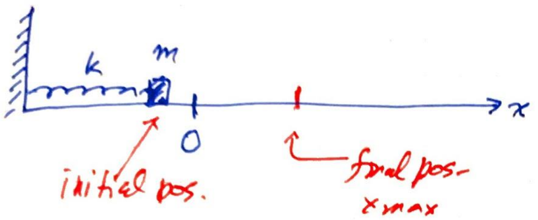
QUIZ Question (FRI 13)



$x_0 = -1$ m

$v_0 = 2$ m/s

$k/m = 3\ s^{-2}$

$$\frac{1}{2} m v^2 + \frac{1}{2} k x^2 = E = \text{constant}$$

$$\frac{1}{2} m v_0^2 + \frac{1}{2} k x_0^2 = \frac{1}{2} k x_f^2$$

$$x_f = \sqrt{x_0^2 + \frac{m v_0^2}{k}} = \sqrt{1 + \frac{4}{3}}\ m = \sqrt{\frac{7}{3}}\ m$$

Continuing Chapter 4 - Energy

Starting now, PHY 321 becomes more difficult.

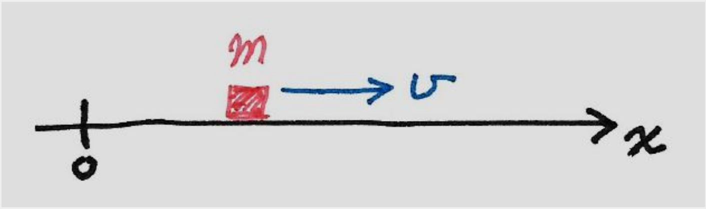
SECTION	TITLE	
4.6	Energy in linear motion	MONDAY
4.7	Curvilinear motion, and 1D systems	MONDAY
4.8	Central forces	WEDNESDAY
4.9	Energy for a system of 2 particles	FRIDAY
4.10	Energy for many particles, and rigid bodies	FRIDAY

Homework Assignment 8 includes four computer problems.

Section 4.6. Energy in linear motion (Review)

We went over this last time, and you have already read Section 4.6.

Here we are concerned with *linear motion of a particle in a potential.*



- the coordinate is x
- the equation of motion is

$$m\ddot{x} = F(x) = -\,dU/dx$$

- the energies are $T = \frac{1}{2} m \dot{x}^2$ and $U(x)$

Solve the equation of motion

The first integral **($x'' \Rightarrow x'$)** comes from conservation of energy

$$\dot{x} = \pm\sqrt{\frac{2}{m}\left[E - U(x)\right]}$$

The second integral **($x' \Rightarrow x$)** comes from the time calculation

$$dt' = \frac{dx'}{v} = \frac{dx'}{\dot{x}}$$

Details – *the sign and the turning points and the energy* – need to be worked out …

$$\int_{t_0}^{t} dt' = \int_{x_0}^{x} \frac{dx'}{\pm\sqrt{\frac{2}{m}\left[E - U(x')\right]}}$$

$$t - t_0 = \pm\sqrt{\frac{m}{2}} \int_{x_0}^{x} \frac{dx'}{\sqrt{E - U(x')}}$$

A trivial example: the free fall problem; we did it last time.

A less trivial example: a mass on a spring; Taylor Problem 4.28. (assigned)

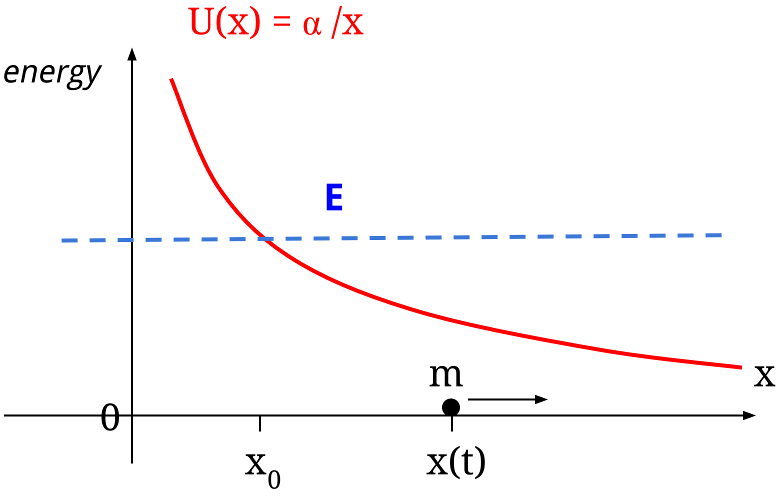
Here is a nontrivial example:

A particle moves on the x axis, with $x > 0$, and the potential energy is $U(x) = \alpha / x$. Assume α is positive, so the particles is repelled from the origin.

For example, consider a fixed charge at the origin and a moving charge (the "particle") on the positive x axis.

Suppose the particle is released from rest at x_0 . Calculate x(t).

Sketch a picture.



The problem is to calculate x(t).

Equations

$$m\ddot{x} = \alpha / x^2$$

$$x(0) = x_0 \quad \text{and} \quad \dot{x}(0) = 0$$

We can't solve them directly.
We'll use the conservation of energy to get the first integral.

1. Conservation of energy

$$E = \frac{1}{2} m \dot{x}^2 + \frac{\alpha}{x} = \frac{\alpha}{x_0}$$

$$\dot{x}^2 = \frac{2\alpha}{m}\left(\frac{1}{x_0} - \frac{1}{x}\right)$$

$$\dot{x} = + \sqrt{\frac{2\alpha}{m x_0}} \left(1 - \frac{x_0}{x}\right)^{1/2}$$

2. The time calculation

$$dt' = \frac{dx'}{v} = \frac{dx'}{\dot{x}}$$

$$\int_0^t dt' = \int_{x_0}^{x} \frac{dx'}{\sqrt{\frac{2\alpha}{m x_0}} \left(1 - x_0/x'\right)^{1/2}}$$

$$t = \sqrt{\frac{m x_0}{2\alpha}} \int_{x_0}^{x} \left(\frac{x'}{x' - x_0}\right)^{1/2} dx'$$

$$t = \sqrt{\frac{m x_0^3}{2\alpha}} \int_1^{x/x_0} \left(\frac{u}{u-1}\right)^{1/2} du$$

3. Final Result

$$t = \sqrt{\frac{m x_0^3}{2\alpha}} \left\{ \left[\frac{x}{x_0}\left(\frac{x}{x_0} - 1\right)^{1/2} + \operatorname{arcsinh}\left[\left(\frac{x}{x_0} - 1\right)^{1/2}\right] \right] \right\}$$

$$t = \sqrt{\frac{m x_0^3}{2\alpha}} \left\{ \frac{1}{2} \sinh 2\psi + \psi \right\}$$ ← parametric equations

where $\sinh \psi = \left(\frac{x}{x_0} - 1\right)^{1/2}$; $x = x_0 \cosh^2 \psi$

Section 4.7. Curvilinear motion and other examples of one-dimensional motion

A system is called "one-dimensional" if the configuration is determined by a single dynamical variable.

Linear motion is strictly one-dimensional; but it's not the only kind of "one-dimensional" motion in the generalized sense.

Curvilinear Motion

Consider a particle that is constrained to move on a curve. That is "one-dimensional" because only a single variable is required to specify the position of the particle.

s = arclength

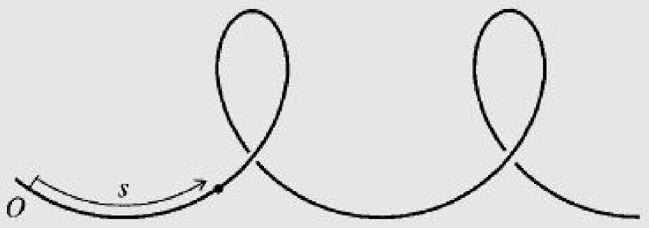


Figure 4.13 An object constrained to move on a curved track can be considered to be a one-dimensional system, with the position specified by the distance s (measured

Even this train moving on a curve is an example of *one-dimensional motion* in the generalized sense;

the configuration of the moving object is determined by a single variable, $s(t)$.

Curvilinear Motion

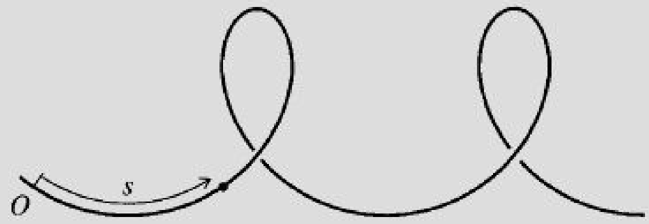
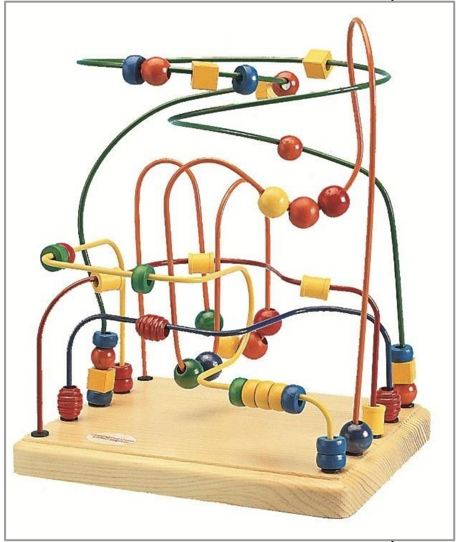


Figure 4.13 An object constrained to move on a curved track can be considered to be a one-dimensional system, with the position specified by the distance s (measured

- The variable is s = arclength ; $s = s(t)$
- The force equation is $m\ddot{s} = F_{tangential}$
- The energies are $T = \frac{1}{2} m \dot{s}^2$

  and $U(s)$ where $F_{tang.} = -dU/ds$

- But what keeps the particle on the curve?



Picture m as a bead threaded on a stiff wire.

Normal force

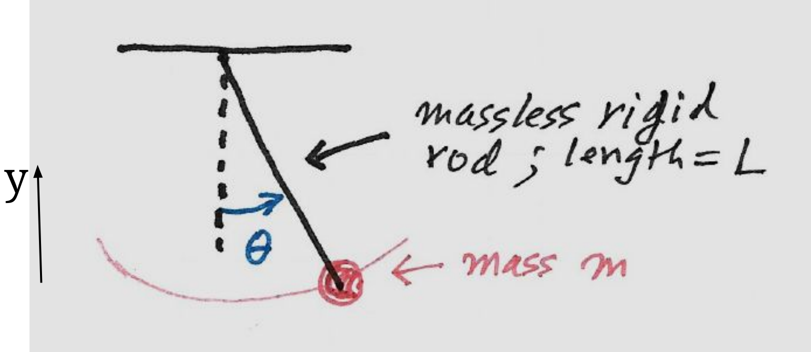
≡ force of constraint ;

≡ keeps the particle on the curve ;

≡ *but it does no work!!!* ;

≡ ΔT comes only from $F_{tang.}$.

The Simple Pendulum
(Taylor Problems 4.34 and 4.38)



The mass moves on a circular arc.

- The variable is θ = angle; $\theta = \theta(t)$
- The force equation ? *requires torque*
- The energies are $T = \frac{1}{2} m L^2 (\dot{\theta})^2$

and $U(\theta) = mgy = mg L (1 - \cos \theta)$

- The equation of motion.

We could write $d\boldsymbol{l}/dt$ = torque;

or, $dE/dt = 0$ implies

$$\frac{1}{2} m L^2 \, 2\dot{\theta}\ddot{\theta} + m g L \sin\theta \, \dot{\theta} = 0$$

$$\ddot{\theta} = -\frac{g}{L} \sin\theta$$ *the simple pendulum*

This is "1-dimensional" in the generalized sense:
$\exists$ *a single dynamical variable .*

Assigned in the homework.

Example 4.7

STABILITY OF A CUBE BALANCED ON A CYLINDER

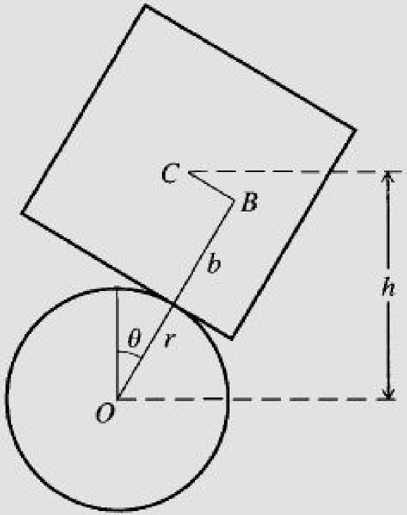
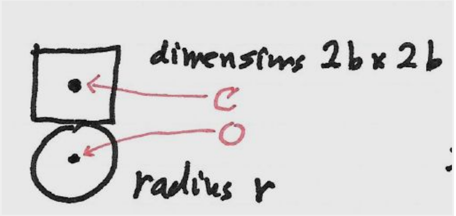


Figure 4.14 A cube, of side $2b$ and center C, is placed on a fixed horizontal cylinder of radius r and



The cylinder is fixed.

The cube is free to roll from side to side, not slipping on the cylinder. {*center directly above center for θ=0*}

Calculate U(θ).

This is a "1d" problem; the variable is θ. Analyze the potential energy. (See the Figure.)

Let h = the height of the center of mass of the cube. Then U = m g h . Now express h in terms of the angle θ .

Example 4.7

STABILITY OF A CUBE BALANCED ON A CYLINDER

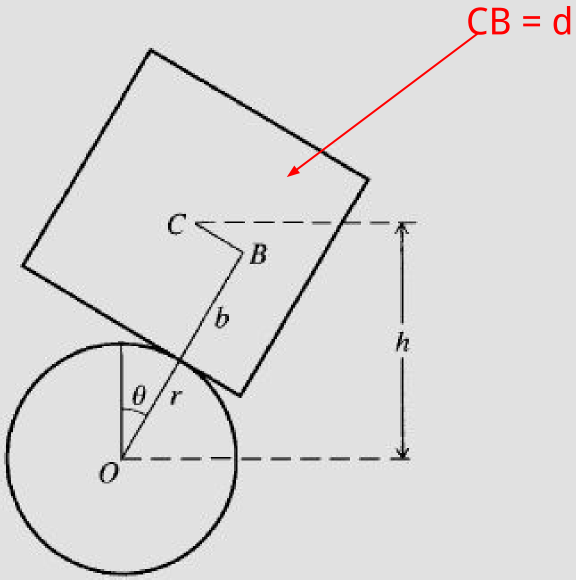


Figure 4.14 A cube, of side $2b$ and center C, is placed on a fixed horizontal cylinder of radius r and

Geometrical analysis

$$r\theta = d = CB$$

$$h = (r+b)\cos\theta + d\sin\theta$$

$$U(\theta) = mgh = mg\left[(r+b)\cos\theta + r\theta\sin(\theta)\right]$$

Condition for equilibrium

$$\frac{dU}{d\theta} = mg\left[-(r+b)\sin\theta + r\sin\theta + r\theta\cos\theta\right]$$

(cancel)

$U'(0) = 0$ so $\theta = 0$ is an equilibrium configuration

Condition for stability

$$\frac{d^2U}{d\theta^2} = mg\left[-b\cos\theta + r\cos\theta - r\theta\sin\theta\right]$$

$$U''(0) = mg(r-b)$$

The condition for stability is $U''(0) > 0$; $\boxed{b \le r}$ stable equilibrium

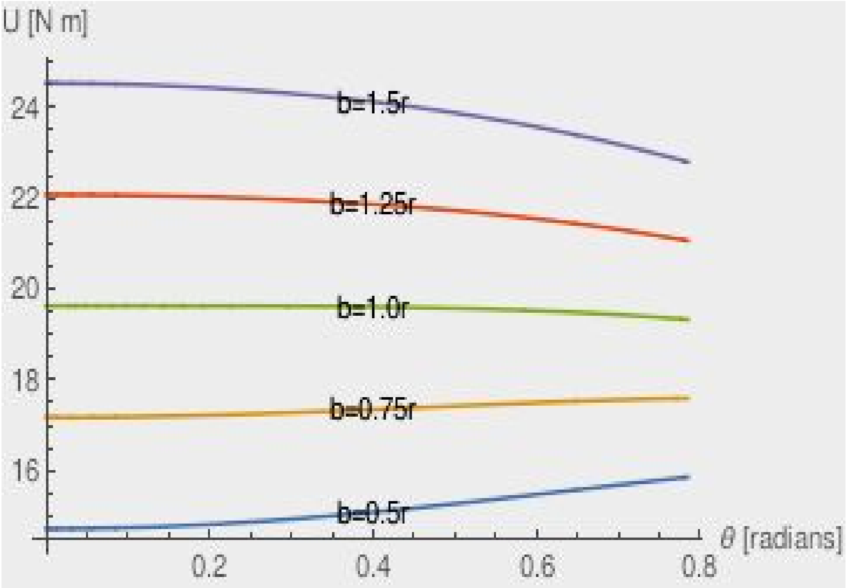
STABILITY ANALYSIS

Plot U(θ) for different values of b/r .
(2r=diameter;   2b = width)

The cube balanced at θ = 0 is stable
if b ≦ r ;

i.e., θ = 0 is a ***stable equilibrium***
if the width of the cube is smaller than
the diameter of the cylinder.

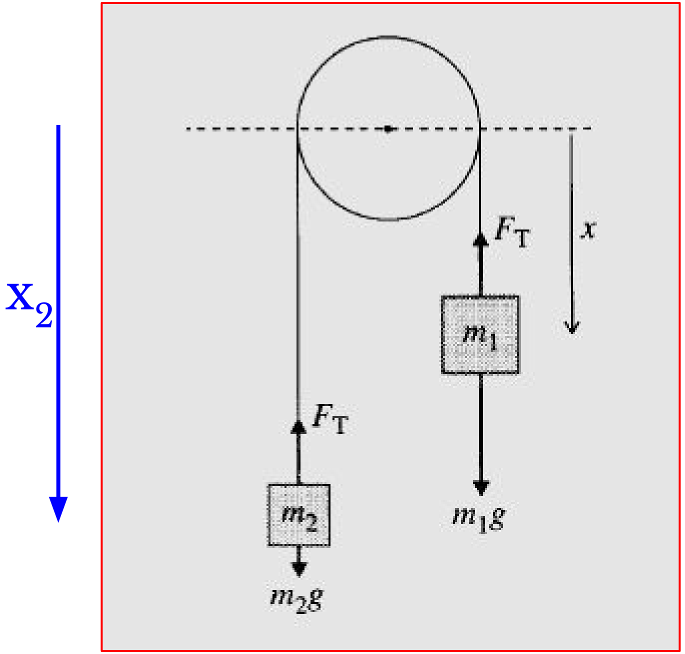
Assigned Problem 4.33.



Another example:

The Atwood machine

Figure 4.15



Atwood machine

The configuration depends on a single variable (x) because the length of the string is constant (L);

$$L = x_2 + \pi R + x \; ; \qquad \text{or} \qquad x_2 = L - \pi R - x$$

Analysis by energies:

$T = T_1 + T_2 = ½ (m_1+m_2) x'^2$

$U_1 = - m_1 g x$ *(x is downward)*

$U_2 = - m_2 g x_2 = + m_2 g x + \textit{constant}$

$$E = ½ (m_1 + m_2) x'^2 + (m_2 - m_1) g x$$

Energy is constant, so

$dE/dt = 0 = (m_1 + m_2) x' x'' + (m_2 - m_1) g x'$

$$(m_1 + m_2) x'' = (m_1 - m_2) g$$

Result : constant acceleration,

$$a = (m_1 - m_2) / (m_1 + m_2) \; g$$

Homework Assignment #8

due in class Wednesday, October 25

[37] Problem 4.26 *

[38] Problem 4.28 ** and Problem 4.29 ** [Computer]

[39] Problem 4.33 ** [Computer]

[40] Problem 4.34 **

[41] Problem 4.37 *** [Computer]

[42] Problem 4.38 *** [Computer]

Use the cover page.

- *This is a long assignment, so start working on it now.*
- *Work together with 1 or 2 other students on the computer calculations.*