PHY410 Homework Set 5

1. [5 pts] Kittel-Kroemer, problem 4-19.

2. [10 pts] In the class we arrived at the energy U of a solid within Debye theory and considered that energy and the heat capacity in the two limiting cases of high and low temperatures. Different results in the Debye theory are commonly expressed in terms of the Debye function

$$D(x) = \frac{3}{x^3} \int_0^x \mathrm{d}z \, \frac{z^3}{\mathrm{e}^z - 1} \, .$$

   (a) Find the lowest terms of expansion for $D(x)$ around $x = 0$, i.e. represent D there as $D = D_0 + D_1 \, x + D_2 \, x^2 \ldots$.

   (b) Show that the energy of a solid can be expressed as

$$U = 3N \, \tau \, D(\tau_D/\tau) \, .$$

   (c) Express the heat capacity C_V in terms of the Debye function and its derivative. Produce a numerical plot of $C_V/3N$ in terms of τ/τ_D. Attach evidence for the numerical effort you carried out.

   (d) It is common to expand C_V in the high-temperature limit in the form

$$C_V = 3N \left[1 - \sum_i a_i \left(\frac{\tau_D}{\tau} \right)^i \right] .$$

   Determine the lowest nonvanishing a_i in this expansion and include the corresponding approximation to C_V in the numerical plot above.

3. [10 pts] The velocity of longitudinal sound waves in liquid helium ^{4}He, at temperatures below 0.6 K, is $v = 238.3 \, \mathrm{m/s}$. Within the liquid there are no transverse displacement waves. The density is $\rho = 145 \, \mathrm{kg/m^3}$.

   (a) Using the data above, determine the Debye temperature in liquid helium in K. Note that the number of modes describing displacements in one direction should be equal to N, rather than $3N$.

   (b) Calculate the low-temperature heat capacity for helium, per mass, within the Debye theory, and compare to the experimental result of $C_V/m = 20.4 \, T^3$, where T is in K and C_V/m is in J/(kg·K).