PHY410 Homework Set 2

1. [5 pts] In class we saw that the additivity of entropy for systems in thermal contact emerges within the approximation where we compute the entropy for the most likely state of subsystems, $\log(\overline{g}_I\,\overline{g}_{II})$, rather than for the convolution of all possible states of subsystems, $\log\big[\sum_{U_I} g_I(N_I,U_I)\,g_{II}(N-N_I,U-U_I)\big]$. The error is expected to be small when both N_I and N_{II} are large. For N_I, $N_{II} \gg 1$, we found

$$\log g(N,s) = \log(\overline{g}_I\,\overline{g}_{II}) + \frac{1}{2}\log\left(\frac{\pi}{2}\,\frac{N_I\,N_{II}}{N}\right).$$

   Consider now the two cases of $s = U = 0$, one with $N_I = N_{II} = 0.5\times 10^{22}$ spins, and another with $N_I = 10^{22}$ and $N_{II} = 10$. For each of those cases compute the relative error made to the entropy when assuming the additivity above. Comment on your findings.

2. [5 pts] Kittel-Kroemer, problem 2-1.

3. [10 pts]

   (a) First solve the problem 2-2 in Kittel-Kroemer. The result you are likely to find for the magnetization is known as Curie's law. Note that $U < 0$, as the aligned magnetic dipoles contribute negative energy, $-m\,B$, and anti-aligned dipoles contribute positive, $m\,B$. The derived expression for τ may be rearranged into

$$-\frac{U}{N}\,\tau = (m\,B)^2\,.$$

   The latter says that the average energy per dipole U/N times the energy scale set by the temperature τ produces a constant. This result can be interpreted in the following way. The tendencies in the system are of reducing energy, i.e. of making U as negative as possible, but also of increasing entropy, i.e. making the dipole alignment as random as possible. The temperature controls which is more important. At high temperatures, entropy wins and $U \to 0$. At low temperatures, the energy wins. The expression above gives $U \to -\infty$ as $\tau \to 0$, but that pathology is due to the breakdown of the applied approximations.

   (b) Using the net magnetization $M = 2m\langle s\rangle$, find the magnetic susceptibility $\chi = (\partial M/\partial B)_N$ as a function of temperature τ.

   (c) Express the entropy σ in terms of τ, B, and N. Consider now a process where the magnetic field B is gradually *reduced* from its initial value B_i to the final value $B_f < B_i$. The temperature in the initial state is τ_i and the system is enclosed so that N does not change. Assuming that the entropy $\sigma(\tau, B, N)$ remains constant during this process find final temperature τ_f. How is the temperature ratio τ_f/τ_i related to the magnetic field ratio B_f/B_i? The so-called adiabatic demagnetization is employed in magnetic refrigeration.

4. [5 pts] Kittel-Kroemer, problem 2-3. Regarding the multiplicity for a set of harmonic oscillators, read the appropriate portion of Chapter 1.