CHAPTER 9 - INTERACTIONS

OUTLINE

Section 9.1 ▸ Quantum electrodynamics

- ➢ The Lagrangian
- ➢ Gauge invariance
- ➢ QED for charged leptons
- ➢ Nuclear particles

Section 9.2 ▸ Low energy weak interactions

Homework ▸ Problem 9.1

Section 9.1 ▸ Quantum electrodynamics

➢ The Lagrangian

Recall the Lagrangian density for free electrons (+ positrons) and free photons,

$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} (i \not{\partial} - m) \psi$$

Now include interactions by "minimal substitution" ; $p^\mu \to p'^\mu = p^\mu - qA^\mu$;

$$i\partial^\mu \longrightarrow i\partial^\mu - qA^\mu$$

So,

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} (i\not{\partial} - q\not{A} - m) \psi$$
$$= \mathcal{L}_0 - qA_\mu \bar{\psi} \gamma^\mu \psi$$

For the electron f $\mathcal{L}_{int} = eA_\mu \bar{\psi} \gamma^\mu \psi$

➢ Gauge invariance

$$\psi(x) \to e^{i\alpha(x)}\psi(x) \ ; \quad \bar\psi(x) = e^{-i\alpha(x)}\bar\psi(x)$$

$$A^\mu(x) \to A^\mu(x) + \frac{1}{e}\partial^\mu \alpha(x)$$

$$\mathcal{L} \to \mathcal{L} \ ; \text{ but } \mathcal{L}_{int} \to \mathcal{L}_{int} + (\partial^\mu\alpha)\bar\psi\gamma^\mu\psi$$

"Covariant derivative" $D^\mu\psi \equiv (\partial^\mu - ieA^\mu)\psi$

$$D^\mu\psi \to e^{i\alpha} D^\mu\psi$$

An example of a non-minimal interaction

$$\mathcal{L}^{(NM)}_{int} = \frac{-e\kappa}{4m}\bar\psi\sigma_{\mu\nu}\psi F^{\mu\nu}$$

obviously gauge invariant and Lorentz inv.

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \qquad \text{"spin"}$$

$$\Rightarrow [i\partial\!\!\!/ + eA\!\!\!/ - m]\psi = \frac{e\kappa}{2m}\psi \qquad (9.9?)$$

probably wrong.

Non relativistic limit $\psi = \begin{pmatrix} e^{-imt}\varphi \\ \text{small} \end{pmatrix}$

where

$$i\frac{\partial}{\partial t}\varphi = \left[\frac{(\vec p + e\vec A)^2}{2m} + \frac{e}{2m}\vec\sigma\cdot\vec B\right]\varphi + \frac{e\kappa}{2m}\vec\sigma\cdot\vec B\,\varphi$$

$$H = \frac{\vec p^{\,2}}{2m} + \frac{e}{2m}(\vec L + 2\vec S)\cdot\vec B \qquad + \frac{e\kappa}{m}\vec S\cdot\vec B$$

Spin

Magnetic moment $H = -\vec\mu\cdot\vec B$

$$\Rightarrow \quad \vec\mu = -\frac{e}{m}\vec S\cdot\vec B - \frac{e\kappa}{m}\vec S\cdot\vec B$$

Define g by $\vec\mu = g\left(\frac{-e}{2m}\right)\vec S$

$$\Rightarrow \quad g = 2 + 2\kappa$$

$\therefore \kappa$ implies an "anomalous mag. moment $= (g-2)/2$

➢ QED for the charged leptons

e, μ, τ are fundamental particles (or, *fields*) with charge −e and spin ½ .

Table 9.1

	m [MeV]	g
e	0.511	2.00231…
μ	105.7	2.00233…
τ	1777	2.000(58)

The field theory is

$$\mathcal{L}_{QED} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \sum_{\ell = e,\mu,\tau} \bar{\psi}_\ell \left(i\partial\!\!\!/ - m_\ell \right) \psi_\ell$$

$$i\partial\!\!\!/ \to i\partial\!\!\!/ + eA\!\!\!/$$

$$= \mathcal{L}_{free} - A_\lambda J^\lambda_{lept}$$

$$J^\lambda_{lept} = -e \sum_\ell \bar{\psi}_\ell \gamma^\lambda \psi_\ell$$

➢ Nuclear particles

The magnetic dipole moments of the nucleons are given by

$$\boldsymbol{\mu}_{\mathbf{p,n}} = g_{p,n}\ \mu_0\ \mathbf{S}$$

where $\mu_0 = e\hbar/(2\ M_p\ c)$. Expt'l values are

$$g_p/2 = 2.793 \ldots \text{ and } g_n/2 = -1.913 \ldots$$

which imply these anomalous magnetic moment couplings

$$\kappa_p = 1.793 \ldots \text{ and } \kappa_n = -1.913 \ldots$$

Therefore, the electromagnetic field theory, including nucleons is

$$\mathcal{L}_{NUCL} = \bar{\psi}_p (i\not\partial - e\not A - M_p)\psi_p + \bar{\psi}_n (i\not\partial - M_n)\psi_n + \frac{e}{4M_p}\{\kappa_p \bar{\psi}_p \sigma_{\mu\nu}\psi_p + \kappa_n \bar{\psi}_n \sigma_{\mu\nu}\psi_n\} F^{\mu\nu}$$

or $$\mathcal{L}_{NUCL} = \mathcal{L}_{free} - A_\lambda J^\lambda_{NUCL.} \quad (+\ 4\text{-divergence})$$

$$J^\mu_{NUCL.} = e\bar{\psi}_p \gamma^\mu \psi_p + \frac{e\partial^\nu}{2M}\{\kappa_p \bar{\psi}_p \sigma_{\mu\nu}\psi_p + \kappa_n \bar{\psi}_n \sigma_{\mu\nu}\psi_n\}$$

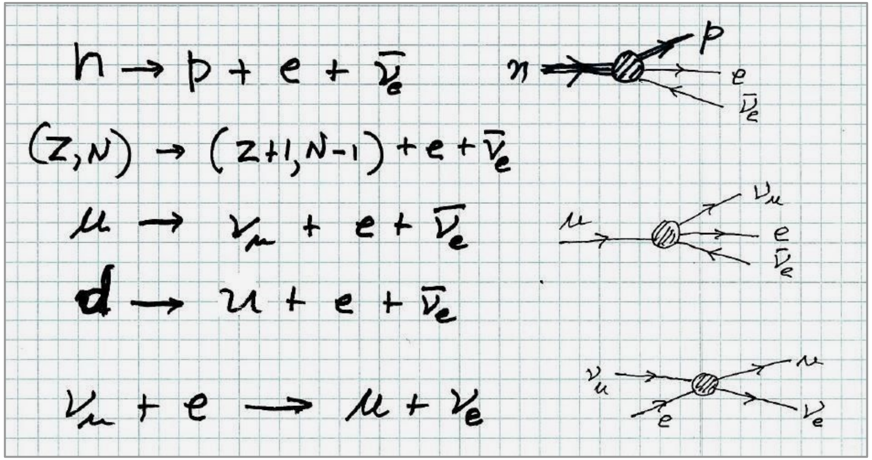
Here we are treating the nucleons as point-like elementary particles. Therefore the nucleon part of the theory is only valid for low energies.

Section 9.2 ▸
The Fermi interaction for beta-decay ;

or, more generally,

Low-energy weak interactions

We know many weak interactions



The low-energy theory for weak interactions is

Fermi's 4-fermion coupling

$$£ = £_{\text{QED}} + £_{\text{WEAK}}$$

$$£_{\text{WEAK}} = -G_F/\sqrt{(2)}\ J_W{}^{\mu}\ J_{W\mu}$$

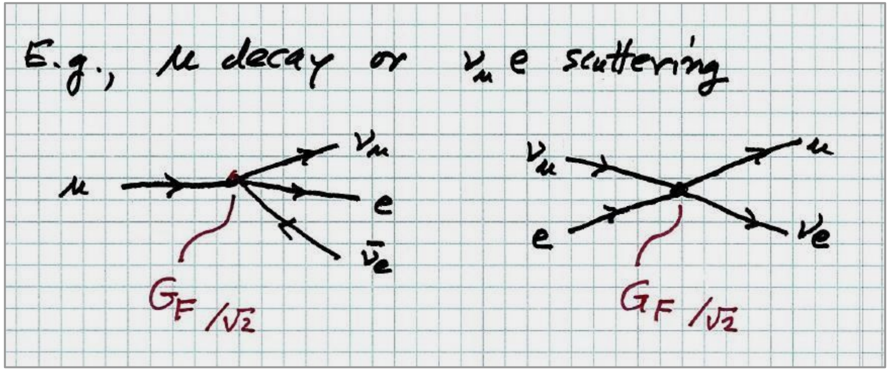
where

$$J_W{}^{\mu} = (J_W{}^{\mu})_{\text{LEPT}} + (J_W{}^{\mu})_{\text{NUCL}}$$

$$(J_W{}^{\mu})_{\text{LEPT}} = \psi_e\, \gamma^{\mu}\, [1 - \gamma_5]\, \psi_\nu + h.c.$$

$$(J_W{}^{\mu})_{\text{NUCL}} = \psi_p\, \gamma^{\mu}\, [1 + (g_A/g_V)\gamma_5]\, \psi_n + h.c.$$

$£_{\text{WEAK}}$ leads to *weak-interaction vertices* for Feynman diagrams.



The Fermi coupling constant, G_F .
See page 146.

- Sign is positive; known from interference between weak and QED interactions.
- $G_F/\sqrt{(2)} = g^2/M_W^{\ 2}$; from standard model.
- $G_F = 1.16 \times 10^{-5}\ \mathrm{GeV}^{-2}$; from μ decay.

Homework Problems
due Friday March 24

1. Maiani and Benhar, problem 9.1.

Also, read:
Sec. 9.3 Strong interactions
Sec. 9.4 Hadrons, leptons and fields of force