PHY410 Homework Set 12

1. [10 pts] Water boils when vapor pressure equals ambient atmospheric pressure. The latent heat L of water boiling around the sea-level temperature of 100° C is 40.7 kJ/mol or 2.26 MJ/kg. Your task will be to estimate by how many degrees Celsius (i.e. also Kelvin), ΔT, is the temperature of boiling going to be lower on top of Mt. Everest, compared to sea level, assuming an isothermal atmosphere.

   (a) Use the result arrived earlier in the class for pressure changing with elevation, $p = p_0 \exp\big(-mgh/k_B T\big)$, to estimate the pressure at Mt. Everest compared to that at sea level, p/p_0. What is $\Delta p/p_0$, where $\Delta p = p - p_0$? For m take the mass of nitrogen molecules.

   (b) Next, use the Clausius-Clapeyron (CC) equation to estimate the change ΔT in the temperature of boiling with the elevation. Assume that the molar volume of liquid is negligible compared to that of vapor and use the ideal-gas equation of state for the vapor, as common in the context of the CC equation. The measured temperature of boiling on Mt. Everest is 71° C.

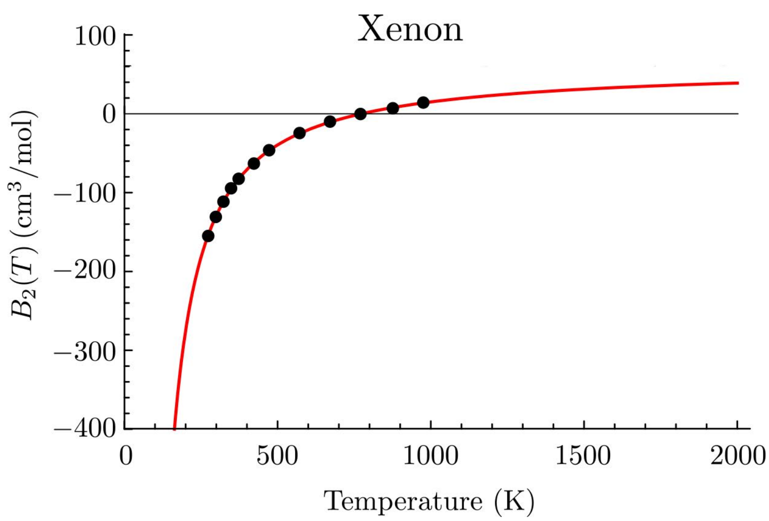
2. [10 pts] In the so-called virial expansion, the pressure of a low density gas is written as

$$\frac{p}{RT} = \frac{\mathcal{N}}{V} + B_2(T)\left(\frac{\mathcal{N}}{V}\right)^2 + B_3(T)\left(\frac{\mathcal{N}}{V}\right)^3 + \ldots$$

   Here, $\mathcal{N}$ is the mole number and B_k are so-called virial coefficients that can be calculated theoretically and/or determined experimentally by studying deviations from ideal-gas pressure.

   (a) Use $N = \mathcal{N}\, N_A$ and $\tau = k_B\, T$ in the van der Waals (vdW) equation of state and expand pressure to find B_2 for the vdW gas.

   (b) Using data for $B_2(T)$ of Xenon in the figure, find $N_A\, a/k_B$ and $N_A\, b$ in the vdW equation of state. For this you may consider two measurements at sufficiently different temperatures and compare them to your vdW prediction. With two linear equations containing two unknowns, you should be able to find $N_A\, a/k_B$ and $N_A\, b$. Be careful with dimensions.



(c) The coefficient b in the vdW equation of state is supposed to represent excluded volume around the center of a particle, where the center of another particle cannot fall. Assuming that the Xenon atoms cannot come closer to each other than $2R$ where R is the atomic radius, i.e. $b = (4\pi/3)\,(2R)^3$, estimate the radius R of a Xenon atom.

(d) Predict the critical temperature T_c for Xenon. The experimental value is $T_c^{\text{exp}} = 289.8\,\text{K}$. How does your result compare? When you coarsely read off values from the plot, expect just a coarse validity of your prediction.

3. [10 pts] The Dieterici equation of state is

$$p = \frac{N\tau}{V - Nb}\exp\left(-\frac{aN}{V\tau}\right).$$

(a) Under what conditions does the Dieterici equation reduce to the ideal-gas equation?

(b) Show that the Dieterici equation can be approximated to yield the van der Waals equation. What condition(s) must N, V and τ meet for such an approximation to be valid?

(c) Give a physical interpretation for the coefficients a and b.

(d) Find the critical parameter values p_c, τ_c and V_c. Note: There exist different strategies for finding parameters of the critical point. These strategies usually involve consideration of isotherms. One strategy relies on finding an inflection point as in the book and class. Another looks for different locations of a vanishing derivative $(\partial p/\partial V)_\tau = 0$ on an isotherm, there in the liquid-gas region. The different locations merge at the critical point.

4. **Note:** Since we did not get far enough in the lecture, this problem is moved to the next-week's set.

   [5 pts]

   (a) *In the context of the Landau theory of phase transitions, fill out the missing entries in the Table.*

Name of phase transition	*Order parameter*	*Order of phase transition*
Liquid-gas		
Liquid-solid		
	Magnetization	
	Occupation of single-particle ground state	

   (b) *List differences between first-order and second-order phase transitions.*