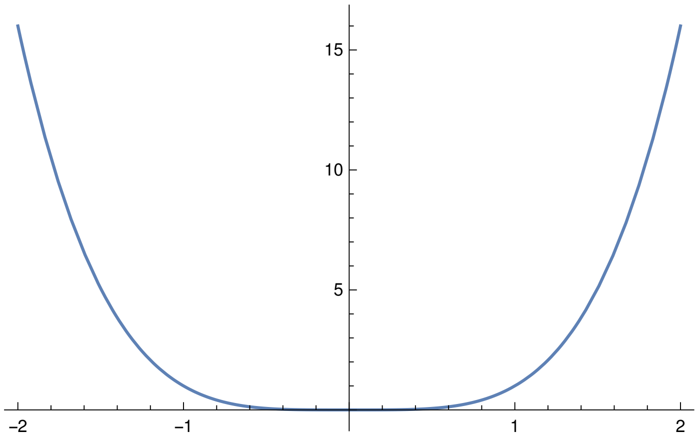
COMPUTER CALCULATIONS FOR HOMEWORK ASSIGNMENT 8

[38] PROBLEM 4.29 : U = k x^4

In[1]:=
```
Plot[x^4, {x, -2, 2}]
```

Out[1]=



For the potential $U(x) = k\,x^4$, first derive the equation for one quarter cycle,

$$\tau / 4 = \sqrt{m/(2k)} \int_0^A dx' \Big/ \sqrt{A^4 - x'^4}\ .$$

Now calculate the period of oscillation τ, for $m = k = A = 1$.

```
f = 4 Sqrt[m/(2 k)] * Power[1 - u^4, -1/2]
f = f /. {k → 1, m → 1}
NIntegrate[f, {u, 0, 1}]
```

$$\frac{2\sqrt{2}\sqrt{\frac{m}{k}}}{\sqrt{1-u^4}}$$

$$\frac{2\sqrt{2}}{\sqrt{1-u^4}}$$

3.70815

$\tau = 3.708$ in time units.

[39] PROBLEM 4.33 : cube on a cylinder

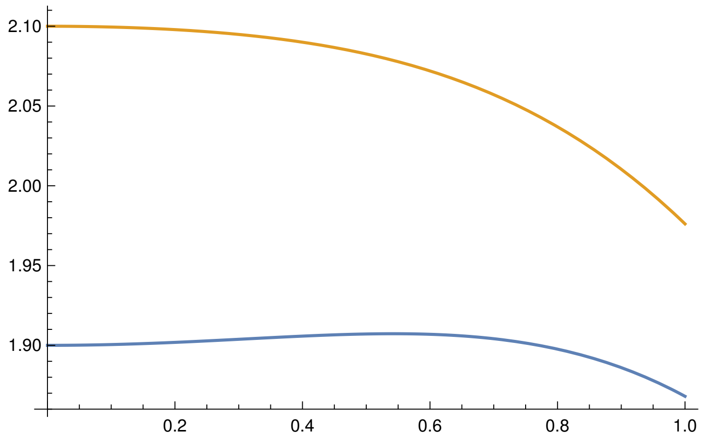
The potential energy is

$U(\theta) = mg\{(r+b)\cos\theta + r\theta\sin\theta\}$.

Now plot $U(\theta)$ for $b = 0.9\,r$ (stable) and $b = 1.1\,r$ (unstable), with r = m = g = 1.

```
{r, m, g} = {1, 1, 1}
U[b_, θ_] := m * g * ((r + b) * Cos[θ] + r * θ * Sin[θ])
Plot[{U[0.9, θ], U[1.1, θ]}, {θ, 0, 1}]
```

```
{1, 1, 1}
```



[41] PROBLEM 4.37 (wheel and two masses)

First, derive the potential energy,

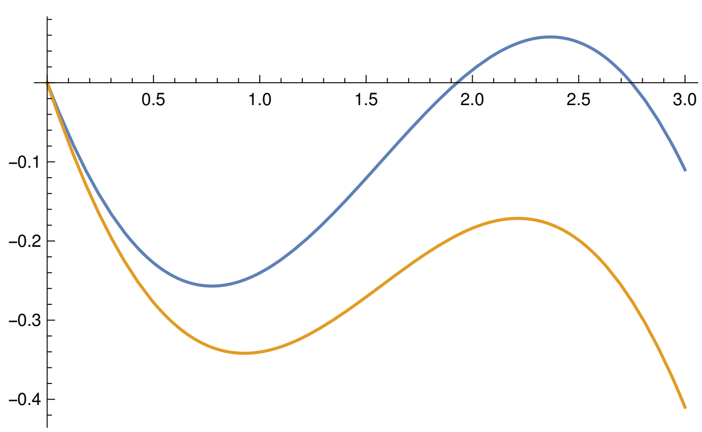
$U(\varphi) = MgR\,(1 - \cos\varphi) - mgR\,\varphi$.

Now plot $U(\varphi)$ for $m = 0.7\,M$ (blue; angle is bounded)

and for $m = 0.8\,M$ (orange; angle in unbounded).

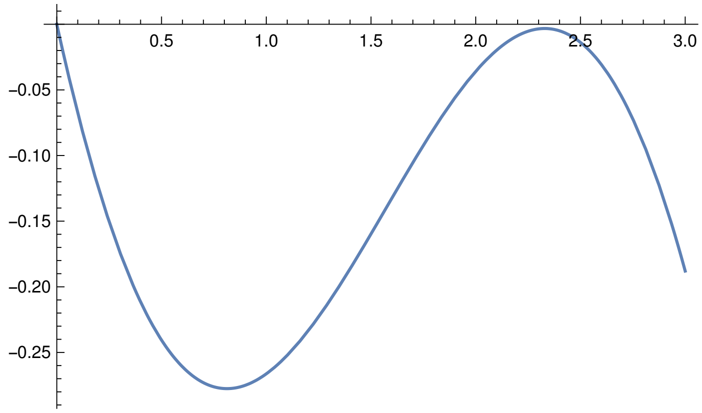
```
{M, g, R} = {1, 1, 1}
U[m_, φ_] := M * g * R * (1 - Cos[φ]) - m * g * R * φ
Plot[{U[0.7, φ], U[0.8, φ]}, {φ, 0, 3}]
```

```
{1, 1, 1}
```



The critical value of m/M is 0.726.

```
Plot[U[0.726, φ], {φ, 0, 3}]
```



[42] PROBLEM 4.38 (simple pendulum)

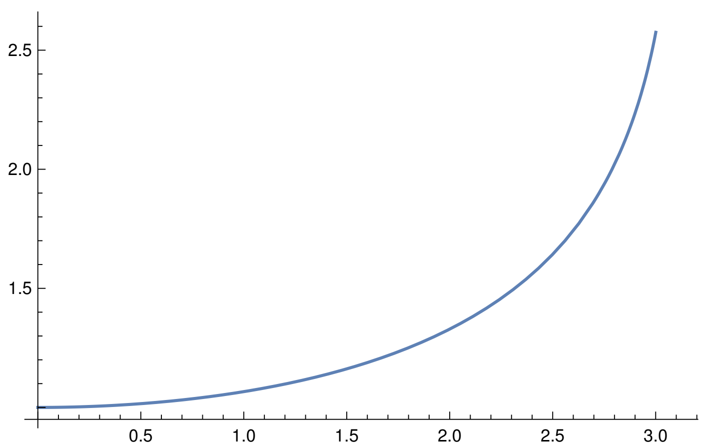
First, derive the period of oscillation.

$$\tau = (2\,\tau 0\,/\,\pi) \int_0^1 du\;\left(1 - u^2\right)^{-1/2}\left(1 - A^2\,u^2\right)^{-1/2}$$

where A = sin (Φ/2).

Now plot $\tau/\tau 0$ as a function of Φ,

```
K[Asq_] := (2 / π) * NIntegrate[Power[(1 - u^2) * (1 - Asq * u^2), -1 / 2],
   {u, 0, 1}]
Plot[K[Sin[Φ / 2]^2], {Φ, 0, Pi}]
```



τ goes to infinity as Φ goes to π,
because Φ is an equilibriujm configuration
(inverted pendulum).