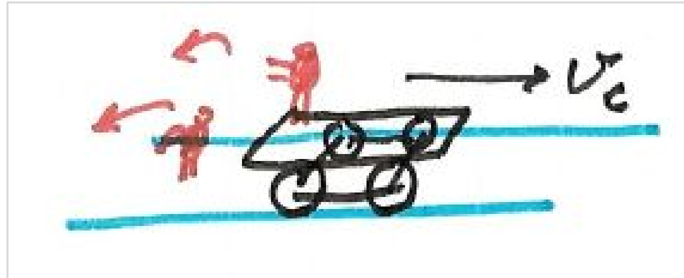
Homework Assignment #5

Problem 3.4

Two hobos on a hand cart ...



(a) If they jump simultaneously then

momentum is conserved $\Rightarrow$ $M_c v_c + 2m_h (v_c - u) = 0$

thus

$$\boxed{v_c = \frac{2m_h u}{M_c + 2m_h} = \frac{m_h u}{M_c + 2m_h}[2]}$$

(b) If they jump one after the other then

1. first jump $\Rightarrow$ $(M_c + m_h) v_1 + m_h (v_1 - u) = 0$
2. second jump $\Rightarrow$ $M_c v_2 + m_h (v_2 - u) = (M_c + m_h) v_1$

These are two equations for two unknowns (v_1 and v_2).

We want the final speed of the car = v_2 .

After a bit of algebra, the answer is

$$\boxed{v_2 = \frac{m_h u}{M_c + 2m_h}\left[\frac{2M_c + 3m_h}{M_c + m_h}\right]}$$

- The second procedure gives greater speed to the car.

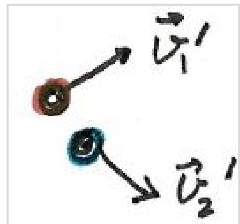
Problem 3.5

An elastic collision with equal masses ...

Equal masses, and one particle is initially at rest.

Before the collision:

After the collision:



Conservation laws

Momentum $\quad m\,\mathbf{v_1} = m\,\mathbf{v_1}' + m\,\mathbf{v_2}' \quad \Rightarrow \quad \mathbf{v_1 = v_1' + v_2'} \quad (1)$

Kinetic energy $\quad \frac{1}{2} m\, v_1{}^2 = \frac{1}{2} m\, v_1{}'^2 + \frac{1}{2} m\, v_2{}'^2$

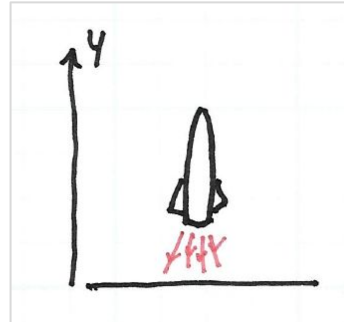
$$\Rightarrow \quad v_1{}^2 = v_1{}'^2 + v_2{}'^2 \quad (2)$$

By eq. (1)

$$v_1{}^2 = (\mathbf{v_1}' + \mathbf{v_2}')^2 = v_1{}'^2 + v_2{}'^2 + 2\,\mathbf{v_1}' \cdot \mathbf{v_2}'$$

Comparison to eq. (2) implies $\mathbf{v_1}'.\mathbf{v_2}' = 0$.

That is, *the final velocities must be perpendicular.*



Problem 3.6

The Saturn V rocket ...

The rocket equations are

$$dm/dt = -K \qquad \text{and} \qquad v = v_{ex} \ln(m_0 / m).$$

We have these parameters:

$$K = 15 \times 10^3 \text{ kg/s}; \qquad v_{ex} = 2500 \text{ m/s} \qquad .$$

The calculation :::

$$\text{Thrust} = m\, dv/dt = K\, v_{ex} = 37.5 \times 10^6 \text{ N} \qquad \times(1 \text{ ton} / 9 \times 10^3 \text{ N})$$

$$= 4170 \text{ tons of force.}$$

Compare that to the initial weight = 3000 tons.

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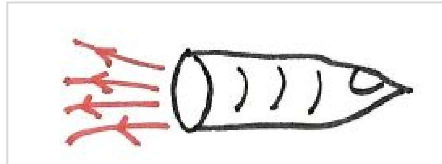
Problem 3.10

A rocket in deep space ...



A rocket accelerates from rest in deep space.

Calculate the *maximum momentum* of the rocket.

The rocket equation (there is no external force) is

$$v(t) = v_{ex} \ln [\, m_0 / m(t) \,].$$

The momentum at time t is $p(t) = m(t)\, v(t)$, so

$$p = m\, v_{ex} \ln [\, m_0/m].$$

To find the maximum momentum, set $dp/dm = 0$.

$$\frac{dp}{dm} = v_{ex} \ln\left(\frac{m_0}{m}\right) - v_{ex}$$

Therefore $dp/dm = 0$ implies

$$\ln\left(\frac{m_0}{m}\right) = 1$$

That is, $m = m_0/e = 0.368\, m_0$.

The maximum momentum is

$$p_{max} = (m_0/e)\, v_{ex} \ln[m_0/(m_0/e)]$$

$$= m_0\, v_{ex}/e$$

$$= 0.368\, m_0\, v_{ex}.$$

⇧

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Problem 3.12

A two - stage rocket ...

(a) First, consider a single-stage rocket;

$$v_{final} = v_{ex} \ln (m_0 / m_{final}) = v_{ex} \ln [m_0 / (0.4\, m_0)]$$

$$= v_{ex} \ln (5/2) \qquad = 0.916\, v_{ex}$$

(b) Now, two stages;

1. the first stage $(m_0 \rightarrow m_1 = 0.7\, m_0)$

   $$v_1 = v_{ex} \ln (m_0/m_1) = v_{ex} \ln [m_0/(0.7\, m_0)] = v_{ex} \ln (10/7)$$

2. the second stage $(m_1' \rightarrow m_2 = 0.3\, m_0)$ [where $m_1' = m_1 - 0.1\, m_0$]

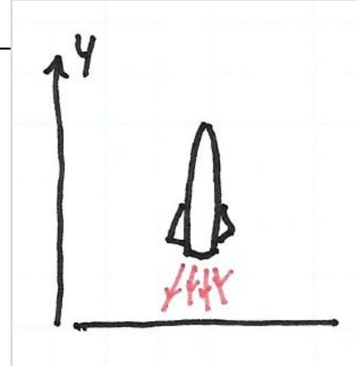
   $$v_2 = v_1 + v_{ex} \ln [(m_1 - 0.1\, m_0)/(m_1 - 0.1\, m_0 - 0.3\, m_0)]$$

   $$= v_{ex} \{ \ln(10/7) + \ln(6/3) \} = v_{ex} \ln (20/7) \qquad = 1.050\, v_{ex}$$

- The two-stage rocket reaches a larger final speed.

Problem 3.13

A rocket taking off in Earth's gravity ...



The velocity of a rocket accelerating upward

in Earth's gravity (near the Earth's surface and starting from rest) is

$$v(t) = v_{ex} \ln [\, m_0 / (m_0 - K\,t)\,] - g\,t.$$

The height at time t is

$$y(t) = \int_0^t v(t')\,dt' = \int_0^t \{ v_{ex} \ln (m_0) - \ln \ (m_0 - K\,t') - g\,t' \}\,dt'$$

$$y(t) = \int_0^t v(t')\,dt'$$

$$= \int_0^t \left[v_{ex} \ln m_0 - v_{ex} \ln(m_0 - Kt') - gt' \right] dt'$$

$$= v_{ex} (\ln m_0) t - \tfrac{1}{2} g t^2 - v_{ex} \underbrace{\int_0^t \ln(m_0 - Kt')\,dt'}_{A}$$

Let $x = m_0 - Kt'$

$dx = -K\,dt'$

$$A = \frac{-1}{K} \int_{m_0}^{m} \ln x \, dx = \frac{-1}{K} \left(x \ln x - x \right)\Big|_{m_0}^{m}$$

$$= \frac{-1}{K} \left[m \ln m - m - m_0 \ln m_0 + m_0 \right]$$

where $m = m_0 - Kt$.

$$y(t) = -\tfrac{1}{2} g t^2 + v_{ex} (\ln m_0) t$$

$$+ \frac{v_{ex}}{K} \left[m_0 \ln m - Kt \ln m - m_0 \ln m_0 + Kt \right]$$

$$= (\text{algebra}) = -\tfrac{1}{2} g t^2 + v_{ex} t - \frac{m v_{ex}}{K} \ln\left(\frac{m_0}{m}\right)$$

<u>The numerical calculation for Space Shuttle parameters</u>

$m_0 = 2 \times 10^6$ kg ; $K = 1 \times 10^6$ kg/(120 s) ; $v_{ex} = 3000$ m/s ;

then the height at time t = 120 s is

$y(120 \text{ s}) = 39.9$ km.