Name______*SOLUTION KEY*__________________ **4.1**

Homework Assignment #4 due in class Wednesday, September 27

Cover sheet : Staple this page in front of your solutions.

INSTRUCTIONS: Write the requested *answers* (without calculations) on this page; write the detailed *solutions* (your work written clearly) on your own paper.

[17] Problem 2.23.* *Answer: the terminal speed for the parachutist is ...*

107 m/s */1 point/*

[18] Problem 2.31.** *Answer: the time for the basketball to fall to the ground from a 30 m tower is*

2.78 s */2 points/*

[19] Problem 2.41.** *Answer: the calculated value of* y_{max} *is ...*

20.4 m */2 points/*

[20] Problem 2.53.* *Answer: describe the particle's motion ...*

The trajectory is a helix with increasing pitch. */1 point/*

[21] Problem 2.43.*** [computer]

Hand in the computer program (2 points) , calculations, and plots (2 points).

Answer here: the horizontal distance where the ball hits the ground is ...

17.71 m */5 points total/*

[22] A mathematical exercise. Define $f_n(x) = (1 + x/n)^n$.

(A) What is the limit of $f_n(x)$ as $n \to \infty$. *Give a proof of the result.*

(B) Hand in a graph that shows, on one graph, $f_1(x)$, $f_2(x)$, $f_5(x)$ and $f_\infty(x)$ versus x for x from –2 to 2. (Use a computer.)

Answer here: what is $f_\infty(x)$ ***?*** */5 points total/*

exp(x)

⇧

4.2

Homework Assignment #4

17 Problem 2.23

Terminal speeds of falling objects ...

For quadratic air resistance, $m v' = mg - c v^2$. $(v' = dv/dt)$

The terminal speed (F = 0) is

$$v_{ter} = \sqrt{mg/c}\,.$$

Here $c = \gamma D^2$ where $\gamma = 0.25\ \mathrm{Ns^2/m^2}$ and D = diameter.

Also, $m = \rho V = \rho (\pi/6) D^3$.

case	D	ρ	v_{ter}
(a)	3 mm	8×10^3 kg/m^3	22.2 m/s
(b)	0.12 m	8×10^3 kg/m^3	140.4 m/s
(c)	0.56 m	1×10^3 kg/m^3	107.0 m/s

18 Problem 2.31

A basketball falling through air ...

Parameters: $m = 0.6$ kg ; $D = 0.24$ m

(a) Terminal speed

$v_{ter} = \sqrt{mg/c}$ where $c = (0.25\ Ns^2/m^2)\ D^2$

$v_{ter} = 20.2$ m/s

(b) It falls distance 30 m, from rest. Calculate t_{final} and v_{final} .

- The distance is $y = (v_{ter}^{\ 2}/g)\ \ln[\cosh(gt/v_{ter})]$

  Thus $t = (v_{ter}/g)\ \mathrm{arccosh}[\exp(gy/v_{ter}^{\ 2})]$

  $y = 30$ m implies $t = 2.78$ s. So t_{final} = **2.78 s**

- The velocity is $v = v_{ter} \tanh(gt/v_{ter})$

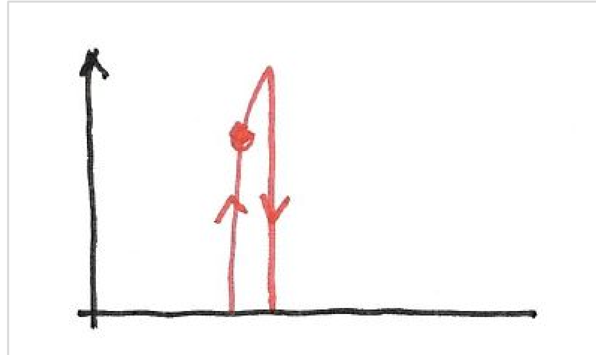
  so $v_{final} = y(t_{final})$ = **17.6 m/s.**

Compare in vacuum,

$t_{final} = \sqrt{2y/g}$ = **2.47 s** and $v_{final} = g\, t_{final}$ = **24.2 m/s.**

19 Problem 2.41

A baseball is thrown upward ...



$mv' = -mg - c\,v^2 \qquad (v' = dv/dt)$

Terminal speed $\qquad v_{ter} = \sqrt{mg/c}$

so $\qquad c = mg / v_{ter}^2$

$dv/dt = -g - g\,v^2/v_{ter}^2$

Now calculate v as a function of height;

- Separation of variables

$$\frac{dv}{dt} = \frac{dv}{dy}\frac{dy}{dt} = \frac{dv}{dy}\,v$$

$$\frac{dv}{dy} = \frac{1}{v}\frac{dv}{dt} = \frac{-g}{v}\left(1 + v^2/v_{ter}^2\right)$$

$$\frac{v\,dv}{v_{ter}^2 + v^2} = \frac{-g}{v_{ter}^2}\,dy$$

- Integration

$$\int_{v_0}^{v} \frac{v'\,dv'}{(v_{ter}^2 + v'^2)} = \frac{-g}{v_{ter}^2}\int_0^y dy' = \frac{-gy}{v_{ter}^2}$$

$$\hookrightarrow = \frac{1}{2}\ln\frac{v^2 + v_{ter}^2}{v_0^2 + v_{ter}^2}$$

- Thus

$$v^2(y) = -v_{ter}^2 + (v_0^2 + v_{ter}^2)\,e^{-2gy/v_{ter}^2}$$

$$y_{max} = \frac{v_{ter}^2}{2g}\ln\frac{v_{ter}^2 + v_0^2}{v_{ter}^2}$$

At maximum height, v = 0.

Therefore, $\qquad v_{ter}^2 = (v_{ter}^2 + v_0^2)\exp(-2gy_{max}/v_{ter}^2)$.

Solve for y_{max}: $\qquad y_{max} = (v_{ter}^2/2g)\ln[(v_{ter}^2+v_0^2)/v_{ter}^2] \qquad$ eq. 2.89

Numerical

Set v_0 = 20 m/s and v_{ter} = 35 m/s; then y_{max} = **17.7 m.**

Compare in vacuum, $\qquad y_{max} = v_0^2/(2g)$ = **20.4 m.**

20 Problem 2.53

A charged particle in parallel E and B fields ...

Set up coordinates with $\mathbf{B} = B\,\boldsymbol{e}_z$ and $\mathbf{E} = E\,\boldsymbol{e}_z$.

The force on q is

$$\mathbf{F} = q\,\mathbf{E} + q\,\mathbf{v} \times \mathbf{B} = qE\,\boldsymbol{e}_z + q\,(\boldsymbol{e}_x v_y B - \boldsymbol{e}_y v_x B)$$

Check:

$$\mathbf{v} \times \mathbf{B} = \begin{vmatrix} \mathbf{e_x} & \mathbf{e_y} & \mathbf{e_z} \\ v_x & v_y & v_z \\ 0 & 0 & B \end{vmatrix}$$

The equation of motion is

$$m\,d\mathbf{v}/dt = m\,v'_x\,\boldsymbol{e}_x + m\,v'_y\,\boldsymbol{e}_y + m\,v'_z\,\boldsymbol{e}_z = \mathbf{F}\ ;$$

(notation : ' = d/dt)

so

$$v'_x = (qB/m)\,v_y \quad ; \quad v'_y = -\,(qB/m)\,v_x \quad ; \quad v'_z = 0\ .$$

z component

$$v_z = v_{0z} + (qE/m)\,t$$

constant acceleration.

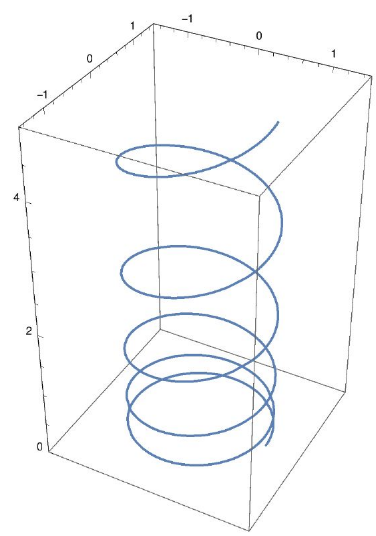
x and y components

$$v_x = A\cos\omega t \qquad \omega = qB/m$$

$$v_y = A\sin\omega t$$

uniform circular motion.

- The trajectory is a helix with increasing pitch.



21 Problem 2.43

Trajectory of a basketball ...

This problem is a computer problem. Hand in the program and plots.

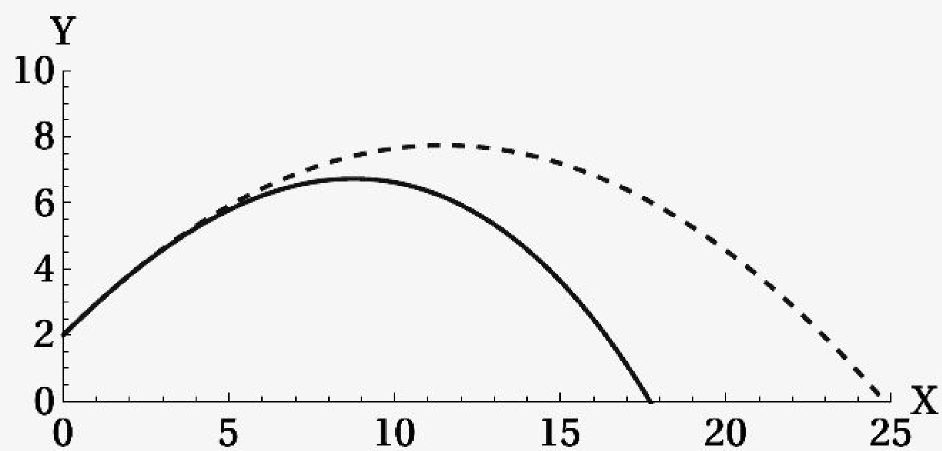
(a) Basketball throw

basketball parameters, initial conditions and air resistance

```
In[162]:= {mass, diam, g} = {0.6, 0.24, 9.81};
          {x0, y0, v0x, v0y} = {0, 2, 15 / Sqrt[2], 15 / Sqrt[2]};
          {γ, c} = {0.25 , 0.25 * diam^2};

In[177]:= eqns = {
              mass * x''[t] == -c * Sqrt[x'[t]^2 + y'[t]^2] * x'[t],
              mass * y''[t] == -c * Sqrt[x'[t]^2 + y'[t]^2] * y'[t] - mass * g,
              x[0] == x0, y[0] == y0, x'[0] == v0x , y'[0] == v0y};
           sols = NDSolve[eqns, {x, y}, {t, 0, 5}];
          X = x /. sols[[1]]; Y = y /. sols[[1]];
          p1 = ParametricPlot[{X[t], Y[t]}, {t, 0, 3},
              PlotRange → {{0, 25}, {0, 10}},
              BaseStyle → {FontFamily → "Times", FontSize → 16},
              AxesLabel → {"X", "Y"}] ;
          p2 = ParametricPlot[{x0 + v0x * t, y0 + v0y * t - 0.5 * g * t^2}, {t, 0, 3},
              PlotStyle → Dashing[{0.01, 0.02}]];
          fig243 = Show[p1, p2]
```

Out[182]=

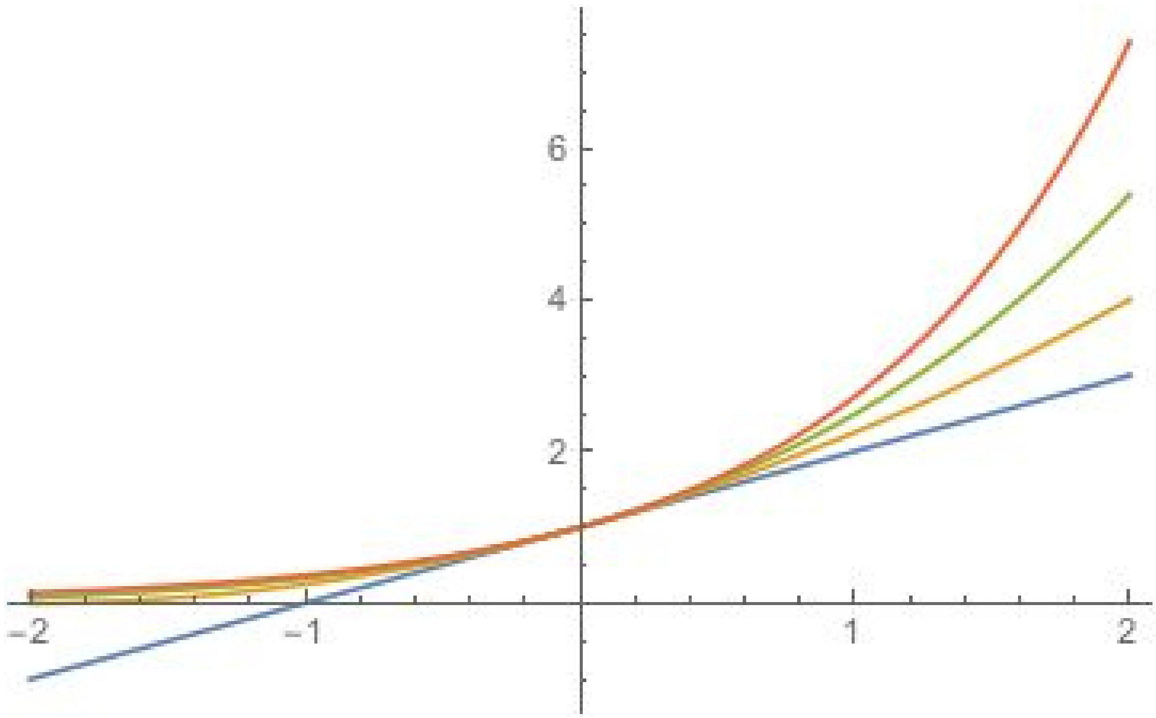


(b) Range calculation

```
In[195]:= tfinal = t /. FindRoot[Y[t] == 0, {t, 3}];
          {X[tfinal], Y[tfinal]}
          RangeNoAir = v0x * (2 * v0y / g)
```

Out[196]= $\{17.712, 6.10623 \times 10^{-16}\}$

Out[197]= 22.9358



6
4
2
−2
−1
1
2