Homework due Friday January 20

Problem 1. For the harmonic oscillator, derive the ground state wave function $\Phi_0(x)$ from this property of the ket, $a|0\rangle = 0$.

Problem 2. For the coherent state given in Lecture 1,
i.e., $c_n = \exp(-\alpha^2/2)\ \alpha^n\ /\ \mathrm{Sqrt}(n!)$:
(a) Calculate $\langle t \mid x \mid t \rangle = A \cos \omega t$. (Determine A.)
(b) Calculate $\langle t \mid x^2 \mid t \rangle$; and show that the uncertainty of x is small in the classical limit.
(c) Calculate $\langle t \mid H \mid t \rangle$. Compare the result to the classical energy.
Hint: the coherent state $|\psi,t\rangle$ is an eigenstate of a.

Problem 3. The quantum field for the free electromagnetic field A(x,t) *in the Heisenberg picture* is written as an expansion in plane waves, with the annihilation and creation operators, $a_{k\sigma}$ and $a^\dagger_{k\sigma}$.

(a) Let $H = \sum \hbar\omega\, a^\dagger_{k\sigma} a_{k\sigma}$ + const. Show that A(x,t) obeys the Heisenberg equation of motion.

(b) Show that $H = 1/2 \int (E^2 + B^2)\, d^3x$.

Problem 4.

Start with the equations for time dependence in the Schroedinger and Heisenberg pictures.

Prove $\langle \alpha,t \mid O \mid \beta,t \rangle_{Schr.} = \langle \alpha \mid O(t) \mid \beta \rangle_{Heis.}$

Problem 5.

WKAR-FM is a public radio station in East Lansing, Michigan; broadcasting on the FM dial at 90.5 MHz. It is owned by Michigan State University, and is sister station to the AM radio and television stations with the same call letters. The station signed on for the first time on October 4, 1948 as the Lansing area's first FM station.

The station's 85,000-watt signal, combined with a 269.3 meter antenna can be heard as far east as Flint and the Detroit suburbs, and as far west as Grand Rapids and Kalamazoo. WKAR-FM is a "Superpower Grandfathered" Class B FM station, providing a signal 7.6 db stronger than would be granted today under current U.S. Federal Communications Commission (FCC) rules.

(A) Estimate the mean number of photons emitted per second.

(B) Estimate the RMS variation of the number of photons emitted per second.

Problem 6.

Show that the photon coherent state is an eigenstate of the annihilation operator.