- **Chapter 3: Momentum and Angular Momentum**
  - 3.1 Conservation of Momentum
  - 3.2 Rockets
  - 3.3 The center of mass
  - 3.4 Angular momentum for a single particle
  - 3.5 Angular momentum for several particles

Read Chapter 3 during the next two weeks.

Homework Assignment #5
due in class Wednesday, October 4

[21] Problem 3.4 **
[22] Problem 3.5 **
[23] Problem 3.6 *
[24] Problem 3.10 *
[25] Problem 3.12 **
[26] Problem 3.13 **

Use the cover sheet!

Section 3.1
Conservation of Momentum

First, define momentum for a single particle,

$$\mathbf{p} = m\,\boldsymbol{v}\,.$$

Now consider a system containing N particles. For each particle there is momentum,

$$\mathbf{p}_\alpha = m_\alpha\,\boldsymbol{v}_\alpha\,.$$

$$(\alpha = 1\ 2\ 3\ \ldots\ N)$$

The total momentum of the system is **P**

$$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \ldots + \mathbf{p}_N\,.$$

$$\mathbf{P} = \sum_{\alpha=1}^{N} \mathbf{p}_\alpha = \sum_{\alpha=1}^{N} m_\alpha \boldsymbol{v}_\alpha$$

This is the crucial result:

- ***Theorem.*** $\dot{\mathbf{P}} = \mathbf{F}^{ext}$
  where $\mathbf{F}^{ext}$ is the sum of all external forces acting on the particles.

(dot over a letter means d/dt)

Proof

$$\vec{P} = \sum_{\alpha} \vec{p}_{\alpha} = \sum_{\alpha} m_{\alpha} \vec{v}_{\alpha}$$

$$\frac{d\vec{P}}{dt} = \sum_{\alpha} m_{\alpha} \frac{d\vec{v}_{\alpha}}{dt} = \sum_{\alpha} \vec{F}_{\alpha}$$

$$= \sum_{\alpha} \vec{F}_{\alpha}^{ext} + \sum_{\alpha} \sum_{\substack{\beta \\ (\beta \neq \alpha)}} \vec{F}_{\alpha\beta}$$

All the internal forces cancel in pairs by Newton's third law;

$\vec{F}_{12} + \vec{F}_{21} = 0$ or $\vec{F}_{\alpha\beta} + \vec{F}_{\beta\alpha} = 0$

$$\therefore \quad \frac{d\vec{P}}{dt} = \sum_{\alpha} F_{\alpha}^{ext} = \vec{F}^{ext}$$

The principle of conservation of momentum

For an *isolated system* of N particles, the total momentum is a constant of the motion.

Proof: Because it is an isolated system, there are no external forces.

Thus the previous theorem states that

$$d\mathbf{P}/dt = 0.$$

Hence **P** is constant in time.

Example 3.1

A perfectly inelastic collision

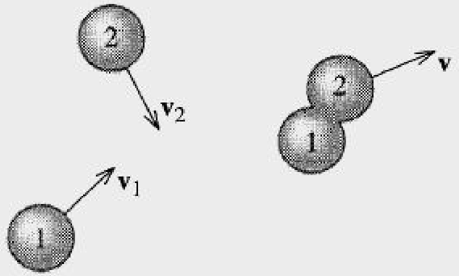


Figure 3.1 A perfectly inelastic collision between two lumps of putty.

The problem is to calculate $\boldsymbol{v}$.

Principle: *The total momentum is conserved.*

Before the collision,

$$\mathbf{P} = m_1\boldsymbol{v_1} + m_2\boldsymbol{v_2} \quad .$$

After the collision (stuck together)

$$\mathbf{P} = (m_1+m_2)\,\boldsymbol{v} \quad .$$

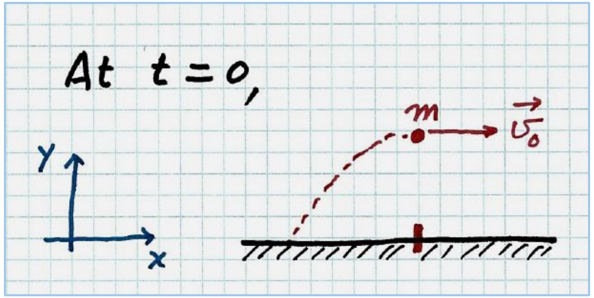
The momentum is conserved (constant) so

$$(m_1+m_2)\,\boldsymbol{v} = m_1\boldsymbol{v_1} + m_2\boldsymbol{v_2}$$

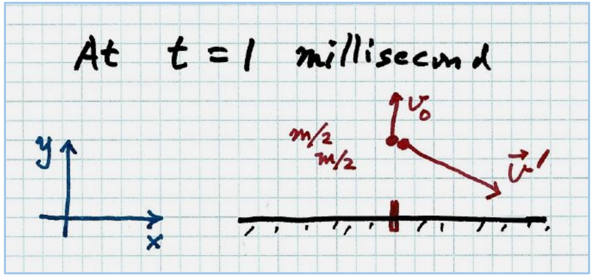
$$\therefore \quad \boldsymbol{v} = (m_1\boldsymbol{v_1} + m_2\boldsymbol{v_2}) \,/\, (m_1 + m_2)$$

Special cases: If $m_1 >> m_2$ then $\boldsymbol{v} \approx \boldsymbol{v_1}$;
if $m_1 << m_2$ then $\boldsymbol{v} \approx \boldsymbol{v_2}$;
If $m_1 = m_2$ then $\boldsymbol{v} = \frac{1}{2}(\boldsymbol{v_1} + \boldsymbol{v_2})$.

Another example:
an exploding projectile (Taylor, Problem 3.2)



Ar t = 1 ms, the shell explodes into 2 equal mass fragments, and one fragment goes straight up with speed v_0



- Calculate the velocity of the second fragment.

By conservation of momentum,

$$m v_0 \hat{e}_x = \frac{m}{2} v_0 \hat{e}_y + \frac{m}{2} \vec{v}'$$

$$\therefore \quad \vec{v}' = 2v_0 \hat{e}_x - v_0 \hat{e}_y$$

- How much energy was released in the explosion?

$$T_f - T_i = \frac{1}{2}\frac{m}{2} v_0^2 + \frac{1}{2}\frac{m}{2} v'^2 - \frac{1}{2} m v_0^2$$

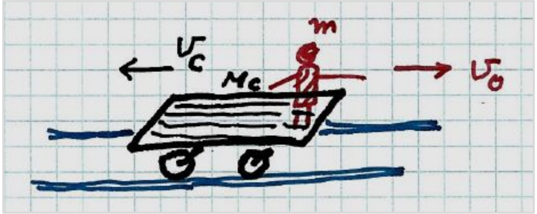
$$= \frac{1}{4} m v_0^2 + \frac{1}{4} m \left(4v_0^2 + v_0^2\right) - \frac{1}{2} m v_0^2$$

$$= m v_0^2$$

← The explosion released that amount of energy, which was converted into kinetic energy of the fragments.

Another example:

a hobo jumps off a railroad flat car
(Taylor, Problem 3.4)



Assuming the car is initially at rest, calculate the increase of kinetic energy.

Principle: Momentum is conserved.

$$p = 0 = m v_0 + M_c v_c$$

$$v_c = -\frac{m v_0}{M_c}$$

$$\Delta K.E. = \frac{1}{2} m v_0^2 + \frac{1}{2} M_c v_c^2$$

$$= \frac{1}{2} m v_0^2 + \frac{1}{2} M_c \left(-m v_0 / M_c\right)^2$$

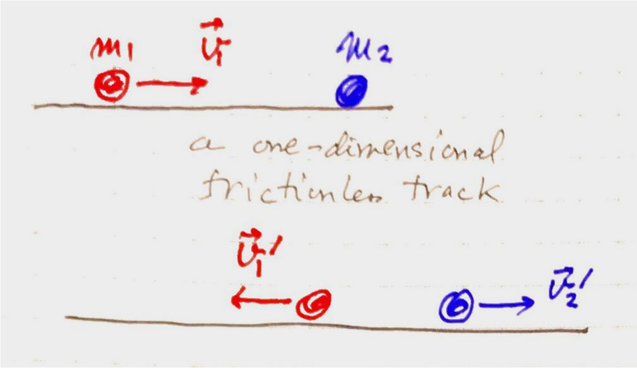
$$= \frac{1}{2} m v_0^2 \left[1 + \frac{m}{M_c}\right] \longleftarrow \begin{cases} \approx \frac{1}{2} m v_0^2 & \text{if } m \ll M_c \\ m v_0^2 & \text{if } m = M_c \end{cases}$$

Comment. In impulsive collisions, momentum is conserved because during the short time of the collision, external forces are negligible.

<u>A head-on *elastic collision* in 1 dimension with one particle at rest</u>

Before

After



$$m_1 v_1 = m_1 v_1' + m_2 v_2'$$

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

(elastic)

Algebra to solve the equations $\Rightarrow$

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 \quad \text{and} \quad v_2' = \frac{2 m_1}{m_1 + m_2} v_1$$

Special cases

- $m_1 = m_2 \Rightarrow v_1' = 0, \; v_2' = v_1$
- $m_1 > m_2 \Rightarrow m_1$ continues forward
- $m_1 < m_2 \Rightarrow m_1$ bounces back

Conservation of momentum and Newton's third law

• *Is momentum always conserved?*

■ **We've already seen that *kinetic energy* is not always conserved.**

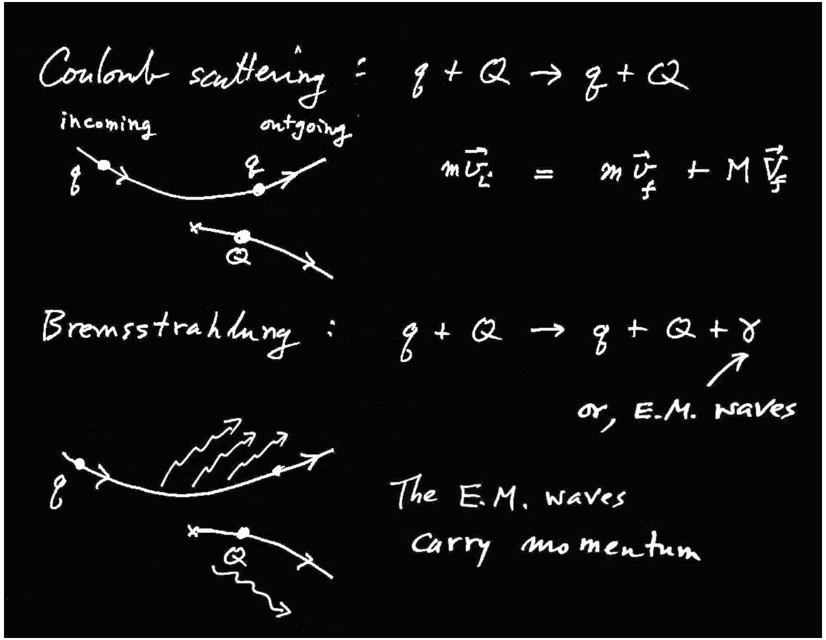
Chapter 4 will introduce potential energy. But *mechanical energy* (T+U) is not always conserved.

THERMODYNAMICS is necessary to understand that *total energy* is always conserved; the first law of thermodynamics. ■

• *Is momentum always conserved?*

Particle momentum is not always conserved because there is field momentum. But *total momentum* is conserved.

• ***Example***



For PHY 321: *In Newtonian mechanics, particle momentum is always conserved.*

Homework Assignment #5
due in class Wednesday, October 4

[21] Problem 3.4 **

[22] Problem 3.5 **

[23] Problem 3.6 *

[24] Problem 3.10 *

[25] Problem 3.12 **

[26] Problem 3.13 **

Use the cover sheet.