PHY410 Homework Set 14

1. [10 pts] For a Maxwell velocity distribution, determine (a) the most probable speed v_{mp}, one for which the distribution maximizes, (b) the average speed $\langle v \rangle$ and (c) the rms speed $v_{\mathrm{rms}} \equiv \langle v^2 \rangle^{1/2}$. Is the average square speed $\langle v^2 \rangle$ consistent with the expectation on the average kinetic energy $\langle \epsilon \rangle = \langle \frac{Mv^2}{2} \rangle$ at a given temperature τ? (d) Compute numerical values for v_{mp}, $\langle v \rangle$ and v_{rms} for oxygen molecules at temperature $T = 20°\mathrm{C}$. (e) Are the characteristic speed values going to be higher or lower for nitrogen molecules at the same temperature and why? By what factor?

2. [10 pts] In class we discussed particle diffusion and found that the particle flux density is related to the concentration gradient according to the transport equation

$$\boldsymbol{J}_n = -D\, \boldsymbol{\nabla} n\,.$$

We assumed then that the system was in a steady state, but the relation turns out to be also valid when the concentration and flux density are functions of time

(a) When particles are conserved, then any change in particle concentration in a region must result from particles crossing the boundary of the region. Show that

$$\boldsymbol{\nabla} \cdot \boldsymbol{J}_n + \frac{\partial n}{\partial t} = 0\,,$$

where the first term represents divergence of $\boldsymbol{J}_n$. The Gauss' theorem may be useful here. The derived equation is called the continuity equation.

(b) By combining the continuity equation with the transport equation above, and assuming, for simplicity, that D is independent of position, demonstrate (see Ch. 15) that the diffusion equation is valid for n:

$$\frac{\partial n}{\partial t} = D\, \Delta\, n\,,$$

where

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\,.$$

(c) Finally consider the diffusion equation in one dimension,

$$\frac{\partial n}{\partial t} = D\, \frac{\partial^2 n}{\partial x^2}\,.$$

Demonstrate that, according to this equation, the average square position of particles, $\langle x^2 \rangle_t = \frac{1}{N} \int \mathrm{d}x\, n(x,t)\, x^2$, where $N = \int \mathrm{d}x\, n(x,t)$, grows linearly with time, with the growth coefficient proportional to D:

$$\langle x^2 \rangle_t = \langle x^2 \rangle_{t=0} + 2Dt\,.$$

Multiplying both sides of the diffusion equation by x^2, integrating over x and employing partial integration, may be helpful here. You may assume that n vanishes at far-away distances. Is N conserved according to the diffusion equation? A similar result is found in three dimensions.

3. [10 pts] The average collision cross section σ_c for molecules in a nitrogen gas around 0°C is about $0.43\,\mathrm{nm}^2$.

   (a) If the N_2 molecules were represented as spheres of radius R, what radius would this cross section correspond to?

   (b) Estimate the mean free path λ_{mfp} for the molecules in nitrogen at 0°C and 1 atm.

   (c) Estimate the mean free flight time between the collisions $\tau_f = \lambda / v_{\mathrm{rms}}$ in nitrogen at the above conditions. As air is dominated by nitrogen and the relevant characteristics of oxygen are not very different, your estimates should be relevant for air too.

   (d) Determine the corresponding diffusivity D for nitrogen. The traditional unit for diffusivity is cm^2/s. The measured value is $0.155\,\mathrm{cm}^2/s$.

   (e) Estimate the heat conductivity K for nitrogen. Around 0°C the rotational degrees of freedom in the molecules are fully active. The measured conductivity, both for nitrogen and air, is about $24\,\mathrm{mW}/(\mathrm{m}\cdot\mathrm{K})$.

   (f) Finally estimate the viscosity η for nitrogen in $\mathrm{Pa}\cdot\mathrm{s}$. The measured viscosity value, both for nitrogen and air around 0°C, is $1.7 \times 10^{-5}\,\mathrm{Pa}\cdot\mathrm{s}$.

4. [5 pts] Fill out the missing entries in the Table representing irreversible transport.

Transported quantity	Driving gradient	Coefficient name	Approximate expression
	$\nabla\Phi$	Conductivity	
		Viscosity	
Energy			
Particle number			$D = \frac{1}{3}\lambda\langle v\rangle$