Light Scattering by Electrons

This subject has an interesting history.

§ Rayleigh scattering (1871)
-classical theory;
-before the discovery of the electron;
-why the sky is blue.

§ Thomson scattering (1906)
-classical theory;
-after the discovery of the electron;
-light scattering by plasmas.

§ Compton scattering (1923)
-experimental;
-the failure of the classical wave theory, and a triumph of the photon theory.

§ Dirac (1927)
-quantum field theory of photons.

§ Raman scattering (1928)
-experimental;
-inelastic scattering of light by atoms.

§ Klein-Nishina formula (1928)
-quantum field theory;
-cross section for relativistic γe scattering

<u>The Klein-Nishina formula</u>

This is the formula for photon-electron scattering, in first order perturbation theory in relativistic Q.E.D.

(We'll derive it in PHY 955.)

$$\left(\frac{d\sigma}{d\Omega}\right)_{lab} = \frac{\alpha^2}{2m^2}\left(\frac{\omega'}{\omega}\right)^2\left\{\frac{\omega}{\omega'} + \frac{\omega'}{\omega} - \sin^2\theta\right\}$$

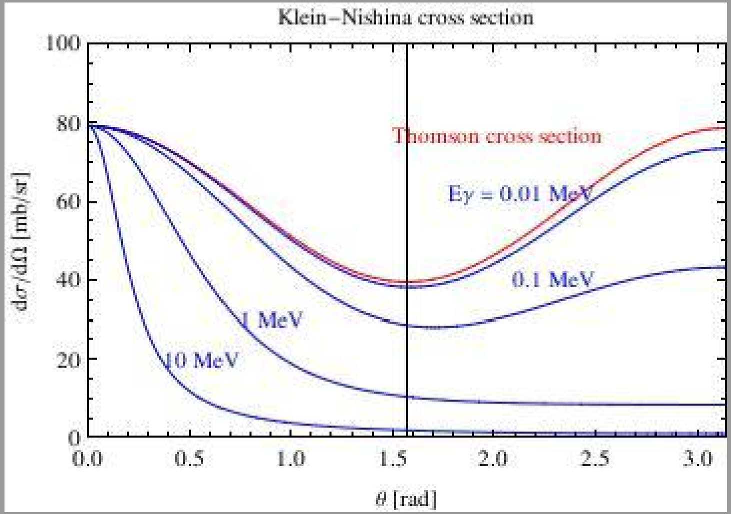
where $\frac{1}{\omega'} = \frac{1}{\omega} - \frac{1}{m}(\cos\theta - 1)$

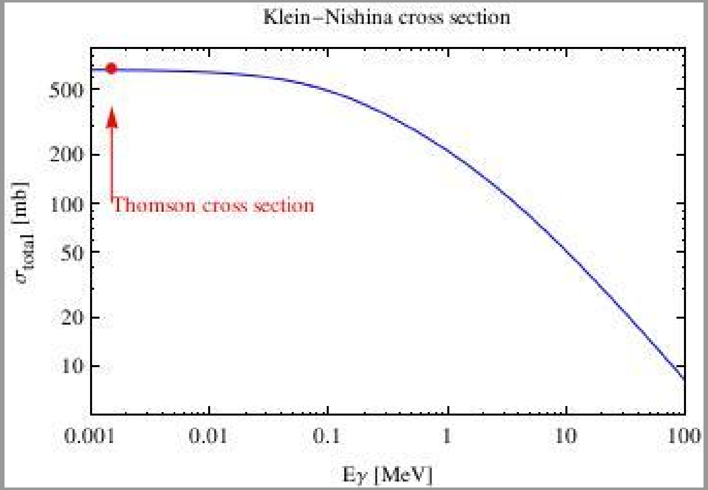
$\hbar = 1$ and $c = 1$

Constants *(we use rationalized c.g.s. units)*

$\alpha = e^2 / (4\pi\hbar c) = 1/137$

$\lambda_{Compton} = 2\pi\hbar / (mc) = 386$ fm

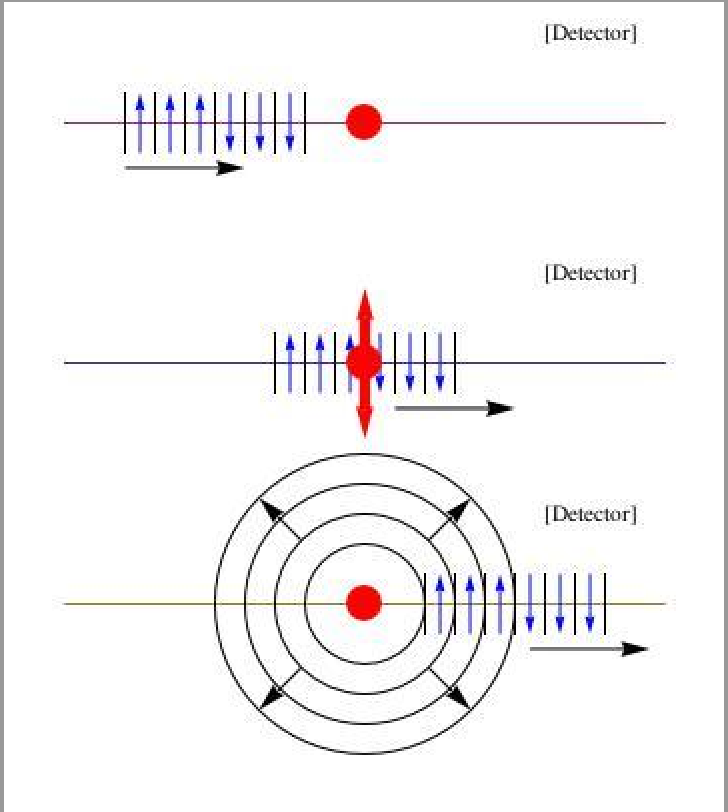




Light scattering by electrons -- sketches

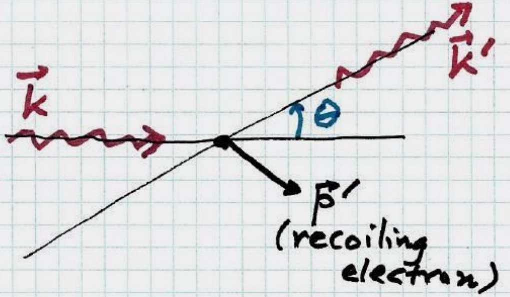
The classical wave theory:
-- the wave fronts represent a physical wave;
they carry energy;
the detector must register some scattered energy.

The photon theory:
-- the wave fronts represent a photon;
they only carry probability;
the total energy is $\hbar\omega$;
the detector will either register energy $\hbar\omega$, or nothing.



Photon scattering by a free electron – ***KINEMATICS***

energy and momentum are conserved



$$\gamma + e \rightarrow \gamma' + e'$$

$$\hbar\vec{k} + \vec{p} = \hbar\vec{k}' + \vec{p}'$$

$$\hbar c k + E_p = \hbar c k' + E_{p'}$$

"Lab frame" = rest frame of e

$$\vec{p} = 0$$

$$E_p = mc^2$$

$$E_{p'} = \sqrt{m^2c^4 + \hbar^2c^2(\vec{k}-\vec{k}')^2}$$

$$mc^2 + \hbar c(k - k') = E_{p'} = \sqrt{m^2c^4 + \hbar^2c^2(\vec{k}-\vec{k}')^2}$$

KINEMATICS IN THE REST FRAME OF e

$$mc^2 + \hbar c\,(k-k') = \sqrt{m^2c^4 + \hbar^2c^2(\vec{k}-\vec{k}')^2}$$

$$m^2c^4 + \hbar^2c^2(k^2+k'^2-2kk') + 2mc^2\,\hbar c\,(k-k')$$

$$= m^2c^4 + \hbar^2c^2(k^2+k'^2-2kk'\cos\theta)$$

$$-\hbar c\cdot 2kk' + 2mc^2(k-k') = \hbar c\,(-2kk'\cos\theta)$$

$$\boxed{k' = \frac{mc^2\,k}{mc^2 + \hbar c k\,(1-\cos\theta)}} = kF$$

$$\therefore\ \lambda' = \lambda + \frac{2\pi\hbar}{mc}(1-\cos\theta)$$

For $E_\gamma \ll mc^2$, $k' \simeq k$; i.e., $F \simeq 1$.

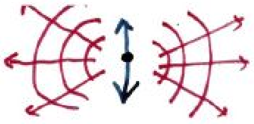
COMPTON EFFECT

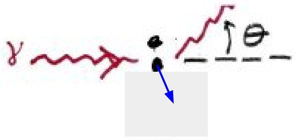
THOMSON scattering

Thomson scattering

= *light scattering by a free electron, in the low-energy limit.*

Thomson (1906) provided the calculation ***for the cross section***, *using classical wave theory* , valid in the long wavelength limit.





Dirac (1927) rederived the result using quantum field theory.

$$\gamma + e \longrightarrow \gamma' + e' \qquad \text{with} \qquad E_\gamma << mc^2$$

In this limit, we may approximate $E'_\gamma \approx E_\gamma$,
so it was called "elastic scattering".
The electron does recoil, so $\mathbf{p}' \neq \mathbf{p}$;
however we may approximate $E'_e \approx E_e = m\,c^2$.

The scattering cross section (see Section 1.4.4)

$$\frac{d\sigma}{d\Omega_{k'}} = \frac{\text{differential rate}}{\text{incident flux}}$$

We could use time-dependent perturbation theory.
Recall Fermi's Golden Rule,

$$\text{Rate} = \frac{2\pi}{\hbar} \sum_f |\langle f | T | i \rangle|^2 \ \delta(E_f - E_i)$$

$| \, i \, \rangle = | \, \mathbf{k} \, , \varepsilon \, \rangle_{\text{photon}} \ \ | \, \mathbf{p} \, , s \, \rangle_{\text{electron}}$,

with $\mathbf{p} = \mathbf{0}$ in the rest frame;

$| \, f \, \rangle = | \, \mathbf{k}' \, , \varepsilon' \, \rangle_{\text{photon}} \ \ | \, \mathbf{p}' \, , s' \, \rangle_{\text{electron}}$

See Section 1.4.4.

This calculation is OK, but it's not ideal:

- the photon states have infinite zero-point energy ;
- what about electron spin? ;
- the electron states have the "filled Dirac sea" ;
- the calculation is not manifestly covariant.

In PHY 955 we'll calculate σ using Feynman diagrams. (⇒ Klein-Nishina formula).

Results from Section 1.4.4

Eq. 1.69 differential cross section for polarized photons

$$\left(\frac{d\sigma}{d\Omega}\right)_{pol.} = r_0^2 \left(\vec{\epsilon}_{\vec{k}} \cdot \vec{\epsilon}\,'_{\vec{k}'}\right)^2$$

$$r_0 = \frac{e^2}{4\pi mc^2} = 2.82 \text{ fm}$$

Eq. 1.69a unpolarized differential cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{unpol.} = \frac{1}{2} r_0^2 \left(1 + \cos^2\theta\right)$$

Eq. 1.72 total cross section

$$\sigma_{total} = \frac{8\pi}{3} r_0^2$$

Examples of Thomson scattering (Wikipedia)

- The cosmic microwave background is linearly polarized as a result of Thomson scattering, as measured by DASI and more recent experiments.
- The solar K-corona is the result of the Thomson scattering of solar radiation from solar coronal electrons. NASA's STEREO mission generates three-dimensional images of the electron density around the sun by measuring this K-corona from two separate satellites.
- In tokamaks, corona of ICF targets and other experimental fusion devices, the electron temperatures and densities in the plasma can be measured with high accuracy by detecting the effect of Thomson scattering of a high-intensity laser beam.

  inertial confinement
- Inverse-Compton scattering can be viewed as Thomson scattering in the rest frame of the relativistic particle.
- X-ray crystallography is based on Thomson scattering.

Problems due Friday, January 27

Homework Problem 9
Calculate the total cross section for Thomson scattering, and express the result in millibarns (mb).

Homework Problem 10
"In fact, it can easily be demonstrated that it takes a photon emitted in the solar core many thousands of years to fight its way to the surface because of Thomson scattering."
Prove it.