{lin}} = \frac{Dv\rho}{48\eta} = 5\times10^{-6}$$ Very small

[~14] Problem 2.18 TAYLOR'S THEOREM

$$f(x+\delta) = f(x) + f'(x)\,\delta + f''(x)\,\frac{\delta^3}{2!} + f'''(x)\,\frac{\delta^3}{3!} \;\ldots$$

(a) $\ln(1+\delta)$; let $f = \ln$ and $x = 1$.

$$\ln(1+\delta) = \delta - \frac{1}{2}\delta^2 + \frac{1}{3}\delta^3 + \ldots$$

(b) $\cos\delta$; let $f = \cos$ and $x = 0$.

$$\cos\delta = 1 - \frac{1}{2}\delta^2 + \frac{1}{24}\delta^4 + \ldots$$

(c) $\sin\delta$; let $f = \sin$ and $x = 0$.

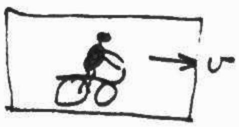
$$\sin\delta = \delta - \frac{1}{6}\delta^3 + \frac{1}{120}\delta^5 + \ldots$$

(d) $$\exp\delta = 1 + \delta + \frac{1}{2}\delta^2 + \frac{1}{6}\delta^3 + \ldots$$

$$+ \ldots + \frac{\delta^n}{n!} + \ldots$$

[redacted]

[15] Problem 2.26 A BICYCLE RIDER, COASTING TO A STOP



horizontal motion with air resistance

Initial velocity $v_0 = 20\ \text{m/s}$; mass = 80 kg.

Use quadratic air resistance,

$$f_{quad} = c v^2 \text{ where } c = 0.20\ \frac{N s^2}{m^2}$$

The characteristic time is $\tau = \frac{M}{c v_0} = 20$ seconds.

The velocity as a function of time is

$$v(t) = \frac{v_0}{1 + t/\tau} ;$$

therefore $t = \tau \left(\frac{v_0}{v} - 1 \right)$

v	t
20 m/s	0
15 m/s	6.33 s
10 m/s	20 s
5 m/s	60 s

[16] (4 points)

The terminal velocity is given by Fnet = 0.

$$mg - b\,v_t - c\,v_t^{\,2} = 0.$$

Constants b and c depend on diameter D.

Solve for v_t and plot a graph of v_t versus D.

Small cloud droplets have terminal speed less than air currents so they just are carried around by the air currents, not falling from gravity.

[17] (4 points)

Use the computer program from the lecture of September 15.

The ball should travel about 100 meters.