Chapter 4. Energy

- ❑ 4.1 kinetic energy and work
- ❑ 4.2 potential energy and conservative forces
- ❑ 4.3/4 force and potential energy
- ❑ 4.5 time-dependent potential energy
- ❑ 4.6 energy for linear 1D systems
- ❑ 4.7 curvilinear 1D systems
- ❑ 4.8 central forces
- ❑ 4.9/10 energy of interaction

2 weeks

Homework Assignment #7
due in class Wednesday, October 18

[31] Problem 4.3 **
[32] Problem 4.8 **
[33] Problem 4.9 **
[34] Problem 4.10 *
[35] Problem 4.18 **
[36] Problem 4.23 **
[37] three exam questions *(write full solutions)*

Use the cover page.

Hints for the exam questions:

(1) $\mathbf{r}(t) = \mathbf{e}_x R \cos(\varphi) + \mathbf{e}_y R \sin(\varphi)$

(2) answer a : vT = SQRT[(mg sinθ)/c]

(3) answer b : v = 2.93 km/s at burnout

What is energy?

From the Oxford Dictionary of Physics ...

> ***Energy***: "A measure of a system's ability to do work."

OK, then, what is "work"?

Oxford Dictionary of Physics …

> ***Work:*** "the scalar product of force and displacement vectors".

General comments

- Energy is an abstract concept;
  - "abstract" meaning mathematical.
- The concept of energy originates in mechanics; Chapter 4.
- Use of the word:
  - Thomas Young, 1802;
  - G G Coriolis, "kinetic energy", 1829;
  - William Rankine, "potential energy", 1853
- For many years in the 19th century, scientists argued whether energy is a substance ("the *caloric"*) or merely a mathematical concept.
- But the concept of energy goes beyond mechanics.

The concept of energy is a unifying principle in physics, and in all the sciences.

Math Concepts in Mechanics

- Momentum $\mathbf{p} = m\,\mathbf{v}$

  $d\mathbf{p} = \mathbf{F(r)}\,dt$

  | *an eq. of motion*

- Kinetic energy $T = ½\, m\, v^2$

  $dT = \mathbf{F(r)} \cdot \mathbf{v}\, dt$

  | *an eq. of motion*

- Potential energy $U(\mathbf{r})$

  $dU = -\,\mathbf{F(r)} \cdot d\mathbf{r}$

  | *an eq. of* position

4.1. KINETIC ENERGY AND WORK

The kinetic energy of a particle (i.e., due to translational motion) ; notation = T ;

$$T = ½\, mv^2$$

Calculate dT/dt

$$\frac{dT}{dt} = \frac{1}{2} m\, 2\vec{v} \cdot \frac{d\vec{v}}{dt}$$

$$= m\vec{v} \cdot \vec{a}$$

$$= \vec{v} \cdot \vec{F}$$

Figure 4.1

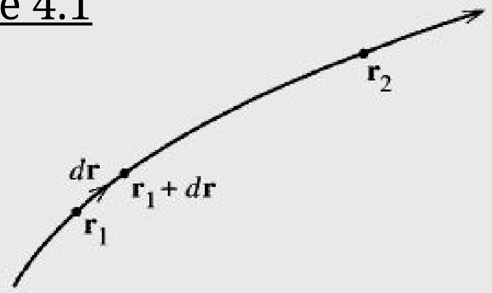


Figure 4.1 Three points on the path of a particle: $\mathbf{r}_1$, $\mathbf{r}_1 + d\mathbf{r}$ (with $d\mathbf{r}$ infinitesimal) and $\mathbf{r}_2$.

$$\frac{dT}{dt} = \vec{v}\cdot\vec{F} = \frac{d\vec{r}}{dt}\cdot\vec{F}$$

$$dT = \vec{F}\cdot d\vec{r}$$

$$\Delta T = T_2 - T_1 = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$$

along the trajectory of the motion

an example of a "line integral"

The Work–Kinetic Energy Theorem

$dT = \mathbf{F}\,.\,d\mathbf{r}$ = work

or, $\Delta T = \int \mathbf{F}\,.\,d\mathbf{r}$

or, $dT/dt = \mathbf{F}\,.\,\mathbf{v}$

In these equations, d**r** means a displacement along the trajectory of the particle's motion.

4.2. POTENTIAL ENERGY

For a conservative force, we may define a potential energy function; notation $U(\boldsymbol{r})$.

- Definition of a "conservative force":

A force is conservative if
(i) $\boldsymbol{F}$ depends only on $\boldsymbol{r}$; ***and*** (ii) the work done by $\boldsymbol{F}$ when the particle moves from $\boldsymbol{r}_1$ to $\boldsymbol{r}_2$ is independent of the path from $\boldsymbol{r}_1$ to $\boldsymbol{r}_2$.

Definition of Work

$\mathbf{W} = \int_\Gamma \boldsymbol{F} \cdot d\boldsymbol{r}$ ← *a line integral*

Γ is an arbitrary path in space from $\boldsymbol{r}_1$ to $\boldsymbol{r}_2$;
Γ is not necessarily the "trajectory"!

For a conservative force the Work depends on the endpoints, but the Work is the same for all paths connecting the endpoints.

Figure 4.3

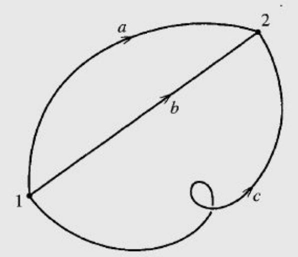


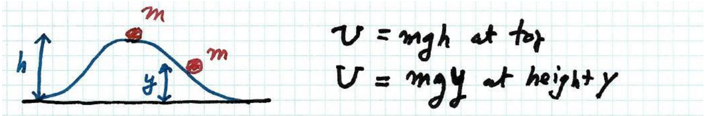
Figure 4.3 Three different paths, a, b, and c, joining the same two points 1 and 2.

- Definition of the potential energy function: $\Delta U = -\mathbf{W}$ (*sign is important*)
$U(\boldsymbol{r}_2) - U(\boldsymbol{r}_1) = -\int_\Gamma \mathbf{F} \cdot d\boldsymbol{r}$

Or, $U(\boldsymbol{r} + d\boldsymbol{r}) - U(\boldsymbol{r}) = -\mathbf{F} \cdot d\boldsymbol{r}$

Note that $U(\boldsymbol{r})$ is a scalar.

- Think of a ball on a hill.



$U(h) - U(y) = -\int_y^h -mg\ dy' = mgh - mgy$

F points from high U to low U.

Example 4.1

Three line integrals

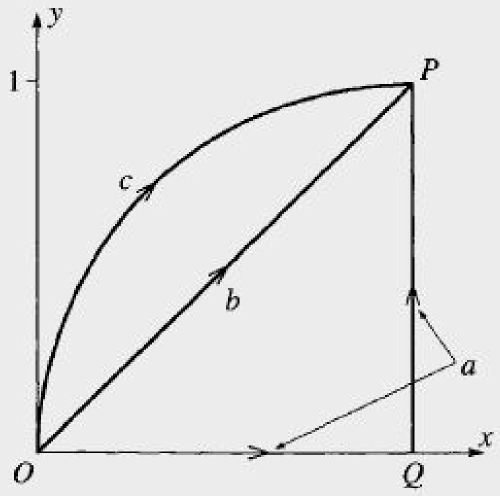


Figure 4.2 Three different paths, a, b, and c, from the origin to the point $P = (1, 1)$.

For a conservative force,

$$W_a = W_b = W_c = -[\,U(P) - U(0)\,]$$

where $W_\alpha = \int_\alpha \boldsymbol{F} \cdot d\boldsymbol{r}$ (def. of work)

For a conservative force,

- $W_{i \to f} = U(i) - U(f)$
  for any path from i to f ;
- W = the work done by **F**;
- U(**r**) = the potential energy corresponding to **F**.

Taylor's Example 4.2

The potential energy for a charge q in a static (*time independent*) electric field $\mathbf{E}(\mathbf{r})$.

$$U(\mathbf{r}_1) - U(\mathbf{r}_2) = \int_\Gamma q\, \mathbf{E} \,.d\mathbf{r}$$

Recall from PHY 184: for a static field,

$$\mathbf{E} = -\nabla V; \qquad \textit{(V = voltage)}$$

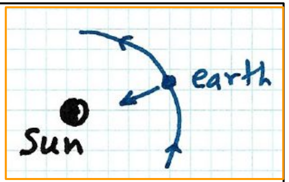
so

$$U(\mathbf{r}_1) - U(\mathbf{r}_2) = -[\, q\, V(\mathbf{r}_2) - q\, V(\mathbf{r}_1)\,]$$

$$U(\mathbf{r}_i) = q\, V(\mathbf{r}_i)$$

Suppose that q is positive; then the potential energy is high where the voltage is high; the force points from higher potential energy to lower potential energy.

Example

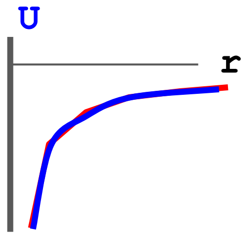


The gravitational potential energy of the Earth, ***due to the gravitational force exerted by the Sun ...***

Newton's theory,

$$\boldsymbol{F(r)} = -\frac{GMm}{r^2}\, \boldsymbol{e_r}$$

$$U(\boldsymbol{r}) = -\frac{GMm}{r}$$



Why is $U(\boldsymbol{r})$ negative?

(1) $U(r) - U(\infty) = \int_r^\infty (-GMm)/r'^2\, dr'$

$= GMm/r' \,\big|_r^\infty = -\,GMm/r$

(2) Or: increasing r is like "going up hill" .

Mechanical energy of a particle

$$E = T + U$$

$$E = \tfrac{1}{2}\, m\, v^2 + U(\mathbf{r})$$

Why is this important?

Theorem:

If the force is conservative then E is a constant of the motion.

Proof:

$$\frac{dE}{dt} = \frac{dT}{dt} + \frac{dU(\vec{r})}{dt}$$

$$= \vec{F}\cdot\vec{v} + \frac{\partial U}{\partial \vec{r}}\cdot\frac{d\vec{r}}{dt}$$

$$= \vec{F}\cdot\vec{v} - \vec{F}\cdot\vec{v} = 0$$

Several Forces

Suppose $\quad m\dfrac{d\vec{v}}{dt} = F_1 + F_2 + F_3 \ldots + F_n$

where all the forces are conservative.

Each force has a corresponding P.E.,

$$\vec{F}_i \Rightarrow U_i(\vec{r})$$

Then $\quad \dfrac{dT}{dt} = \displaystyle\sum_{i=1}^{n} \vec{F}_i\cdot\vec{v} = \sum_{i=1}^{n} \vec{F}_i\cdot d\vec{r}/dt$

$$= \sum_{i=1}^{n} - \frac{dU_i}{dt}$$

$E = T + \displaystyle\sum_{i=1}^{n} U_i$ is constant. $\quad \boxed{\dfrac{dE}{dt} = 0}$

"total mechanical energy"

Note: $\Delta U = -W$; so $dU = -\mathbf{F} \,.\, d\mathbf{R}$; that is, $\partial U/\partial \mathbf{r}_\mathbf{i} = -\mathbf{F}_\mathbf{i}$.

Nonconservative Forces

Suppose $m\frac{d\vec{v}}{dt} = \vec{F}_{cons} + \vec{F}_{nc}$

Define $E = \frac{1}{2}mv^2 + U_{cons.}$

$$dE = m\vec{v}\cdot d\vec{v} + dU_{cons.}$$

$$= \vec{v}\cdot\vec{F}\,dt - \vec{F}_{cons}\cdot d\vec{r}$$

$$= (\vec{F} - \vec{F}_{cons})\cdot d\vec{r} = \vec{F}_{nc}\cdot d\vec{r}$$

$$\Delta E = \int_\Gamma \vec{F}_{nc}\cdot d\vec{r} = W_{nc}$$

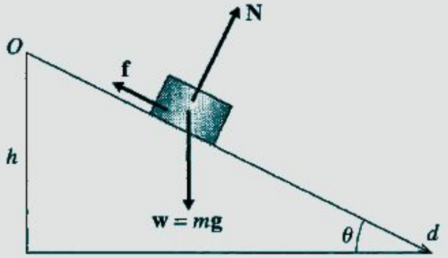


Figure 4.6 A block on an incline of angle θ. The length of the slope is d, and the height is $h = d\sin\theta$.

Example 4.3 ***a block sliding down an incline***

We solved this example before using forces.
Now, use *energies* to find the final velocity.
Friction is not conservative; *so the principle is ,*

$$\Delta(T+U) = W_{nc}$$

$$\Delta T = T_{bottom} - T_{top} = \frac{1}{2}mv^2$$

$$\Delta U = U_{bottom} - U_{top} = -mgh$$

$$W_{nc} = \int_0^d \vec{f}\cdot d\vec{r} = \int_0^d (-)\mu mg\cos\theta\, ds$$

do you understand the minus sign?

Result,

$$\frac{1}{2}mv^2 - mgh = -\mu mg\cos\theta\; d$$

$h = d\sin\theta$

$$v^2_{bottom} = 2gd(\sin\theta - \mu\cos\theta)$$

Example. Calculate the *escape velocity* from the surface of the Earth.

That is, if an object is at the surface of the Earth and moving upward with speed $v > v_{escape}$, then the object will escape from Earth's gravity.

$$E = \frac{1}{2} m v_r^2 - \frac{GMm}{r} \quad \text{(constant)}$$

$$\frac{1}{2} m v_R^2 - \frac{GMm}{R} = \frac{1}{2} m v_\infty^2 > 0$$

$$v_R > \sqrt{\frac{2GM}{R}} = 11{,}200 \text{ m/s}$$

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This is a pretty long assignment, so start working on it today.