PHY 855 Quantum Field Theory

Semester
Spring of every year
Credits
Total Credits: 2   *Lecture/Recitation/Discussion Hours: 2*
Recommended Background
PHY 852
Restrictions
Open to graduate students in the Department of Physics and Astronomy or approval of department.
Description
Introduction to field theory as it pertains to numerous problems in particle, nuclear and condensed matter physics. Second quantization, applications to different fields based on perturbation theory. Offered first half of semester.
Effective Dates
FALL 2014 - Open

(I'll emphasize particle physics.)

SCHEDULE OF CLASSES

M F	4:10 PM - 5:30 PM	Dan Stump	1300 BPS
W	4:10 PM - 5:00 PM	Dan Stump	1300 BPS

TEXTBOOK

Mandl and Shaw,	Quantum Field Theory
Cabibbo, Maiani,	Relativistic Quantum Theory (primarily for PHY 955)

HOMEWORK

6 or 7 assignments

EXAMS

none

GRADING

based on homework and "participation"

Syllabus

Topic	Reading	comments
Theory of the photon	Chapter 1	applied to atoms
Second quantization		quantum many body problem and field theory
Lagrangian field theory	Chapters 2 & 3	canonical quantization
The Dirac field	Chapter 4	second quantization for the Dirac eq.
QED	Chapter 5	covariant photon field ; interactions
Perturbation theory	Chapter 6	Feynman rules

But let's start with something familiar.
<u>The quantum harmonic oscillator</u>

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 q^2 \quad \text{where} \quad [q, p] = i\hbar$$

Remember how we solve the harmonic oscillator.
We define a and a^+ by

$$a = \sqrt{\frac{m\omega}{2\hbar}}\left(q + \frac{ip}{m\omega}\right) \quad \text{and} \quad a^+ = \sqrt{\frac{m\omega}{2\hbar}}\left(q - \frac{ip}{m\omega}\right)$$

a = lowering operator = annihilation operator
a^+ = raising operator = creation operator

Note

$$q = \sqrt{\frac{\hbar}{2m\omega}}\,(a + a^+) \quad \text{and} \quad p = -i\sqrt{\frac{\hbar m\omega}{2}}\,(a - a^+)$$

Creation and annihilation of quanta

The crucial point is the commutator, $[a, a^+] = 1$.

$$[a, a^+] = \sqrt{\frac{m\omega}{2\hbar}}\left(\frac{-i}{m\omega}\right) i\hbar \times 2 = 1$$

That implies $[a^+a, a] = -a$

$$[a^+a, a] = a^+aa - aa^+a$$
$$= [a^+, a]\,a = -a$$

So a is the lowering operator

$$a\,|n\rangle \propto |n-1\rangle$$

and similarly a^+ is the raising operator.

a^+a is the number operator.

Also, the Hamiltonian is

$$H = (a\,a^+ + a^+a)\,\hbar\omega/2 = (a^+a + ½)\,\hbar\omega$$

Energy eigenstates

■ the ground state $|0\rangle$

$a\,|0\rangle = 0$; $H\,|0\rangle = \frac{1}{2}\hbar\omega\,|0\rangle$; $E_0 = \frac{1}{2}\,\hbar\omega$

■ the first excited state $|1\rangle$

$|1\rangle = a^{+}|0\rangle$; $H\,|1\rangle = 3/2\,\hbar\omega\,|0\rangle$; $E_1 = 3/2\,\hbar\omega$

■ all the eigenstates $|n\rangle$ where $n \in \{0, 1, 2, 3, \dots\}$

$|n\rangle = (a^{+})^n\,|0\rangle$/**SQRT(n!)**

$E_n = (n + \frac{1}{2})\,\hbar\omega$

$$\langle n|n\rangle = \frac{1}{n!}\langle 0|\, a^n (a^+)^n |0\rangle$$

$$= \frac{1}{n!}\langle 0|\, a^{n-1}\, \underbrace{a a^+}_{\hookrightarrow\, 1 + a^+ a \,=\, n}\, (a^+)^{n-1} |0\rangle$$

$$= \frac{1}{(n-1)!}\langle 0|\, a^{n-1} (a^+)^{n-1} |0\rangle$$

$$= \dots = \langle 0|0\rangle = 1$$

The energy *eigenfunctions*

Recall the Dirac notation of bra's and ket's.

A state vector is a ket = $|\psi\rangle$;

the wave function is $\langle x \,|\, \psi\rangle = \psi(x)$.

So, the energy eigenfunctions are

$$\Phi_n(x) = \langle x \,|\, n\rangle \qquad n \in \{ 0, 1, 2, 3, \dots \}$$

$$= C \times \text{Hermite polynomial} \times \text{gaussian function}$$

Homework Problem 1. For the harmonic oscillator, derive the ground state wave function $\Phi_0(x)$ from this property of the ket, $a\,|\,0\rangle = 0$.

Time dependent states

Given $|\psi,0\rangle$ (= state at t=0) what is $|\psi,t\rangle$?

The Hamiltonian is the generator of translation in time; i.e.,

$$|\psi,t+\varepsilon\rangle = |\psi,t\rangle - i\,\varepsilon/\hbar\, H\, |\psi,t\rangle \quad ;$$

or,

$$i\hbar\, (\partial/\partial t)\, |\psi,t\rangle = H\, |\psi,t\rangle \quad ;$$

or,

$$|\psi,t\rangle = \exp(-i\, t\, H / \hbar)\, |\psi,0\rangle \quad ;$$

or,

$$|\psi,t\rangle = \sum c_n \exp(-i\, t\, E_n / \hbar)\, |n\rangle \quad .$$

generator

Schroedinger equation

formal solution

expansion in |n>

What is a *classical* oscillator?

$$H = p^2/(2m) + \tfrac{1}{2} m \omega^2 x^2 \quad \Rightarrow \quad \ddot{x}(t) + \omega^2 x(t) = 0.$$

$$x(t) = A \cos \omega t \qquad (\star)$$

How is that related to $|\psi,t\rangle$?

Of course ($\star$) is impossible, *physically*, by the uncertainty principle. What I mean by ($\star$) is that the uncertainty of x is small; i.e., small compared to A.

Comments.

- Define $x(t) = \langle\psi,t\,|\; x \;|\psi,t\rangle$
- By Ehrenfest's theorem, $\ddot{x}(t) + \omega^2 x(t) = 0$.

But that's not good enough, because it doesn't say Δx is small !

The coherent state (R. Glauber)

For sure,

$$|\psi,t> = \sum c_n \exp(- i E_n t / \hbar) \; |n> .$$

The best description of a classical oscillator is

$$c_n = \exp(-\alpha^2/2) \; \alpha^n \; / \, \mathrm{Sqrt}(n!) \qquad .$$

Nobel Prize motivation (2005):
"for his contribution to the quantum theory of optical coherence"

Homework Problem 2. For the coherent state given in Lecture 1,
i.e., $c_n = \exp(-\alpha^2/2) \; \alpha^n \; / \, \mathrm{Sqrt}(n!)$:
(a) Calculate $\langle t \mid x \mid t \rangle = A \cos \omega t$. (Determine A.)
(b) Calculate $\langle t \mid x^2 \mid t \rangle$; and show that the uncertainty of x is small in the classical limit.
(c) Calculate $\langle t \mid H \mid t \rangle$. Compare the result to the classical energy.
Hint: $|t>$ is an eigenstate of a.

Homework due Friday January 20

Problem 1. For the harmonic oscillator, derive the ground state wave function $\Phi_0(x)$ from this property of the ket, $a|0\rangle = 0$.

Problem 2. For the coherent state given in Lecture 1,
i.e., $c_n = \exp(-\alpha^2/2)\ \alpha^n / \mathrm{Sqrt}(n!)$:
(a) Calculate $\langle t \mid x \mid t \rangle = A \cos \omega t$. (Determine A.)
(b) Calculate $\langle t \mid x^2 \mid t \rangle$; and show that the uncertainty of x is small in the classical limit.
(c) Calculate $\langle t \mid H \mid t \rangle$. Compare the result to the classical energy.
Hint: the coherent state $|\psi,t\rangle$ is an eigenstate of a.