

Taylor Problem 1.50

1.50 *** [Computer] The differential equation (1.51) for the skateboard problem of Example 1.2 cannot be solved in terms of elementary functions, but it is easily solved numerically. (a) Use Mathematica to solve the differential equation for the case that the board is released from $\phi_0 = 20$ degrees, using these values: $R = 5$ m and $g = 9.8$ m/s². Make a plot of ϕ against time for two or three periods. (b) On the same picture, plot the approximate solution (1.57) with the same $\phi_0 = 20$ degrees. Comment on the two graphs.

$$\text{Equation (1.51)} \quad \phi'' = - (g/R) \sin \phi$$

$$\text{Equation (1.57)} \quad \phi(t) = \phi_0 \cos(\omega t) \quad ; \quad \omega = \sqrt{g/R}$$

Here is a Mathematica notebook to solve the problem.

```
g = 9.8 (* m/s^2 *)
R = 5.0 (* m *)
ang0 = 20/180*Pi (* radians *)
eqs = {phi''[t] == -g/R*Sin[phi[t]],
       phi[0] == ang0, phi'[0] == 0}
Q = NDSolve[eqs, phi, {t, 0, 15}]
ang = phi /. Q[[1]]
pl1 = Plot[ang[t], {t, 0, 15}]
pl2 = Plot[ang0*Cos[Sqrt[g/R]*t], {t, 0, 15}]
Show[pl1, pl2]
```