1. Homework Assignment 14 is due Friday.

2. Final Exam is Tuesday December 12:

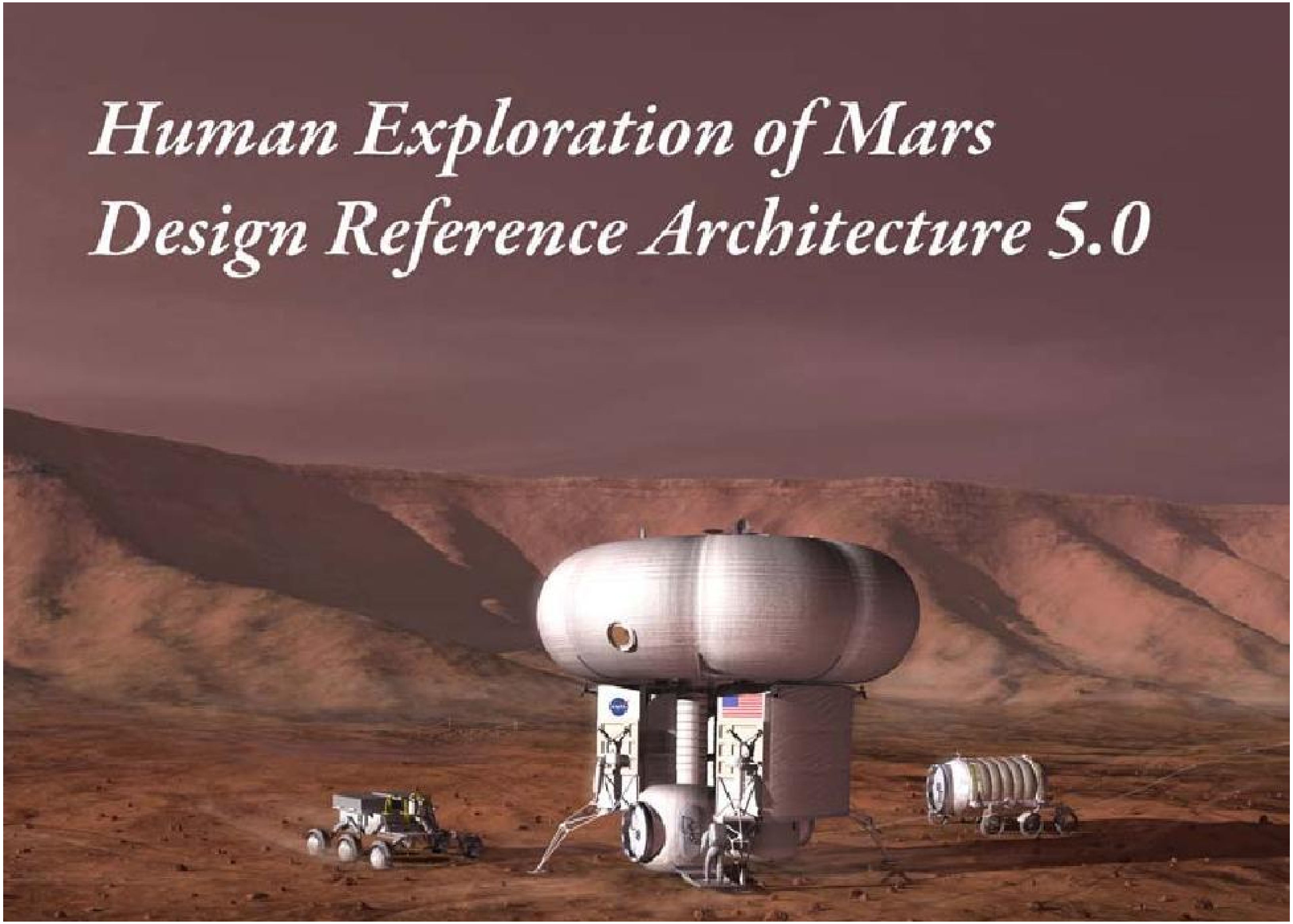
Final Exam: Tuesday, Dec 12 2017
12:45pm - 2:45pm
Room 1420 Biomedical & Physical Sciences Bldg

3. The final exam will be based on Chapter 8.

4. You will be given an equation sheet, which will consist of Taylor's "Principal Equations" on pages 319 and 320.

5. To study for the exam: (i) read and understand Chapter 8; (ii) make sure that you are familiar with pages 319 and 320; (iii) review the chapter 8 lectures and homework.

NASA is making plans to send astronauts to Mars (~ year 2040?)
If you want to read about it, search Google for the report ;
search for "Mars Design Reference Architecture 5.0 - NASA" (2009)



6.2 Decision 1: Mission Type

2 choices

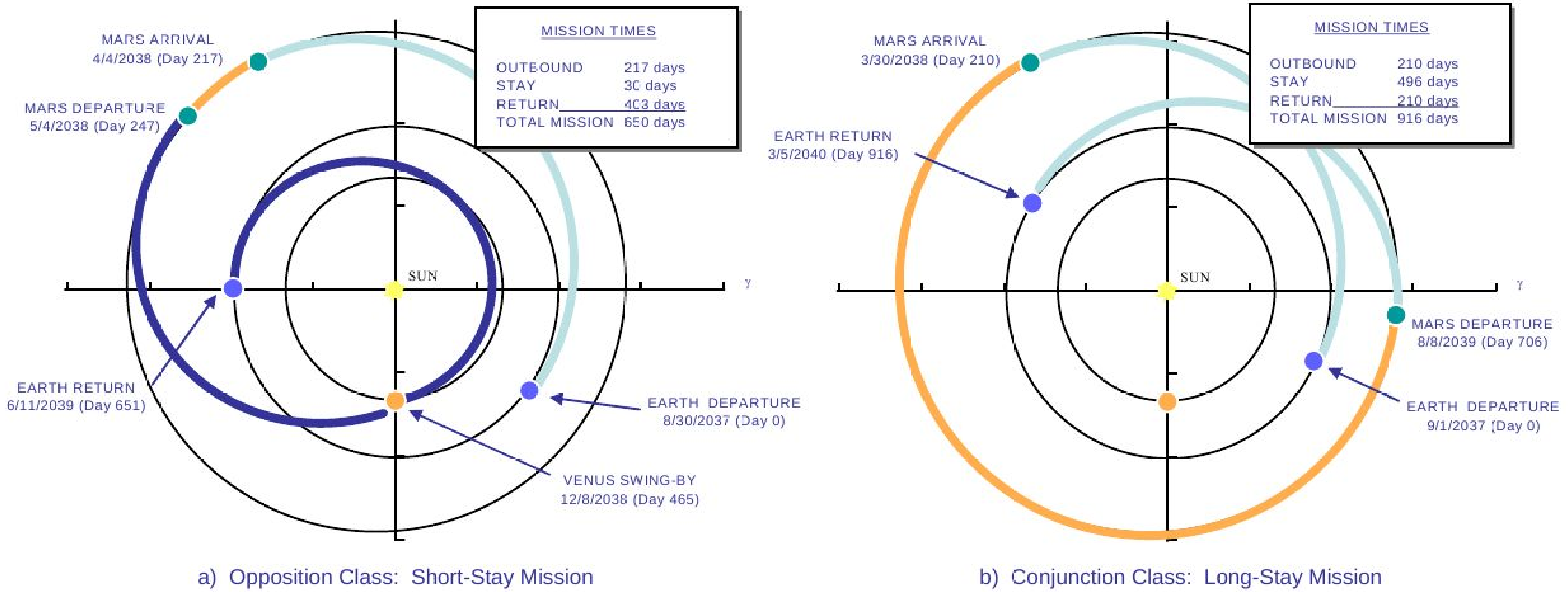


Figure 6-2. Comparison of (a) Opposition-class and (b) Conjunction-class mission profiles.

"short stay mission"
30 days on Mars;
go home via Venus.

"long stay mission"
496 days on Mars

Astrodynamics; Transfer orbits (Section 8.8); Gravity Assist "slingshot"; orbital timing

Parametric equations for Keplerian orbits

Kepler's Equation $M = E - \varepsilon \sin(E)$

This equation was published by Kepler (1619)

M = "mean anomaly"

E = "eccentric anomaly"

ε = eccentricity

The coordinates of the planet are

$x = a [\cos(E) - \varepsilon]$

$y = b \sin(E)$

where

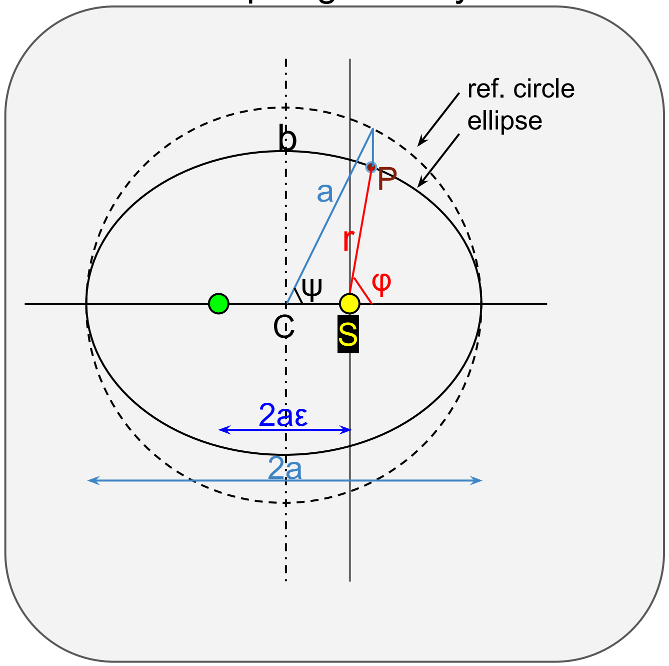
a = semimajor axis

b = semiminor axis

Kepler's equation is transcendental. Numerical analysis is necessary to solve for "E" .

I'll use ψ to denote the "eccentric anomaly".

Ellipse geometry



reference circle; radius = a

orbit ellipse; semimajor axis = a

C = center ; S = Sun ; P = planet

ψ = eccentric anomaly

$x = r \cos \varphi = a \cos \psi - a\varepsilon$

$y = r \sin \varphi = b \sin \psi$ ← *why? b/c*

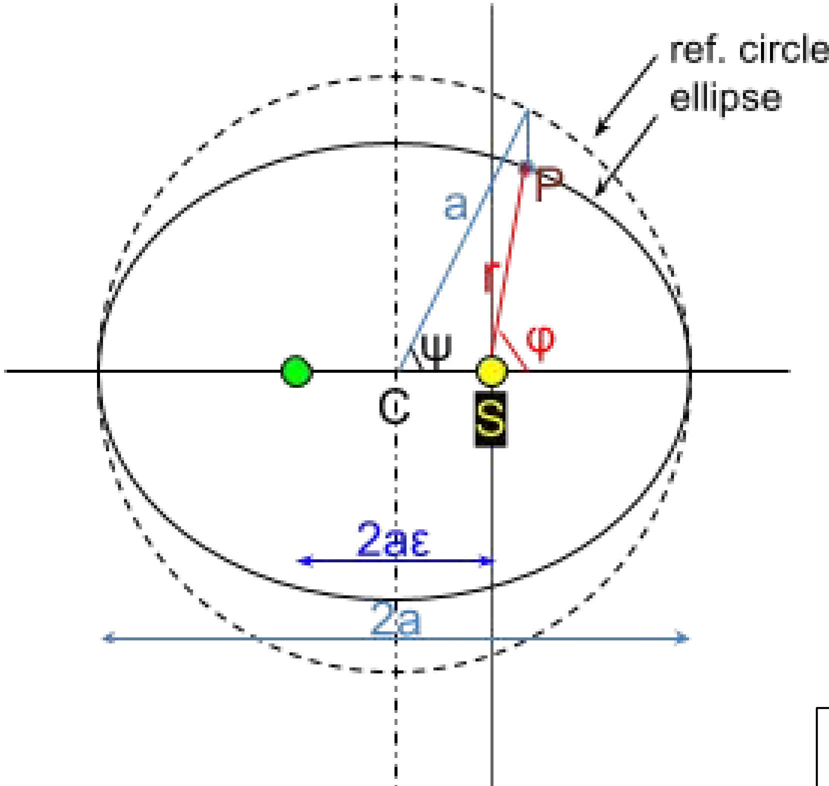
$(x+a\varepsilon)^2 / a^2 + y^2 / b^2 = 1$ *(ellipse)*

Max. $x = a (1 - \varepsilon)$ at $\psi = 0$; ***CHECK***

Max. $y = b$ at $\psi = \pi/2$;

$r = a$ at $\psi = \pi/2$

$(a\varepsilon)^2 + b^2 = a^2$ so $b = a$ SQRT[$1 - \varepsilon^2$]



reference circle; radius = a
orbit ellipse; semimajor axis = a
C = center ; S = Sun ; P = planet

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$x = r \cos \varphi = a \cos \psi - a\varepsilon$

$y = r \sin \varphi = b \sin \psi$

$(x+a\varepsilon)^2 / a^2 + y^2 / b^2 = 1$ *(ellipse)*

The sun is at the origin and the plane of the orbit has Cartesian coordinates x and y. The ***center*** is at $\{x,y\}=\{ -a\varepsilon, 0\}$.

We can write parametric equations for all three variables
(time = t and spatial coordinates = x and y)
in terms of the independent variable ψ :

$$t = T / (2\pi) \; (\psi - \varepsilon \sin \psi) \qquad \textbf{(1)}$$

$$x = a \; (\cos \psi - \varepsilon) \qquad \textbf{(2)}$$

$$y = a \; (1 - \varepsilon^2)^{1/2} \; \sin \psi \qquad \textbf{(3)}$$

The sun is at the origin and the plane of the orbit has Cartesian coordinates x and y.

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(time = t and spatial coordinates = x and y)

in terms of the independent variable ψ :

$t = T / (2\pi)\ (\psi - \varepsilon \sin \psi)$ **(1)**

$x = a (\cos \psi - \varepsilon)$ **(2)**

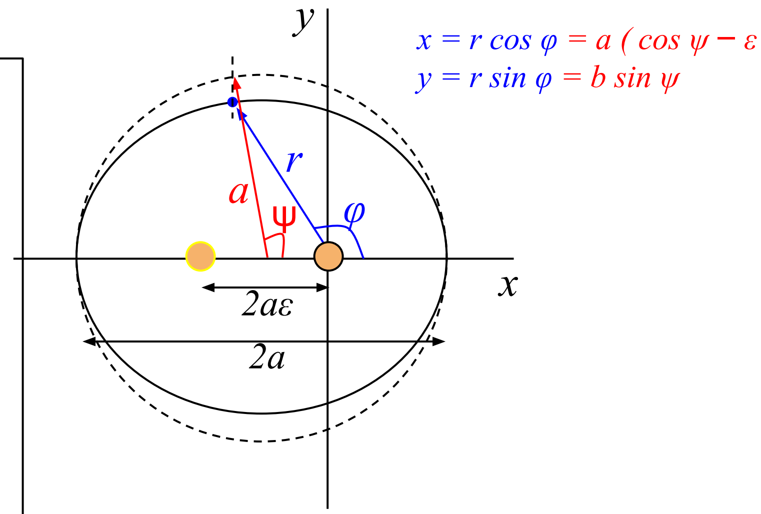
$y = a (1 - \varepsilon^2)^{1/2} \sin \psi$ **(3)**

The parameters T , a and ε are

T = period of revolution; **$\psi \mapsto \psi + 2\pi$**

a = semimajor axis

ε = eccentricity.



In term of Kepler's variables,

$\psi = E$

$t = T/(2\pi)\ M$

$M = E - \varepsilon \sin E$

Proof of the parametric equations.

$$t = T/(2\pi)\,(\psi - \varepsilon \sin\psi) \qquad \textbf{(1)}$$

$$x = a(\cos\psi - \varepsilon) \qquad \textbf{(2)}$$

$$y = a(1-\varepsilon^2)^{1/2} \sin\psi \qquad \textbf{(3)}$$

$$dt/d\psi = T/(2\pi)\,(1 - \varepsilon\cos\psi)$$

We must prove that E (energy) and ℓ (ang. momentum) are constants of the motion.

Theorem 1

The angular momentum (ℓ) is a constant of the motion.

Proof

$$\ell = \mu(x\dot{y} - y\dot{x})$$

$$dx/dt = (dx/d\psi)(d\psi/dt) = (-a\sin\psi)(2\pi/T)(1-\varepsilon\cos\psi)^{-1}$$

$$dy/dt = (dy/d\psi)(d\psi/dt) = (b\cos\psi)(2\pi/T)(1-\varepsilon\cos\psi)^{-1}$$

$$= \mu ab\{(\cos\psi - \varepsilon)\cos\psi + \sin^2\psi\}(2\pi/T)(1-\varepsilon\cos\psi)^{-1}$$

$= \mu ab(2\pi/T)$ which is constant ✓

Also note: $T = (\pi ab)(2\mu/\ell)$ which agrees with Kepler's second law ; $dA/dt = \ell/(2\mu)$ (3.17) ➡ $A/T = \ell/(2\mu)$ ✓

$$t = T / (2\pi) \; (\psi - \varepsilon \sin \psi) \qquad \textbf{(1)}$$

$$x = a (\cos \psi - \varepsilon) \qquad \textbf{(2)}$$

$$y = a (1 - \varepsilon^2)^{1/2} \sin \psi \qquad \textbf{(3)}$$

Theorem 2

The energy (E) is a constant of the motion. Proof

$$E = \tfrac{1}{2} \mu (\dot{x}^2 + \dot{y}^2) - \frac{\gamma}{r}$$

$$= \tfrac{1}{2} \mu \{ a^2 \sin^2 \psi + b^2 \cos^2 \psi \} \left(\tfrac{2\pi}{T} \right)^2 (1 - \varepsilon \cos \psi)^{-2} - \frac{\gamma}{a} (1 - \varepsilon \cos \psi)^{-1}$$

$$= \tfrac{1}{2} \mu a^2 (2\pi / T)^2 \{ 1 - \varepsilon^2 \cos^2 \psi \} (1 - \varepsilon \cos \psi)^{-2} - \frac{\gamma}{a} (1 - \varepsilon \cos \psi)^{-1}$$

$$= \left\{ \tfrac{1}{2} \mu a^2 \left(\tfrac{2\pi}{T} \right)^2 (1 + \varepsilon \cos \psi) - \frac{\gamma}{a} \right\} (1 - \varepsilon \cos \psi)^{-1}$$

$$= \{ c_1 + c_2 \, \varepsilon \cos \psi \} (1 - \varepsilon \cos \psi)^{-1} = E$$

(needs more!)

$$dx/dt = (dx/d\psi)(d\psi/dt)$$
$$= (-a \sin\psi)(2\pi/T)(1 - \varepsilon \cos\psi)^{-1}$$

$$dy/dt = (dy/d\psi)(d\psi/dt)$$
$$= (b \cos\psi)(2\pi/T)(1 - \varepsilon \cos\psi)^{-1}$$

$$r^2 = x^2 + y^2$$
$$= a^2 \{ (\cos \psi - \varepsilon)^2 + (1 - \varepsilon^2) \sin^2\psi \}$$
$$= a^2 \{ 1 - 2\varepsilon \cos\psi + \varepsilon^2 \cos^2\psi \}$$
$$= a^2 (1 - \varepsilon \cos\psi)^2$$

$$b^2 = (1 - \varepsilon^2) a^2$$

$$\frac{A^2 - B^2}{(A - B)^2} = \frac{A+B}{A-B}$$

So, we have this …

$$\{ C_1 + C_2\, \varepsilon \cos\psi \} (1 - \varepsilon \cos\psi)^{-1} = E$$

where

$$C_2 = ½\, \mu\, a^2 (2\pi/T)^2$$

and $C_1 = C_2 - \gamma/a.$

This must be a constant (E).

So, we require $C_2 / C_1 = -1$ and $C_1 = E$.

$C_1 (1 + C_2/C_1\, \varepsilon \cos)(1 - \varepsilon \cos)^{-1} = E$

$C_2 / C_1 = -1$

$C_1 = E$

$C_1 = C_2 - \gamma/a = - C_1 - \gamma/a$

$C_1 = - \gamma / (2a)$

$C_2 = - C_1 = \gamma / (2a)$

$\therefore \gamma / (2a) = ½\, \mu\, a^2 (2\pi/T)^2$

Result

The theorem is true, and E is given by

$$E = - \frac{\gamma}{2a}. \quad ✓$$

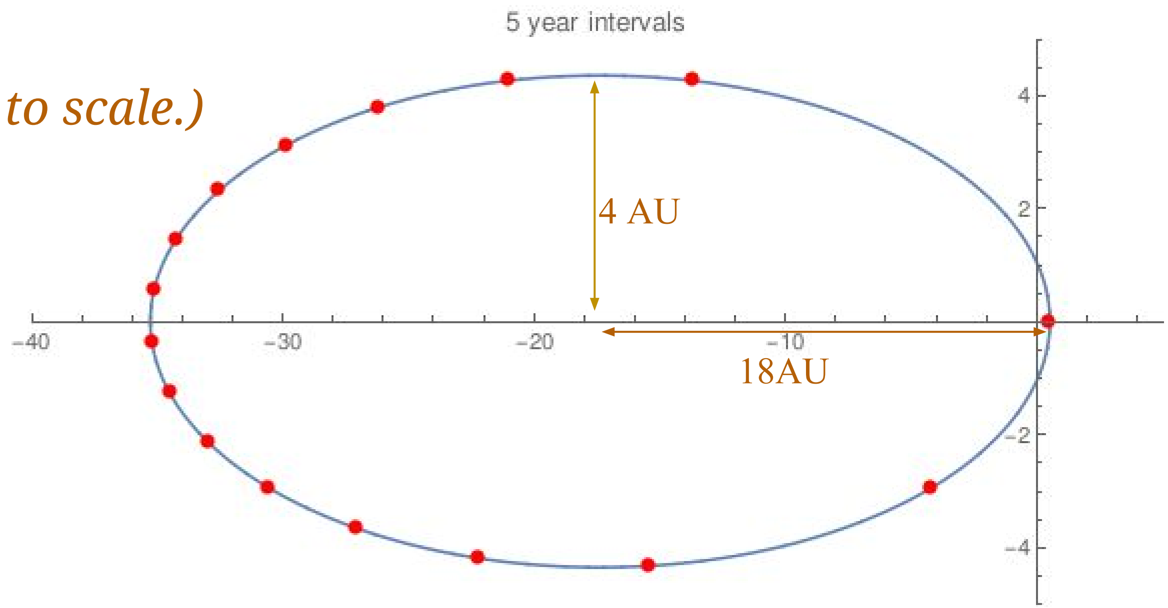
Also, we find $\gamma / (2a) = ½\, \mu\, a^2 (2\pi/T)^2$;

$$\therefore \quad T^2 = \frac{4\pi^2 a^3}{GM} \quad \text{because } \gamma / \mu = GM,$$

which is Kepler's third law.

Example A. The orbit parameters of Halley's comet are a = 17.9 AU and ε = 0.97. Plot of the orbit of Halley's comet.

(Not drawn to scale.)



Example B. Calculate the perihelion distance.

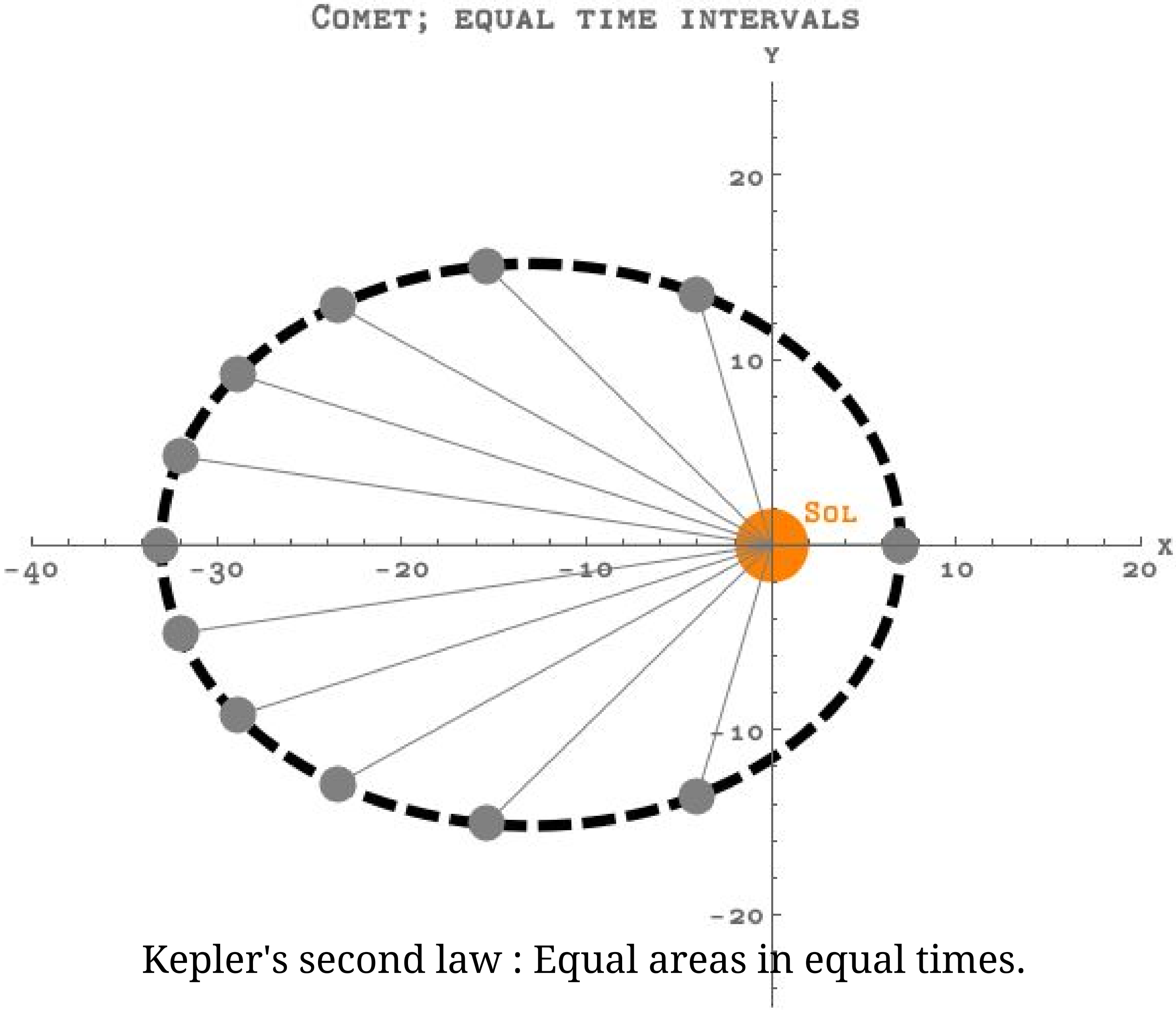
$r_{min} = a\,(1 - \varepsilon) = 0.537$ AU

Example C. Calculate the aphelion distance.

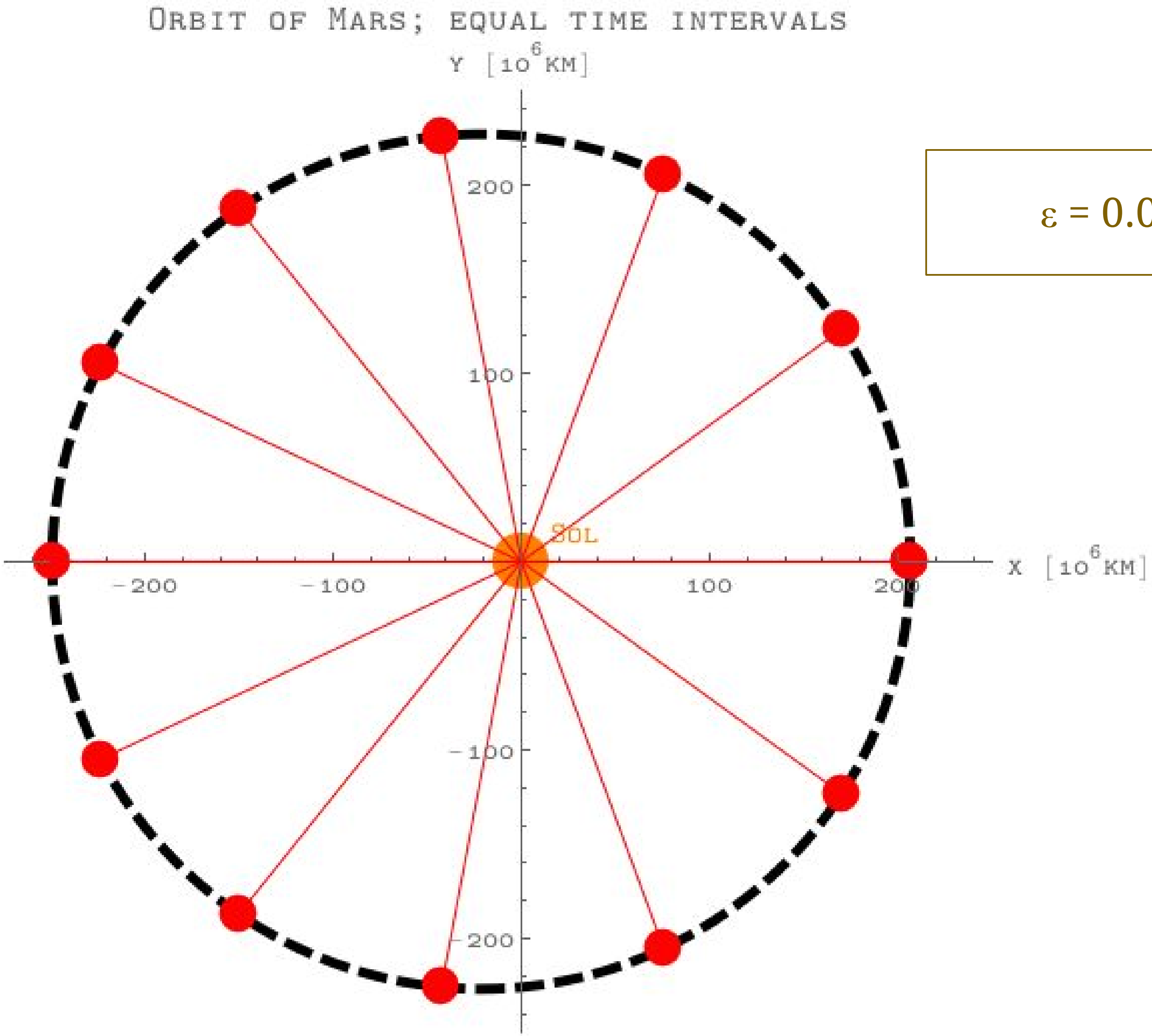
$r_{max} = a\,(1 + \varepsilon) = 35.3$ AU

Example D. Calculate the period of revolution.

$T = 2\pi$ **SQRT** (a^3/GM) = 76 years



Kepler's second law : Equal areas in equal times.


Orbit of Mars; equal time intervals
Y [10^6 km]
X [10^6 km]
Sol
200
100
-100
-200
-200
-100
100
200

ε = 0.0934

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