Homework Problems due Friday February 3

Problem 11.
Three identical spin-0 bosons are in a harmonic oscillator potential. The total energy is $9/2\ \hbar\omega$. From this information alone, write an expression for the 3-particle wave function, $\Psi(x_1 , x_2 , x_3)$.

Problem 12.
Consider two identical spin-0 bosons moving in free space, and interacting with each other. Approximate the 2-particle wave function by products of free waves with momenta $\mathbf{p_1}$ and $\mathbf{p_2}$.

(a) Calculate the expectation value of the two-body potential energy $V(\mathbf{x_1} , \mathbf{x_2})$.

(b) Now suppose $V(\mathbf{x_1} , \mathbf{x_2}) = U(\mathbf{x_1} - \mathbf{x_2})$. Express the result of (a) in terms of the Fourier transform of $U(\mathbf{r})$.

Problem 13.
Prove that $[\Omega, N] = 0$ where Ω is the two-particle interaction potential for identical fermions and N is the total number operator.

Problem 14.
Let $\psi_\alpha(\mathbf{r},t)$ be the *field operator* for spin-½ fermions, in the Heisenberg picture;
α = the spin index.
Derive the field equation for $\psi(\mathbf{r},t)$ in the form

$$i\hbar\ \partial\psi / \partial t = F[\psi]$$

where $F[\psi]$ is a functional—which may involve derivatives and integrals. Simplify the result as much as possible. [[Assume that $\tau = -\hbar^2 \nabla^2 /2m$ and that $v(\mathbf{r_1},\mathbf{r_2})$ is spin independent.]]

Problem 15.
In the second quantized theory of 2 identical fermions, calculate

$$\langle 0|\ c_\alpha\ c_\beta\ \psi^\dagger(\mathbf{r_1})\ \psi^\dagger(\mathbf{r_2})\ |0\rangle$$

where c_α is the annihilation operator for a particle with wave function $u_\alpha(\mathbf{r})$.