Mandl and Shaw

Section 7.3 and Appendix B

THE FEYNMAN RULES FOR QED

Read Mandl and Shaw, Section 7.3, for the derivations.

Mandl and Shaw Appendix B summarizes the rules.

The Feynman amplitude

S_{fi} = the S-matrix element

$$= \quad \langle f|\ T \exp i \int \mathcal{L}_{int}(x)\, d^4x\ |i\rangle$$

We write

$$S_{fi} = \delta_{fi} + (2\pi)^4\, \delta^4(P_{final} - P_{initial}) \left(\prod^{(in)} (2VE_i)^{-1/2}\right) \left(\prod^{(out)} (2VE_f)^{-1/2}\right) \left(\prod^{(fermions)} (2m)^{1/2}\right) \boldsymbol{M}_{\mathbf{fi}}$$

$\boldsymbol{M}_{\mathbf{fi}}$ is the "Feynman amplitude";

it is a Lorentz scalar.

ee scattering ; $e_1 + e_2 \rightarrow e_3 + e_4$

$$< e_3 , e_4 \mid 1/2!\, T \{ [\psi\gamma^\mu\psi\, A_\mu(x_1)]\, [\psi\gamma^\alpha\psi\, A_\alpha(x_2)] \} \mid e_1 , e_2 > d^4x_1 d^4x_2$$

$$= 1/2!\, <0|\, T \{ a_3\, a_4\, [\psi\gamma^\mu\psi\, A_\mu(x_1)]\, [\psi\gamma^\alpha\psi\, A_\alpha(x_2)]\;\; a_1{}^\dagger\;\; a_2{}^\dagger\;\; |0> d^4x_1 d^4x_2$$

Wick's theorem

Two topologically distinct diagrams ;
multiply each one by 2 $\Rightarrow$ cancel 1 /2!

$$= 1/2!\, <0|\, T \{ a_3\, a_4\, [\bar\psi\gamma^\mu\psi\, A_\mu(x_1)]\, [\bar\psi\gamma^\alpha\psi\, A_\alpha(x_2)]\;\; a_1{}^\dagger\;\; a_2{}^\dagger\;\; |0> d^4x_1 d^4x_2$$

$$+ \;1/2!\, <0|\, T \{ a_3\, a_4\, [\bar\psi\gamma^\mu\psi\, A_\mu(x_1)]\, [\bar\psi\gamma^\alpha\psi\, A_\alpha(x_2)]\;\; a_1{}^\dagger\;\; a_2{}^\dagger\;\; |0> d^4x_1 d^4x_2$$

$$+ \;1/2!\, <0|\, T \{ a_3\, a_4\, [\bar\psi\gamma^\mu\psi\, A_\mu(x_1)]\, [\bar\psi\gamma^\alpha\psi\, A_\alpha(x_2)]\;\; a_1{}^\dagger\;\; a_2{}^\dagger\;\; |0> d^4x_1 d^4x_2$$

$$+ \;1/2!\, <0|\, T \{ a_3\, a_4\, [\bar\psi\gamma^\mu\psi\, A_\mu(x_1)]\, [\bar\psi\gamma^\alpha\psi\, A_\alpha(x_2)]\;\; a_1{}^\dagger\;\; a_2{}^\dagger\;\; |0> d^4x_1 d^4x_2$$

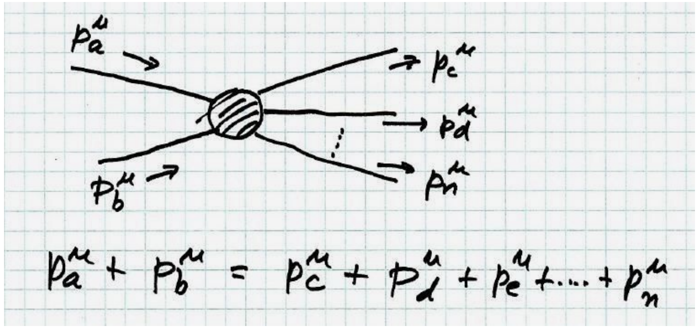
The cross section

For the collision process

$$a + b \rightarrow c + d + e + \ldots + n$$

the differential cross section is

$$d\sigma = (2\pi)^4 \ \delta^4(P_f - P_i)\,(4\,E_a\,E_b\,v_{rel}\,)^{-1}$$
$$\left(\textstyle\prod^{(\text{fermions})} (2m)\,\right)$$
$$\left(\,\left(\textstyle\prod^{(\text{final})} d^3p_f/(2\pi)^3\,/(2E_f)\,\right)\right.$$
$$|\boldsymbol{M}_{\mathbf{fi}}|^2$$

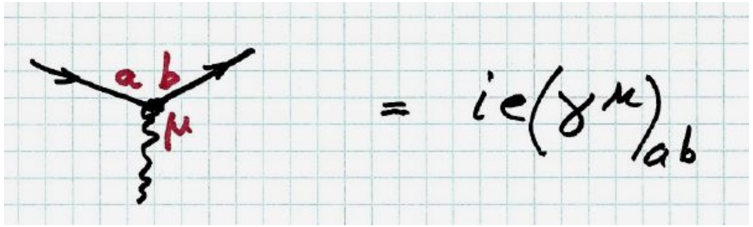


The Feynman rules for QED

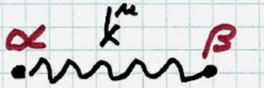
To calculate $\boldsymbol{M}_{\mathbf{fi}}$, *given* $|i>$ *and* $|f>$ ...

(0) Draw all *topologically distinct* graphs with the specified incoming lines $|i>$ and outgoing lines $|f>$.
Each external line attaches to one vertex; each internal line connects two vertices.

(1) Each QED vertex is labeled by a Lorentz index (μ) and two spinor indices (a, b);



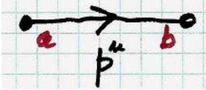
(2) Each internal photon "line" (a squiggle) is labeled by { α , β , k^μ };



$$= i D_F^{\alpha\beta}(k) = \frac{-i g^{\alpha\beta}}{k^2 + i\epsilon}$$

(3) Fermion lines (both internal and external) are *directed* lines; the direction indicated by an arrow on the line; the direction is the direction of flow of the particle charge; antiparticle arrows are *opposite* to the direction of momentum flow.

Each internal fermion line is labelled by { a , b , p^μ } ;

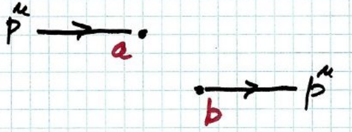


$$= i S_{F\,ab}(p) = \frac{i(\not{p} + m)_{ab}}{p^2 - m^2 + i\epsilon}$$

(4) <u>External lines</u>

fermion in $|i>$

fermion in $|f>$

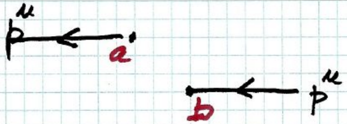


$$= u_{ra}(\vec{p})$$

$$= \bar{u}_{rb}(\vec{p})$$

antifermion in $|i>$

antifermion in $|f>$

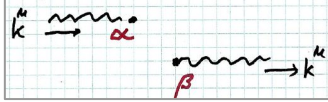


$$= \bar{v}_{ra}(\vec{p})$$

$$= v_{ra}(\vec{p})$$

photon in $|i>$

photon in $|f>$

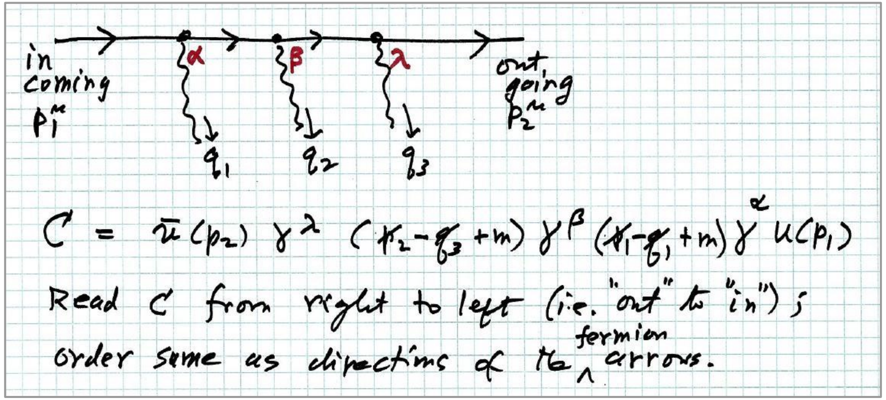


$$= \epsilon_r^{\alpha}(\vec{k}) \quad (k^2 = 0)$$

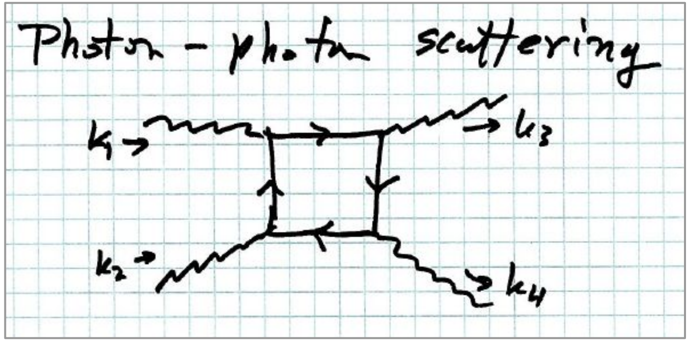
$$= \epsilon_r^{\beta}(\vec{k})$$

Note: $r \in \{1, 2\}$; $a \in \{1, 4\}$

(5) Spinor factors associated with a sequence of connected fermion lines are ordered such that they read from right to left in the same order as specified by the arrows on the fermion lines.



(6) For each closed fermion loop, take the trace and multiply by (– 1).



(7) Four-momentum is conserved at every vertex;

$$\sum p^\mu_{\text{going into the vertex}} = \sum p^\mu_{\text{going out of the vertex}}$$

Integrate (and / $(2\pi)^4$) for all *independent* four-momenta;
i.e., $(2\pi)^{-4}\, d^4q$ occurs for each loop.

(8) Multiply by $\delta_P = +1$ or -1.
δ_P is the sign of the permutation that converts the normal ordered product of fermions into their order in the Feynman diagram.

(9) For external ***static*** electric and magnetic fields there will be a factor $A^\mu_{ext}(\mathbf{q})$ where $\mathbf{q}$ is the three-momentum transfer;
energy (but not 3-momentum) will be conserved.

Example:

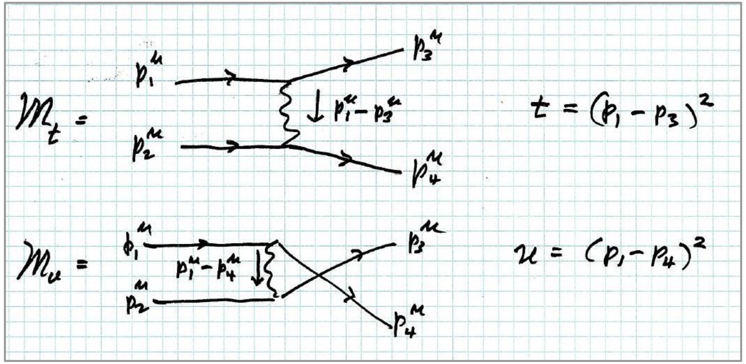
electron-electron elastic scattering

$$\mathbf{e(p_1) + e(p_2) \longrightarrow e(p_3) + e(p_4)}$$

$|\,\boldsymbol{i} > = |\, e_1 , e_2 >$

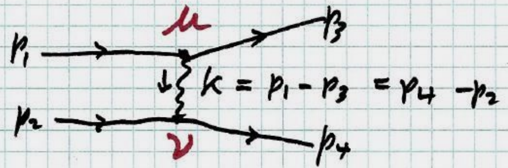
$|\,\boldsymbol{f} > = |\, e_3 , e_4 >$

There are two Feynman diagrams.



$$\boldsymbol{M = M_t + M_u}$$

M_t



$$(\bar{u}_3 \, ie\gamma^\mu u_1)(\bar{u}_4 \, ie\gamma^\nu u_2)\frac{-ig_{\mu\nu}}{t}$$

$$\frac{e^2}{t}(\bar{u}_3\gamma^\mu u_1)(\bar{u}_4\gamma^\nu u_2)\, g_{\mu\nu}$$

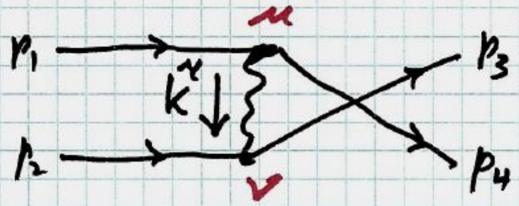
Mandelstam variables,

$s = (p_1 + p_2)^2$

$t = (p_1 - p_3)^2$

$u = (p_1 - p_4)^2$

M_u



$$k^\mu = p_1^\mu - p_4^\mu = p_3^\mu - p_2^\mu$$

$$\frac{-e^2}{u}(\bar{u}_4\gamma^\mu u_1)(\bar{u}_3\gamma^\nu u_2)\, g_{\mu\nu}$$

$M_t = f(p_3, p_1 ; p_4, p_2)$

$M_u = -f(p_4, p_1 ; p_3, p_2)$ "exchange"

Homework Problems

due Friday March 31

7. Mandl and Shaw, problem 7.1.

8. Mandl and Shaw, problem 7.3.

9. Mandl and Shaw, problem 7.5.