Homework Assignment #5
due in class Wednesday, October 4

[21] Problem 3.4 **
[22] Problem 3.5 **
[23] Problem 3.6 *
[24] Problem 3.10 *
[25] Problem 3.12 **
[26] Problem 3.13 **

Use the cover sheet.

The first hour exam will be in class Friday (October 6).

Do the homework now so that you will have some time to study for the exam.

Study: −basic equations;
−lecture notes; −in-class work;
−homework.

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Section 3.3
The Center of Mass

FIRST: The center of mass of a system of N particles

The ***system*** consists of N particles:

masses = m_α

position vectors = $\boldsymbol{r}_\alpha$

($\alpha = 1\ 2\ 3 \ldots N$)

- The total mass , M , is

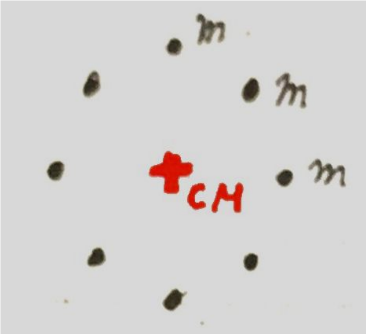
$$M = \sum_{\alpha=1}^{N} m_\alpha \, .$$

- The center of mass position, ***R*** , is

$$\vec{R} = \frac{\sum_{\alpha=1}^{N} m_\alpha \vec{r}_\alpha}{\sum_{\alpha=1}^{N} m_\alpha}$$

$$\vec{R} = \frac{1}{M} \sum_{\alpha=1}^{N} \vec{r}_\alpha m_\alpha$$

I.e., ***R*** =
the "average position",
weighted by the masses



These two theorems shows why ***R*** is important.

<u>Theorem 1</u>

$$M\,\dot{\mathbf{R}} = \mathbf{P} \qquad (\textit{ = total momentum})$$

<u>Theorem 2</u>

$$M\,\ddot{\mathbf{R}} = \mathbf{F}^{ext} \qquad (\textit{ = sum of external forces})$$

In words:

The center of mass position moves under the influence of the external forces,

<u>*independent of internal forces*</u>.

<u>Proofs</u>.

are easy …

$$M\dot{\vec{R}} = (m_1 + m_2)\,\frac{d}{dt}\,\frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

$$= m_1\dot{\vec{r}}_1 + m_2\dot{\vec{r}}_2$$

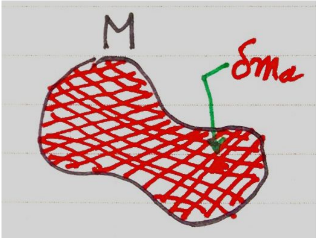
$$= \vec{p}_1 + \vec{p}_1 = \vec{P}$$

and

$$M\ddot{\vec{R}} = \dot{\vec{P}} = \vec{F}_{ext}$$

SECOND : R for a solid body

- Imagine the object divided into an infinite number of infinitesimal parts.



- Recall the definition of an ***integral*** in calculus.

$$\lim_{N\to\infty} \sum_{i=1}^{N} f(x_i)\delta x_i = \int_a^b f(x)\,dx$$

- For example, consider the ***total mass***

$$M = \sum_{\alpha=1}^{N} (\delta m_\alpha)$$

Now take the limit $N \to \infty$ and $\delta m \to 0$ to get the continuum limit,

$$M = \int_{Body} dm = \int_V \rho(\mathbf{r})\, d^3r$$

- <u>Center of Mass position</u>

$$\boldsymbol{R} = (1/M) \sum_{\alpha=1}^{N} \boldsymbol{r}_\alpha (\delta m_\alpha)$$

Now take the limit $N \to \infty$ and $\delta m \to 0$ to get the continuum,

$$\mathbf{R} = (1/M) \int_{Body} \mathbf{r}\, dm$$

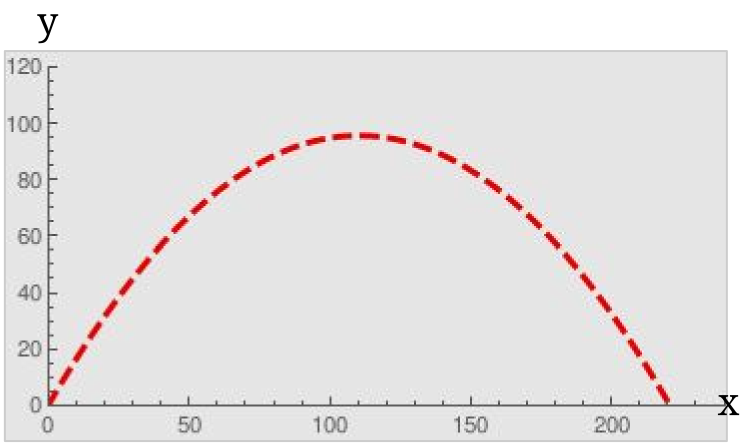
$$= \int_V \mathbf{r}\, \rho(\mathbf{r})\, d^3r \,/\, M .$$

*I.e., **R** is the mean position weighted by the mass density.*

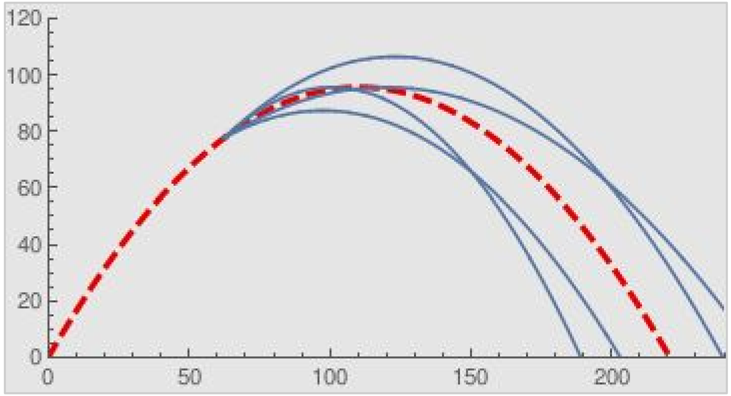
Example

an exploding projectile

Case A. If the projectile does not explode, then the trajectory is a parabola (*ignoring air resistance*) .



Case B. If the projectile explodes into fragments, then *the center of mass point* follows the same parabola as case A.



$$M\,\mathbf{R}'' = \mathbf{F}^{ext} = \sum_{i} m_i\,\mathbf{g} = M\,\mathbf{g}$$

Example.

The center of mass of two particles lies on the line joining the two particles.

Figure 3.3

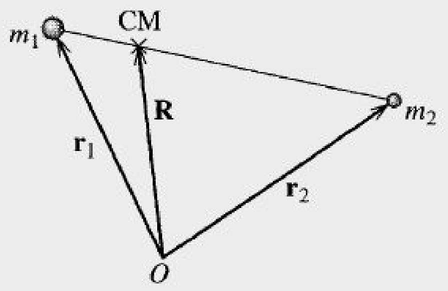


Figure 3.3 The CM of two particles lies at the position $\mathbf{R} = (m_1\mathbf{r}_1 + m_2\mathbf{r}_2)/M$. You can prove that this lies on the line joining m_1 to m_2, as shown, and that the distances of the CM from m_1 and m_2 are in the ratio m_2/m_1.

Example 3.2 - - - CoM of a solid cone

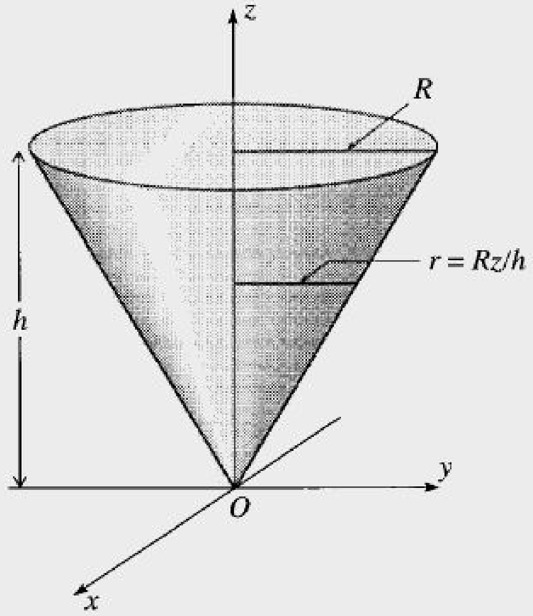


Figure 3.4 A solid cone, centered on the z axis, with vertex at the origin and uniform mass density ϱ. Its height is h and its base has radius R.