Chapter 5. Oscillations

Section 5.1. *Hooke's law*
Section 5.2. *Simple Harmonic Motion*

Read Sections 5.1 and 5.2.

Robert Hooke (1635 – 1703) lived at the same time as Isaac Newton.
(Hooke was a little older.)

They worked on similar topics in physics [mechanics; optics; microscopes (Hooke) and telescopes (Newton)].

But they were not friends, because each one thought that he was superior to the other guy.

5.1. Hooke's law

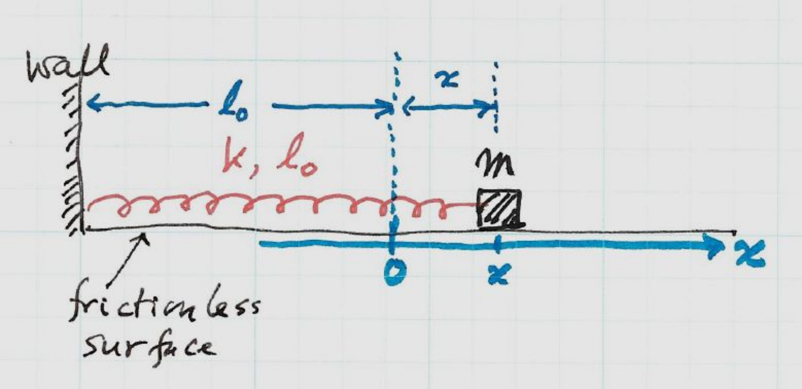
- The force exerted by a spring (stretched or compressed) is $F = -k(l - l_0)$;

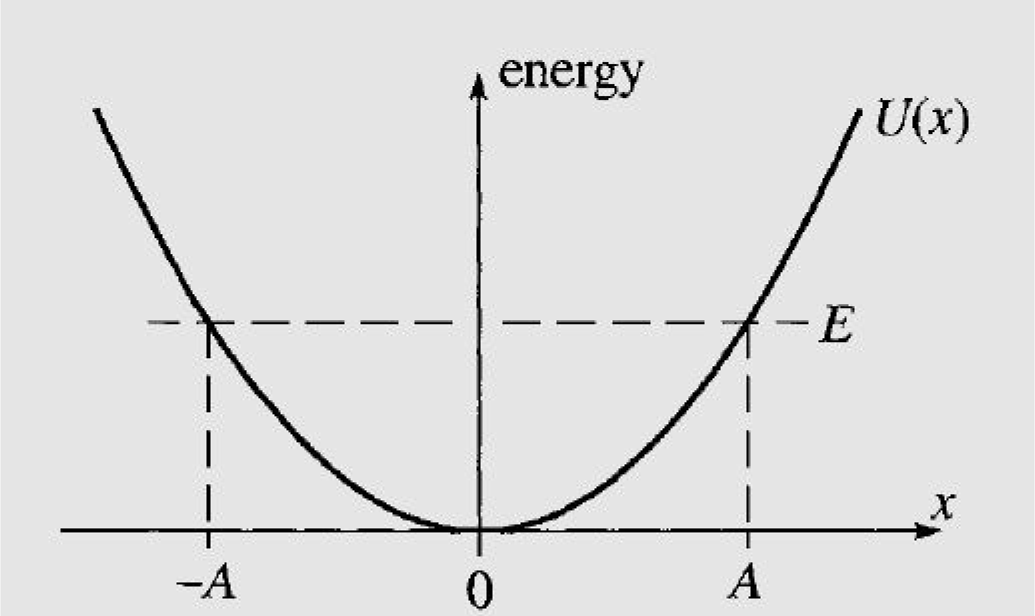
  (l = *length* , l_0 = *equilibrium length*)

- In the figure, let x be the displacement of m from equilibrium; i.e., the length of the spring is $l = l_0 + x$.

- Primary equations

  $F(x) = -k x$; $U(x) = \frac{1}{2} k x^2$; $F = -dU/dx$





You must understand …

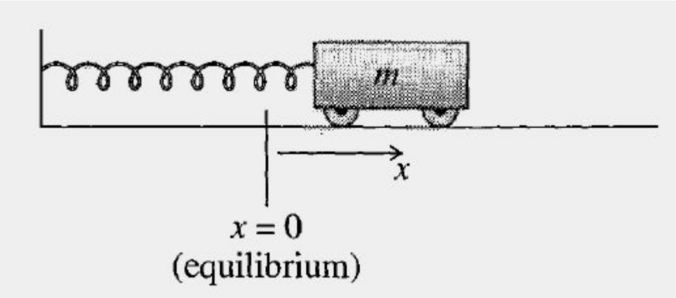
$$E = \tfrac{1}{2} k A^2$$

and $E = \tfrac{1}{2} m v_0^2$

and $E = \tfrac{1}{2} m v^2 + \tfrac{1}{2} k x^2$.

Figure 5.1 A mass m with potential energy $U(x) = \frac{1}{2}kx^2$ and total energy E oscillates between the two turning points at $x = \pm A$, where $U(x) = E$ and the kinetic energy is zero.

5.2. Simple Harmonic Motion



The equation of motion is

$$m \ddot{x} = - k x$$

Or, write

$$\ddot{x} = - \omega^2 x \qquad (1)$$

where $\omega = \sqrt{k/m}$.

Eq. (1) has many solutions …

- sine and cosine solutions;
- *complex* exponential solutions;
- linear combinations of solutions (*the superposition principle*);
- ***Initial conditions*** *are necessary to determine a unique solution.*

We could write the solution in several ways. We could write …

$$x(t) = A \cos(\omega t) + B \sin(\omega t) ;$$

in this form, the initial position is

$$x_0 = x(0) = A$$

and the initial velocity is

$$v_0 = \dot{x}(0) = \omega B .$$

$$(A = x_0 \text{ and } B = v_0 / \omega)$$

Figure 5.3

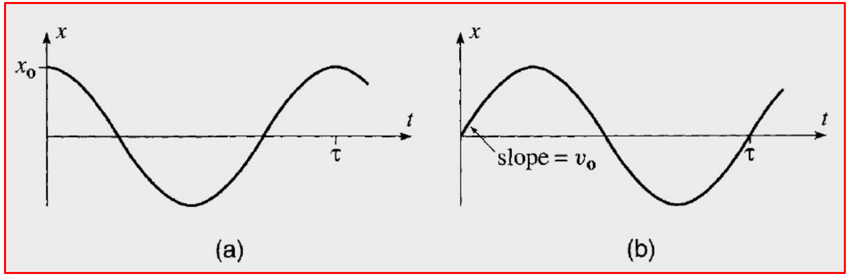


Figure 5.3 (a) Oscillations in which the cart is released from x_0 at $t = 0$ follow a cosine curve. (b) If the cart is kicked from the origin at $t = 0$, the oscillations follow a sine curve with initial slope v_0. In either case the period of the oscillations is $\tau = 2\pi/\omega = 2\pi\sqrt{m/k}$ and is the same whatever the values of x_0 or v_0.

Example (b) is an example of a *phase-shifted cosine solution*, where the phase shift is 90 degrees.

(a) : $x(t) = x_0 \cos(\omega t)$

(b) : $x(t) = (v_0/\omega) \sin(\omega t)$

$= (v_0/\omega) \cos(\omega t - \pi/2)$ 90 deg

∗The general phase-shifted cosine solution is

$$x(t) = A \cos(\omega t - \delta) .$$

A = amplitude;
δ = phase shift .

This is the same as

$$x(t) = B_1 \cos(\omega t) + B_2 \sin(\omega t) ,$$

where $B_1 = A \cos \delta$ and $B_2 = A \sin \delta$.

∗ Or, the general solution could be written as the real part of a complex exponential; e.g.,

$$x(t) = C_1\, e^{\,i\omega t} + C_1^{*}\, e^{\,-i\omega t}$$

//second derivative of $e^{\pm i\omega t} = -\omega^2 e^{\pm i\omega t}$ //

Note: $z + z^{*} = 2\,\mathrm{Re}\,(z)$

Relations between different functional forms

(1) $x(t) = B_1 \cos \omega t + B_2 \sin \omega t$

$$= A \cos(\omega t - \delta)$$

$$= A \cos\delta \cos\omega t + A \sin\delta \sin\omega t$$

So $\begin{cases} B_1 = A\cos\delta \\ B_2 = A \sin\delta \end{cases}$; or: $\begin{array}{l} B_1^2 + B_2^2 = A^2 \\ \tan\delta = B_2/B_1 \end{array}$

(2) $x(t) = C_1 e^{i\omega t} + C_1^* e^{-i\omega t}$

where $C_1 = |C_1| e^{-i\delta}$

$$= |C_1| \left\{ e^{i(\omega t - \delta)} + e^{-i(\omega t - \delta)} \right\}$$

$$= |C_1| \cdot 2 \cos(\omega t - \delta)$$

$$= A \cos(\omega t - \delta)$$

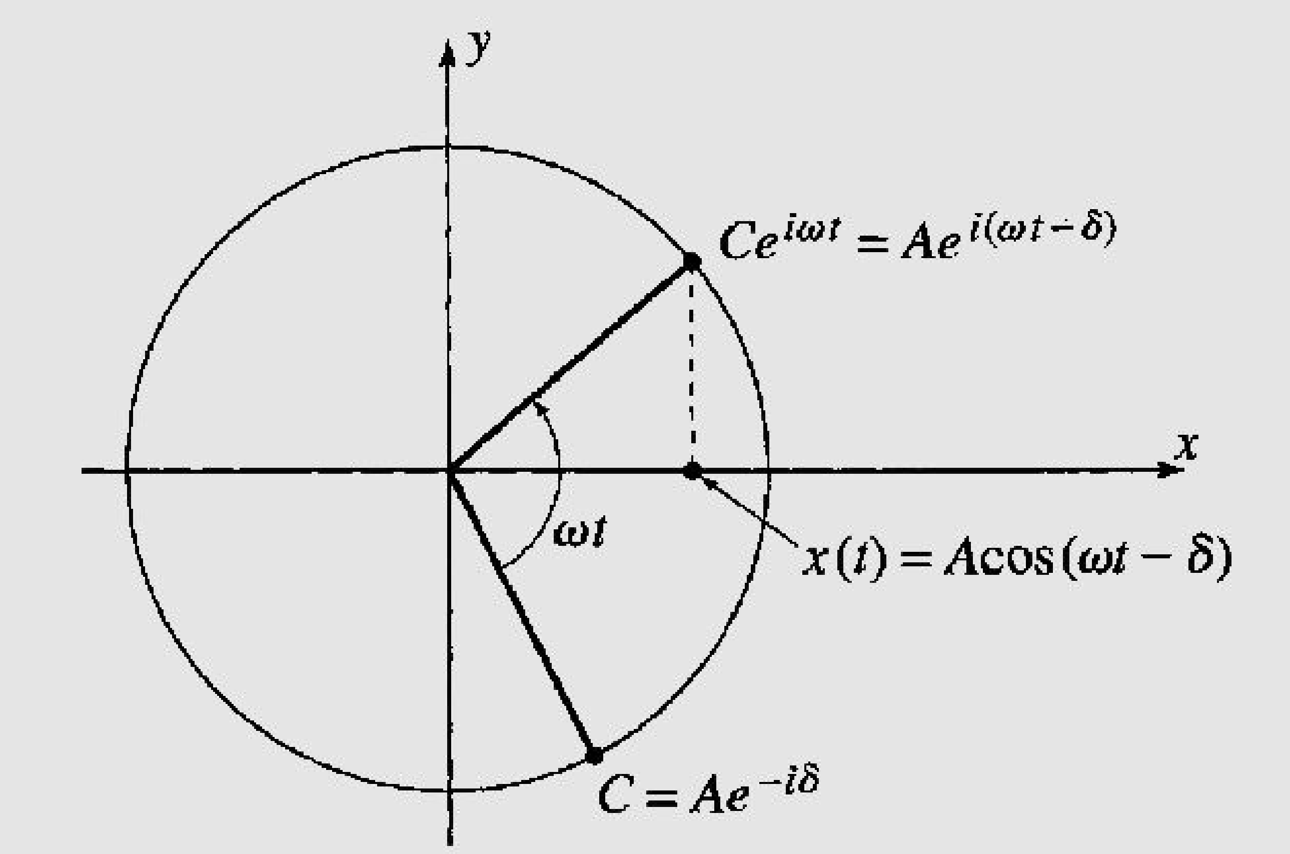
$\therefore A = 2|C_1|$ and $C_1 + C_1^* = A\cos\delta$

Figure 5.5

A geometrical picture of the complex exponential function

$$C\, e^{\,i\,\omega t}$$

- x + i y undergoes clockwise circular motion;
- x undergoes S H M with phase shift δ
- y undergoes S H M with phase shift δ + π/2



- Simple harmonic motion occurs for a mass attached to the end of a spring.

- Simple harmonic motion occurs in many other examples in mechanics.

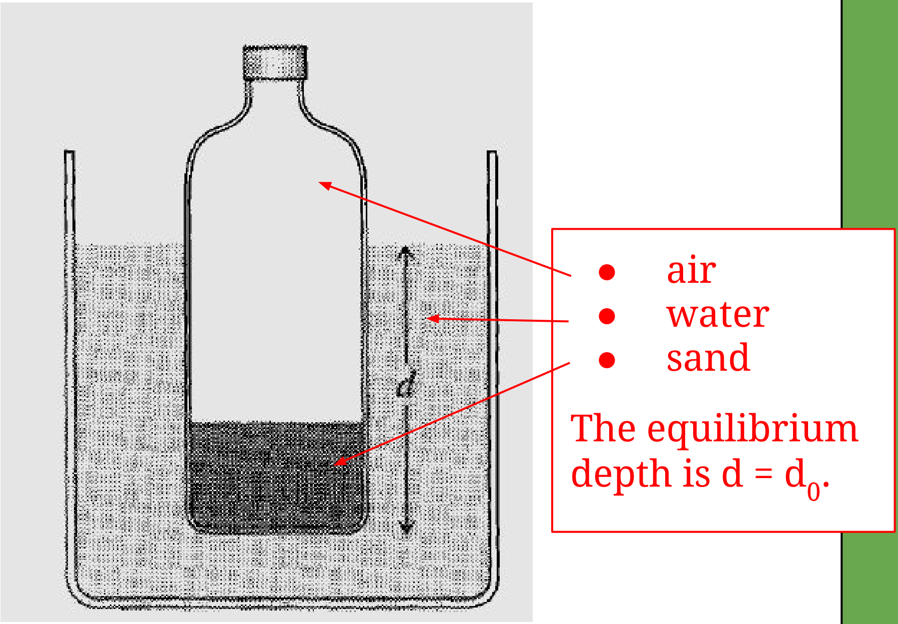
- Harmonic time dependence
  ($\cos \omega t$ or $\exp i\omega t$)
  occurs in many other examples in physics.

- For example, waves,

  $\Phi = \cos (kx - \omega t)$ or $\exp i(kx - \omega t)$

Example 5.2

a bottle in a bucket of water



Show that the bottle undergoes S. H. M.

Let x be the displacement downward from equilibrium.

Then $d = d_0 + x$. **Understand the sign.**

Newton's second law,

$$m\ddot{x} = mg - \rho_w\, g\, A\, (d_0 + x)$$

gravity and buoyancy forces

Equilibrium is at $x = 0$, so

$$mg = \rho_w\, g\, A\, d_0 \ .$$

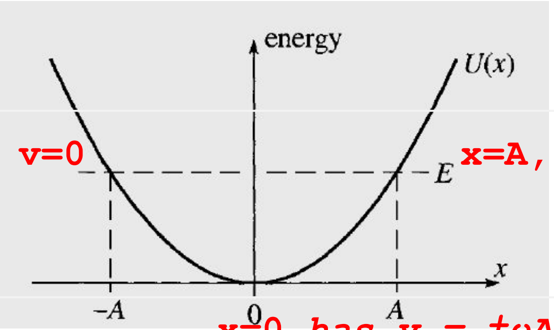
Thus $\ddot{x} = -\omega^2 x$ (★)

where $\omega^2 = \rho_w\, g\, A / m = g / d_0$.

And (★) is the equation for S. H. M.

Taylor: " Try the experiment yourself. But be aware that the details of the flow of water around the bottle complicate the situation. The calculation here is a very simplified version of the truth."

Energy considerations in S.H.M.



$\underline{x(t) = A\cos(\omega t - \delta)}$

Energy is conserved in SHM.

$T = \frac{1}{2} m (\dot{x})^2 = \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t - \delta)$

$U = \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \cos^2(\omega t - \delta)$

Recall, $m\omega^2 = k$.

$T + U = \frac{1}{2} k A^2$ **(constant in t)**

or, $T + U = \frac{1}{2} m\, v(0)^2$ **(same constant)**

Homework Assignment #9
due in class Friday, November 4

[41] Problem 4.41 and Problem 4.43
[42] Problem 5.3 *
[43] Problem 5.5 *
[44] Problem 5.9 *
[45] Problem 5.12 **
[46] Problem 5.18 ***

Use the cover sheet.

Do it now so you will have time
to study for the …

Second Exam: Friday November 3