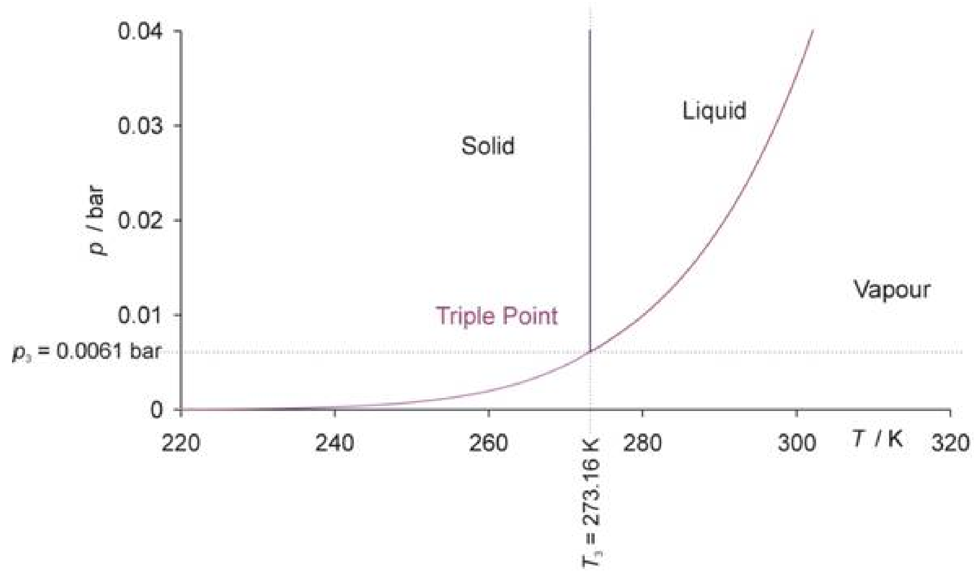
PHY410 Homework Set 13

1. [10 pts] The figure shows the phase diagram for water in the region of the triple point.



   (a) Consider changes in the entropy when moving in a circle around a triple point and demonstrate that, in the vicinity of a triple point, the latent heats are related with

   $$L_{sl} + L_{lv} = L_{sv} ,$$

   where L_{sl}, L_{lv} and L_{sv} are the latent heats, respectively, for the solid-liquid, liquid-vapor and solid-vapor transitions.

   (b) By using the Clausius-Clapeyron equation and carefully comparing the slopes of the coexistence curves in the vicinity of the triple point of water, for the ice-vapor and liquid-vapor transitions, deduce the heat of ice-liquid transition L_{sl}. Other names used for that heat are the heat of melting and heat of fusion. Express your answer in units of J/mol. The measured heat at 0°C is 6.0 J/mol. How does your answer compare?

2. [10 pts] This problem pertains to the mean-field model of a ferromagnet developed in chapter 10 of the textbook and in the lecture.

   (a) When magnetic field B increases by $\mathrm{d}B$ in the direction of net magnetic moment m'_{tot} of a domain of volume V, the energy of the domain changes by

   $$\mathrm{d}U = -m'_{\mathrm{tot}}\, \mathrm{d}B = -V M'\, \mathrm{d}B ,$$

   where M' is the current magnetization of the domain. Show that, when the magnetic field is due to the magnetization itself, $\mathrm{d}B = \lambda\, \mathrm{d}M'$, where λ is some coefficient of proportionality, the energy of a domain that arrives at magnetization M becomes

$$\frac{U}{V} = -\frac{\lambda M^2}{2}\,.$$

(b) The principal characteristic of a second-order phase transition is the fact that the heat capacitance, tied to τ-derivative of entropy, is discontinuous across the transition, while entropy is continuous. This is in contrast to a first-order transitions where already the entropy is discontinuous. Using the result above, express the heat capacitance C in terms of $\mathrm{d}M/\mathrm{d}\tau$.

(c) Next differentiate both sides of the self-consistency relation in the model

$$\frac{MV}{Nm} = \tanh\left(\frac{m\lambda M}{\tau}\right),$$

to express $\mathrm{d}M/\mathrm{d}\tau$ in terms of M and τ. The task may be simplified by using the reduced variables $\hat{\tau} = \tau/\tau_C$ and $\hat{M} = M/M_{\mathrm{sat}}$ where τ_C is Curie temperature and $M_{\mathrm{sat}} = Nm/V$. Relying on the graph of $M(\tau)$ from the book or class, the analytic result you get for $\mathrm{d}M/\mathrm{d}\tau$ or $\mathrm{d}\hat{M}/\mathrm{d}\hat{\tau}$ and the discussion of the heat capacity above, sketch the behavior of C in parallel with the behavior of M (or $\hat{M}$) with temperature τ, from 0 to past τ_C (or $\hat{\tau}$ past 1). *You do not need to solve the self-consistency relation exactly or find exact C.* A careful examination of the results as τ approaches τ_C from below allows to find the jump in C at τ_C, but that examination is not required.

3. [5 pts]

(a) In the context of the Landau theory of phase transitions, fill out the missing entries in the Table.

Name of phase transition	Order parameter	Order of phase transition
Liquid-gas		
Liquid-solid		
	Magnetization	
	Occupation of single-particle ground state	

(b) List differences between first-order and second-order phase transitions.