The Yukawa Theory Problem

■ *Yukawa's theory of the nucleon-nucleon force (1935)*

Nucleons interact through a scalar field $\varphi(\mathbf{x},t)$ with mass m. The range of the force is

$$\text{range} = \hbar/(mc) = 1 \text{ to } 2 \text{ fm};$$

therefore the meson mass is in the range

$$mc^2 = \hbar c/(\text{range}) \text{ from } 100 \text{ to } 200 \text{ MeV}.$$

(Of course Yukawa did not know about pions, which were discovered in 1947.
mass ($\pi^{\pm}$) = 139.6 MeV/c^2
mass (π^{0}) = 135.0 MeV/c^2)

■ *The quantum field theory; ignore isospin*

The Lagrangian density for the theory is

$$\mathcal{L} = \mathcal{L}_{\text{nucleon}} + \mathcal{L}_{\text{meson}} + \mathcal{L}_{\text{interaction}} .$$

Lagrange's equations including the interaction,

$$\mathcal{L}_{\text{interaction}} = g\,\psi^{\dagger}\psi\varphi ,$$

are

$$-\frac{\hbar^2}{2m}\nabla^2\psi - g\phi\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\frac{\partial^2\phi}{\partial t^2} - \nabla^2\phi + m^2\phi = g\,\psi^{\dagger}\psi$$

($\hbar$ = 1 and c = 1)

■ *Calculate the potential energy for a nucleon (N) attracted to a heavy nucleus (U-238).*

First step == calculate the mean field created by the nucleons in the heavy nucleus.

Second step == calculate the potential energy for the external nucleon.

■ *Do a numerical calculation*

r_0 = 1.25 fm — $R = r_o A^{1/3}$
mc^2 = 140 MeV — pion mass
A = 238 — uranium
g = 15 — strong interaction

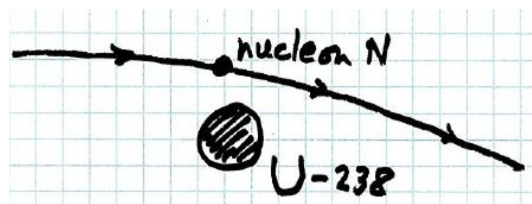
■ *Hand in a plot of the potential energy*

$$V(r) = -\,g\,\varphi_0(r) ,$$

using these units: V in MeV and r in fm.

• Nucleon field $= \psi(\vec{x})$
(Ignore spin and isospin for simplicity.)
e.g., $\psi(\vec{x}) = \sum_{\vec{k}} \frac{1}{\sqrt{V}} e^{i\vec{k}\cdot\vec{x}}\, b_{\vec{k}}$

• Meson field $= \phi(\vec{x})$
(Ignore isospin for simplicity.)
$\phi(\vec{x})$ = real scalar field.



$$-\nabla^2\phi_0 + m^2\phi_0 = \langle g\,\psi^{\dagger}\psi\rangle$$

$$\langle\psi^{\dagger}\psi\rangle_{U\text{-}238} = \text{density of nucleons}$$
$$= \sum_{\alpha=1}^{238} |u_\alpha(\vec{x})|^2$$
$$\approx \frac{A}{4/3\,\pi R^3}\,\Theta(R-r) \text{ where } R = r_0 A^{1/3}.$$

$$\phi_0(\vec{x}) = \int G(\vec{x}-\vec{y})\, n(\vec{y})\, d^3y$$

$$V(\vec{x}) = -g\,\phi_0(\vec{x}) = -g\int G(\vec{x}-\vec{y})\,n(\vec{y})\,d^3y$$
$$= \frac{-3g}{4\pi r_0^3}\,\frac{1}{A}\int \frac{e^{-m|\vec{x}-\vec{y}|}}{4\pi|\vec{x}-\vec{y}|}\,\Theta(r_0A^{1/3}-|\vec{y}|)\,d^3y$$

$\hbar = 1$ and $c = 1$