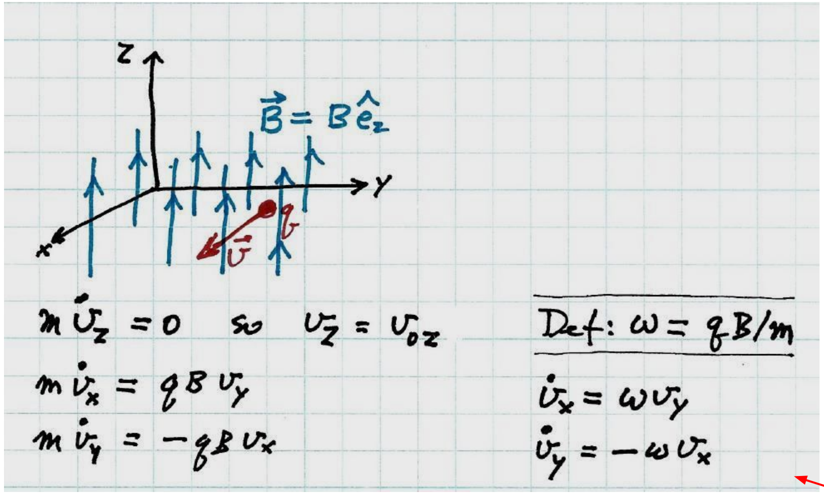
Section 2.7

*Solution for a charge q in a magnetic field **B**.*

Read Section 2.7.

Recall Monday's lecture



η

Define a complex variable η by

$$\eta = v_x + i\,v_y$$

Now note that

$$\dot{\eta} = -\,i\,\omega\,\eta \qquad \textit{(verify it!)}$$

The general solution of the equation of motion is $\eta(t) = A\,e^{-i\omega t}$.

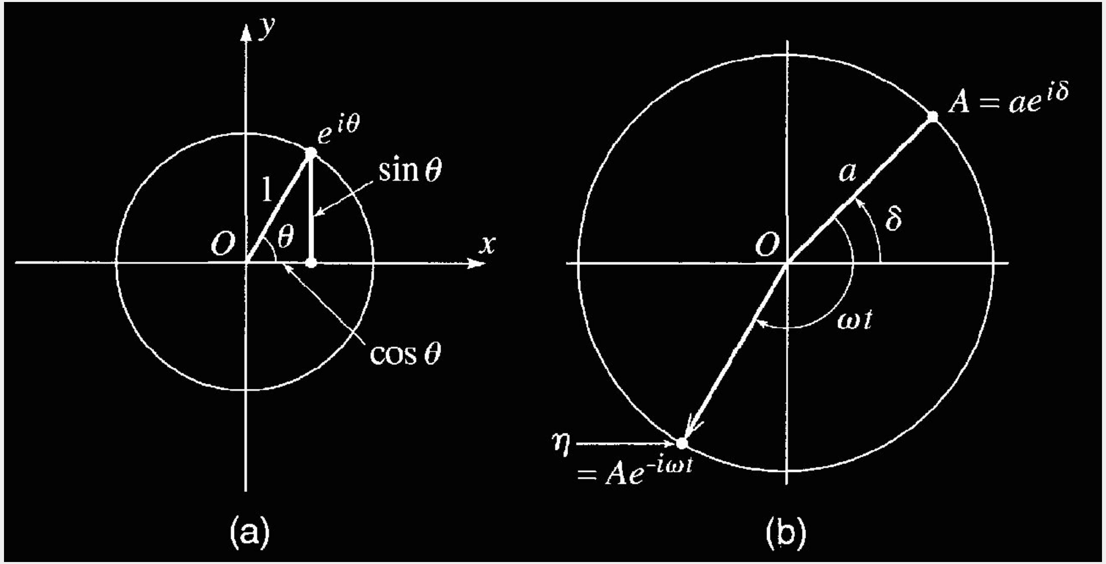
We must allow for A to be complex; write $A = a\,e^{i\delta}$.

Then

$$v_x = \mathrm{Re}\,\eta = a\cos(\omega t - \delta)$$
$$v_y = \mathrm{Im}\,\eta = -\,a\sin(\omega t - \delta)$$

consistent

Figure 2.14 : The transverse velocity components



(a) Illustrates Euler's equation : $e^{i\theta} = \cos\theta + i \sin\theta$

(b) $\eta = A \exp(-i\,\omega\, t)$ and $A = a \exp(i\,\delta)$; ***η(t) rotates clockwise***

The trajectory ; *coordinates vs time*

The transverse motion of a positive charge q in magnetic field **B** = B $\boldsymbol{e}_z$ ...

Define $\xi = x + iy$

Then $\dot{\xi} = \dot{x} + i\dot{y}$

$= v_x + i v_y = \eta$

$\eta = A e^{-i\omega t} \Longrightarrow \xi = \frac{A}{-i\omega} e^{-i\omega t} + \text{constant}$

Or, write

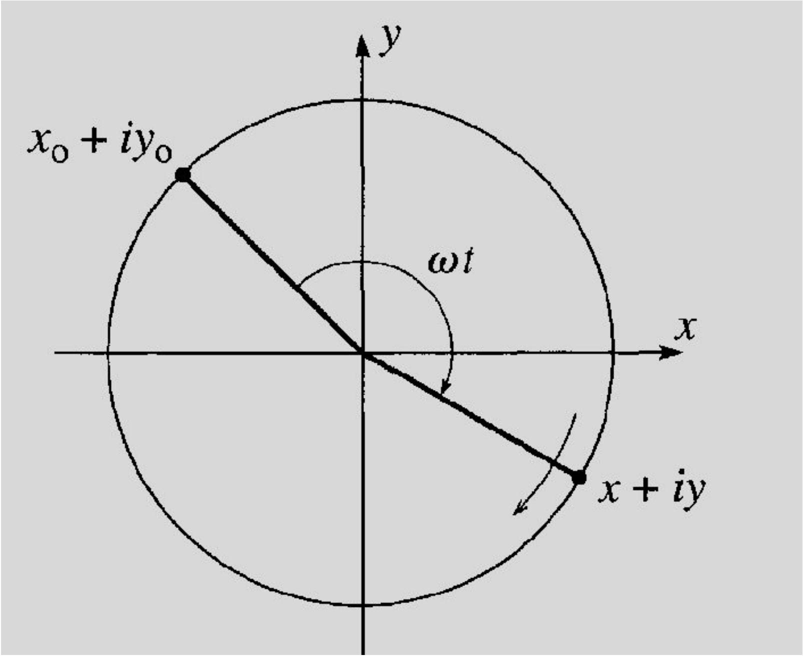
$\xi = C e^{-i\omega t} + a + ib$

W.L.O.G. set $a = 0$ and $b = 0$.

Then $x_0 + i y_0 = C$ (initial values)

$$x + iy = (x_0 + i y_0)(\cos\omega t - i \sin\omega t)$$
$$= x_0 \cos\omega t + y_0 \sin\omega t$$
$$+ i(y_0 \cos\omega t - x_0 \sin\omega t)$$

Figure 2.15



$$x(t) = x_0 \cos\omega t + y_0 \sin\omega t$$
$$y(t) = y_0 \cos\omega t - x_0 \sin\omega t$$
$$x^2 + y^2 = x_0^2 + y_0^2$$

The trajectory is a circle traversed clockwise (for q > 0).

In 3 dimensions, the general trajectory is a cylindrical helix.

Consider $y_0 = 0$.

Then

$$z(t) = v_{0z}\, t$$

$$x(t) = x_0 \cos(\omega t)$$

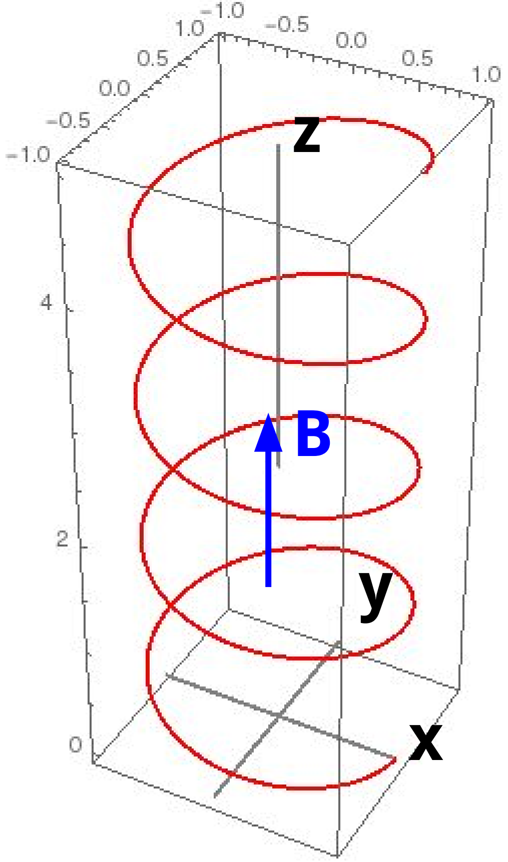
$$y(t) = -\, x_0 \sin(\omega t)$$

Radius $R = x_0$

Period $T = 2\pi / \omega$

where $\omega = qB/m$

Direction = clockwise in xy plane for positive q .



Test your understanding of magnetism: Verify the direction from **F** = q **v** × **B** !

In Class Work
:
2 problems

Homework Assignment #4
due in class Wednesday Sept. 27
[17] Problem 2.23 *
[18] Problem 2.31 **
[19] Problem 2.41 **
[20] Problem 2.53 *
[21] Problem 2.43 *** [computer]
[22] Graph of $f_n(x)$.

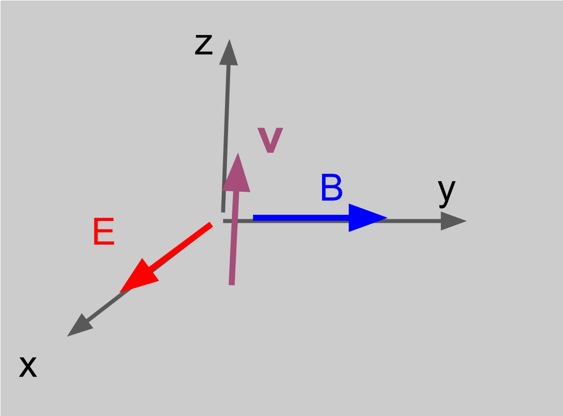
Use the cover sheet!

Wednesday Quiz

Assume: The electric field points in the x direction, and the magnetic field points in the y direction.

Assume: A positive charge is located at the origin, and is moving in the z direction.

What is the direction of the acceleration? There are two possibilities which you should identify.



$$\mathbf{F} = qE_0\,\mathbf{e_x} - qvB_0\,\mathbf{e_x}$$

$$\mathbf{F} = q\,(\,E_0 - vB_0\,)\,\mathbf{e_x}$$

- if $v < E_0/B_0$ then $\mathbf{a} \sim \mathbf{e_x}$
- if $v > E_0/B_0$ then $\mathbf{a} \sim -\mathbf{e_x}$