MANDL and SHAW

Chapter 2 Lagrangian Field Theory

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Chapter 3 The Klein-Gordon Field

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See also **MAIANI and BENHAR**, Chapters 3 and 4.

The Feynman propagator function, $\Delta_F(x - y)$

Mandl and Shaw, Problem 3.3

The propagator is the Green's function for $\Box + \mu^2$

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 = \partial^\mu \partial_\mu$$

$$\mu = \frac{mc}{\hbar} \quad (\text{units} : 1/L)$$

i.e., $\partial^\mu \partial_\mu + m^2$ if we set $\hbar = 1$ and $c = 1$

That is,

$$(\Box + \mu^2)\, \Delta_F(x) = -\delta^4(x)$$

Proof #1 Use the Fourier integral,

$$\Delta_F(x) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik\cdot x}}{k^2-\mu^2+i\epsilon}$$

$$\partial_\mu \Delta_F = \int \frac{d^4k}{(2\pi)^4} (-ik_\mu) \frac{e^{-ik\cdot x}}{k^2-\mu^2+i\epsilon}$$

$$\partial^\mu \partial_\mu \Delta_F = \int \frac{d^4k}{(2\pi)^4} (-k^2) \frac{e^{-ik\cdot x}}{k^2-\mu^2+i\epsilon}$$

$$(\Box+\mu^2)\Delta_F = \int \frac{d^4k}{(2\pi)^4} \frac{(-k^2+\mu^2)\, e^{-ik\cdot x}}{k^2-\mu^2+i\epsilon}$$

$$= -\int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} = -\delta^4(x)$$

$(\epsilon \to 0)$

The propagator is one of the Green's functions of the operator $(\Box + \mu^2)$.

Proof #2 Use the definition,

$$i\hbar c\, \Delta_F(x) = \langle 0 |\, T(\phi(x)\, \phi^\dagger(0))\, | 0 \rangle$$

$$= \theta(x^0) \langle 0 |\, \phi(x)\, \phi^\dagger(0)\, | 0 \rangle + \theta(-x^0) \langle 0 |\, \phi^\dagger(0)\, \phi(x)\, | 0 \rangle$$

$$\frac{\partial}{\partial x^0}(i\hbar c\, \Delta_F) = \delta(x^0) \langle 0 |\, [\phi(x), \phi^\dagger(0)]\, | 0 \rangle \quad \leftarrow \text{ZERO}$$

$$+ \theta(x^0) \langle 0 |\, \frac{\partial \phi}{\partial x^0}\, \phi^\dagger(0)\, | 0 \rangle + \theta(-x^0) \langle 0 |\, \phi^\dagger(0)\, \frac{\partial \phi}{\partial x^0}\, | 0 \rangle$$

$$\frac{\partial^2}{\partial (x^0)^2}(i\hbar c\, \Delta_F) = \delta(x^0) \langle 0 |\, \underbrace{\frac{\partial \phi}{\partial x^0}\, \phi^\dagger(0) - \phi^\dagger(0)\, \frac{\partial \phi}{\partial x^0}}_{-i\delta^3(\vec{x})}\, | 0 \rangle$$

ETCR

$$+ \theta(x^0) \langle 0 |\, (\partial_0^2 \phi)\, \phi^\dagger(0)\, | 0 \rangle + \theta(-x^0) \langle 0 |\, \phi^\dagger(0)\, (\partial_0^2 \phi)\, | 0 \rangle$$

$$(\Box + \mu^2)\, i\hbar c\, \Delta_F = -i\delta^4(x) + \theta(x^0) \langle 0 |\, (\Box + \mu^2)\, \phi(x)\, \phi^\dagger(0)\, | 0 \rangle + \theta(-x^0) \langle 0 |\, \phi^\dagger(0)\, \underbrace{(\Box + \mu^2)\, \phi(x)}_{=\,\text{ZERO}}\, | 0 \rangle$$

$$= -i\delta^4(x)$$

error in factors of $\hbar$ and c? No matter.

Propagator

From Wikipedia, the free encyclopedia

This article is about Quantum field theory. For plant propagation, see Plant propagation. In quantum mechanics and quantum field theory, the **propagator** is a function that specifies the probability amplitude for a particle to travel from one place to another in a given time, or to travel with a certain energy and momentum.

In Feynman diagrams, which serve to calculate the rate of collisions in quantum field theory, virtual particles contribute their propagator to the rate of the scattering event described by the respective diagram.

These may also be viewed as the inverse of the wave operator appropriate to the particle, and are, therefore, often called *(causal) Green's functions* (called "*causal*" to distinguish it from the elliptic Laplacian Green's function).

- sort of correct;
- it's often described like this;
- but it's not really accurate.

Contents

Faster than light?

$$G_F(x,y) = \lim_{\varepsilon\to 0} \frac{1}{(2\pi)^4} \int d^4p \, \frac{e^{-ip(x-y)}}{p^2 - m^2 + i\varepsilon} = \begin{cases} -\frac{1}{4\pi}\delta(s) + \frac{m}{8\pi\sqrt{s}} H_1^{(2)}(m\sqrt{s}) & s \ge 0 \\ -\frac{im}{4\pi^2\sqrt{-s}} K_1(m\sqrt{-s}) & s < 0. \end{cases}$$

The Feynman propagator in coordinate space, depends on s;

s is the "invariant separation";

$s = (x - y)^2 = (x^0 - y^0)^2 - (\mathbf{x} - \mathbf{y})^2$

Recall from special relativity:

$s > 0$ is a timelike separation;

$s < 0$ is a spacelike separation.

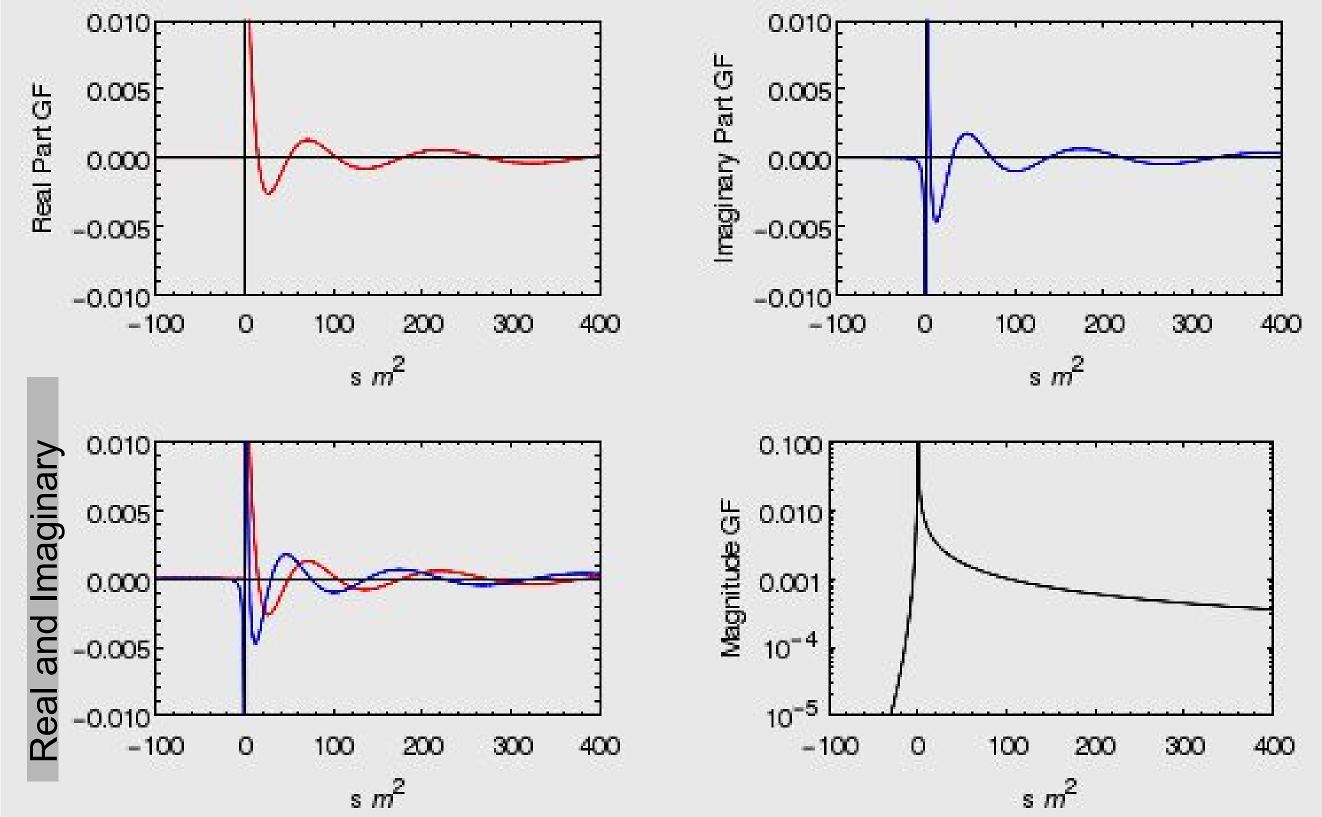
$H_1^{(2)}(z)$ = the Hankel function of the second kind with index 1; *timelike*

$K_1(z)$ = the modified Bessel function with index 1; *spacelike*

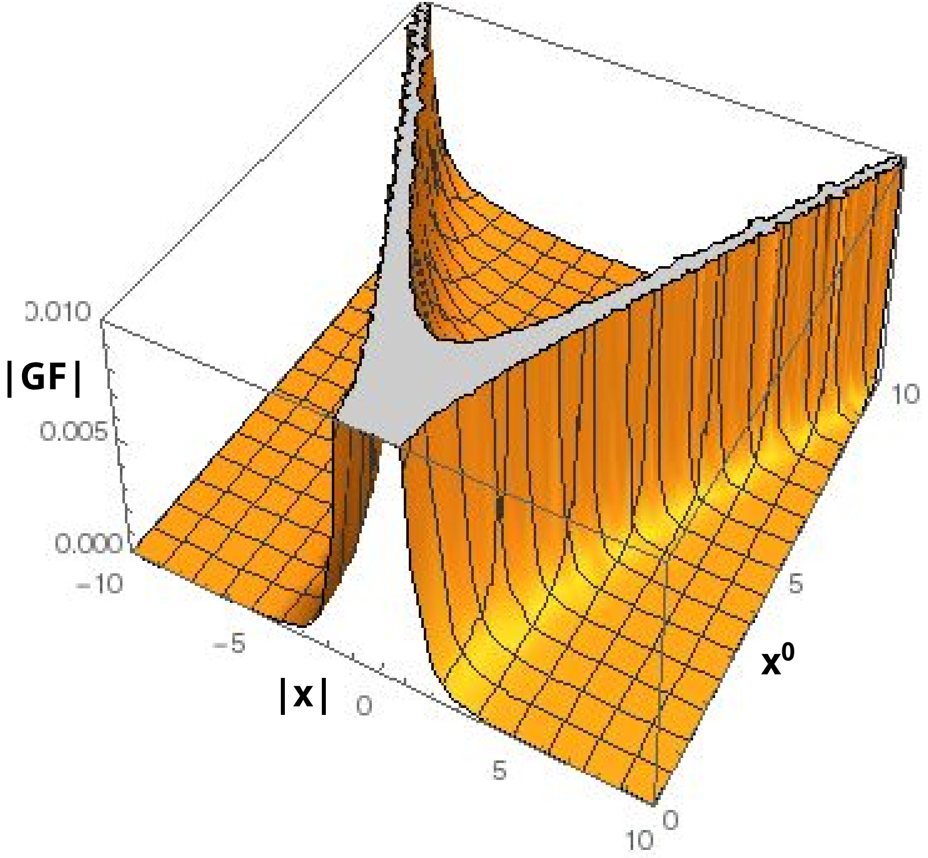
Make a graph of $G_F(x - y)$ versus s .

You can do it with *MATHEMATICA*.

$$G_F(x,y) = \lim_{\varepsilon\to 0} \frac{1}{(2\pi)^4} \int d^4p\, \frac{e^{-ip(x-y)}}{p^2 - m^2 + i\varepsilon} = \begin{cases} -\frac{1}{4\pi}\delta(s) + \frac{m}{8\pi\sqrt{s}} H_1^{(2)}(m\sqrt{s}) & s \geq 0 \\ -\frac{im}{4\pi^2\sqrt{-s}} K_1(m\sqrt{-s}) & s < 0. \end{cases}$$



$$G_F(x,y) = \lim_{\varepsilon\to 0} \frac{1}{(2\pi)^4} \int d^4p \, \frac{e^{-ip(x-y)}}{p^2 - m^2 + i\varepsilon} = \begin{cases} -\frac{1}{4\pi}\delta(s) + \frac{m}{8\pi\sqrt{s}} H_1^{(2)}(m\sqrt{s}) & s \geq 0 \\ -\frac{im}{4\pi^2\sqrt{-s}} K_1(m\sqrt{-s}) & s < 0. \end{cases}$$



- ➔ $G_F(x\text{-}y)$ is peaked at the light cone;
- ➔ it is singular at s = 0 (the light cone);
- ➔ it decreases with time in the forward light cone;
- ➔ it is *NOT ZERO* outside the light cone.

Faster than light?

The Feynman propagator has some properties that seem baffling at first. In particular, unlike the commutator, the propagator is *nonzero* outside of the light cone, though it falls off rapidly for spacelike intervals. Interpreted as an amplitude for particle motion, this translates to the virtual particle traveling faster than light. It is not immediately obvious how this can be reconciled with causality: can we use faster-than-light virtual particles to send faster-than-light messages?

From Mandl and Shaw page 51:

picturing the mathematics. But the reader must be warned not to take this pictorial description of the mathematics as a literal description of a process in space and time. For example, our naive interpretation of the meson propagator would imply that, for $(x - x')$ a space-like separation, the meson travels between the two points with a speed greater than the velocity of light. It is however possible to substantiate the above description if, instead of considering propagation between two points x and x', one calculates the probability for emission and absorption in two appropriately chosen four-dimensional regions.‡

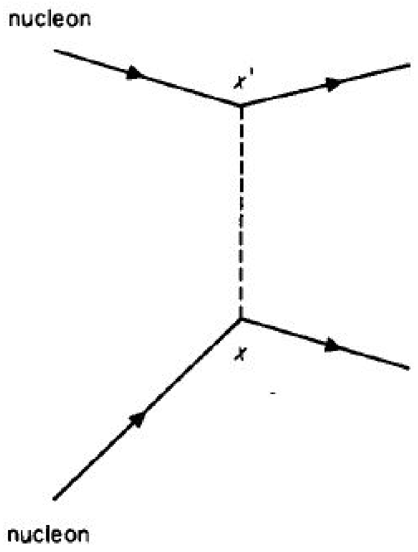


Fig. 3.4. Feynman graph for the one-meson contribution to nucleon–nucleon scattering.

$\Delta_F(x - y)$ is not really a description of motion; rather, it is a *correlation function*.