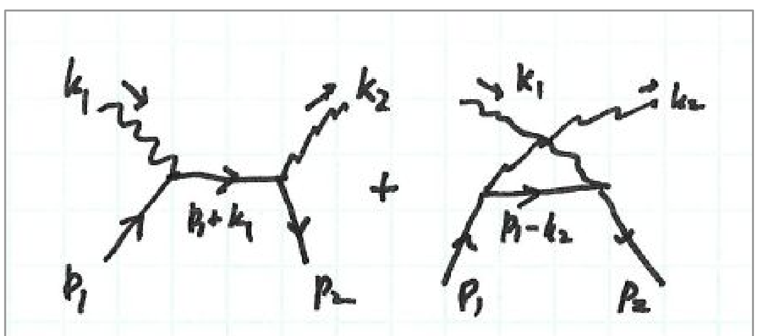
CHAPTER 14 - APPLICATIONS : QED

OUTLINE of the chapter

Section 14.5 ▸ Inverse Compton scattering

We have calculated the magnitude squared of the matrix element, for unpolarized Compton scattering.



Result

$$\overline{|\mathcal{M}|^2} = \frac{2e^4}{m^2}\left(A^2 + A - \frac{1}{4}B\right)$$

$$A = \frac{m^2}{s-m^2} + \frac{m^2}{u-m^2}$$

$$B = \frac{u-m^2}{s-m^2} + \frac{s-m^2}{u-m^2}$$

$$s = (k_1 + p_1)^2 \quad \text{and} \quad u = (k_1 - p_2)^2$$

$|\mathcal{M}|^2$ is Lorentz invariant.

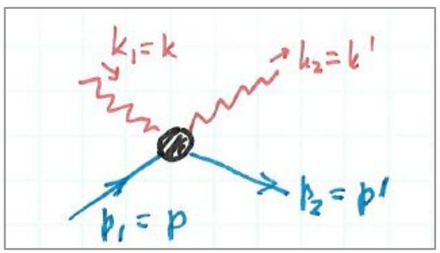
Then we calculated the cross section in the lab frame, $p_1{}^\mu = (\, m, 0, 0, 0 \,)$; this is relevant for a photon (*acting as a projectile*) hitting an electron (*acting as a target*).

Now we'll consider ***inverse*** Compton scattering : the electron is the projectile, hitting a photon which is the target!

It's the same process :

$$\gamma(k) + e(p) \rightarrow \gamma(k') + e(p')$$

but looked at in a different frame of reference, where e moves toward **γ.**



But what frame of reference should we use? There is no such thing as the rest frame for a photon! A photon is never at rest.

We'll use the lab frame for collinear colliding electron and photon. *(More about that later.)*

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<u>There is an interesting **application** of this process.</u>

Nuclear physicists wanted to do *photon−nucleus scattering*.

You can think of different examples:

- ❑ elastic scattering
- ❑ resonant scattering
- ❑ highly inelastic scattering

There would be something to learn about the nucleus from each type of collision.

But it would require a high-energy photon beam; i.e., photons energies of at least 10 MeV , or, better yet > 100 MeV.

But how can we make a photon beam with photon energies in the range 10 – 1000 MeV?

Terminology: The band of the electromagnetic spectrum that we call "X-rays" have …

- ❏ wavelengths from 0.01 to 10 nm;
- ❏ frequencies from 3×10^{16} Hz to 3×10^{19} Hz;
- ❏ photon energies from 100 eV to 100 keV.

How can we make an intense beam of *gamma rays*?

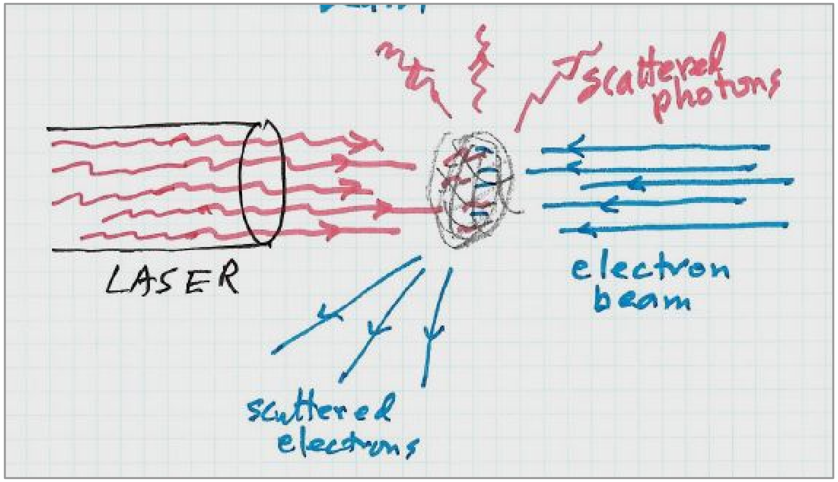
* _ * _ *

Inverse Compton scattering

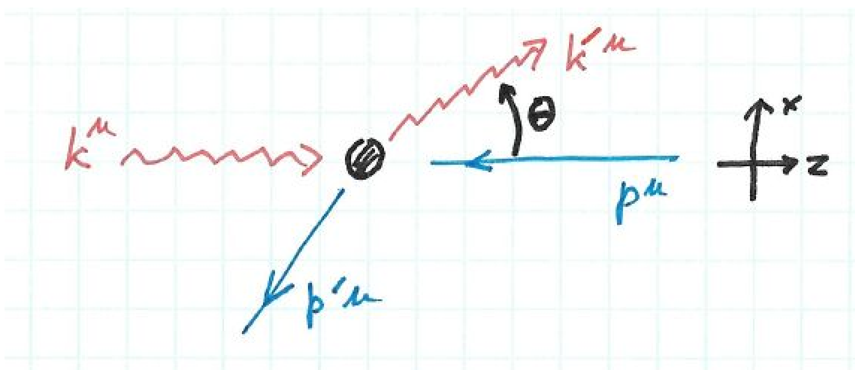
$$e(p) + \gamma(k) \rightarrow e(p') + \gamma(k')$$

where

- ➔ e(p) has very high energy;
- ➔ γ(k) has low energy (optical);
- ➔ we want γ(k') to have high energy (i.e., gamma ray photons)



THE LAB FRAME OF REFERENCE



θ = scattering angle of the photon; angle of deflection

$k^\mu = (\ \omega\ ,\ 0\ ,\ 0\ ,\ k\ )$; $\omega = k$

$p^\mu = (\ E\ ,\ 0\ ,\ 0\ ,\ -p\ )$; $E = \sqrt{(m^2 + p^2)}$; $p>0$

$k'^\mu = (\ \omega'\ ,\ k' \sin\theta\ ,\ 0\ ,\ k' \cos\theta)$; $\omega' = k'$

$p'^\mu = (\ E'\ ,\ - k' \sin\theta\ ,\ 0\ ,\ k - p - k' \cos\theta\ )$

(Note: 3 momentum is conserved.)

$E' = \mathrm{SQRT}[m^2 + k'^2 + (k-p)^2 - 2(k-p)k'\cos\theta]$

(Energy must also be conserved.)

KINEMATICS in the lab frame ...

We'll need the Mandelstam variables,

- $s = (\ k + p\ )^2 = m^2 + 2\ k.p$
  $= m^2 + 2\ \omega\ E + 2\ kp$
  $= m^2 + 2k\ (E+p)$
- $t = (\ k - k'\ )^2 = -\ 2\ k.k'$
  $= -\ 2\ \omega\ \omega' + 2\ k\ k' \cos\theta$
  $= -\ 2\ k\ k'\ (1 - \cos\theta)$
- $u = (\ k - p'\ )^2 = (\ k' - p\ )^2$
  $= m^2 - 2\ k'.p$
  $= m^2 - 2\omega'\ E - 2\ pk' \cos\theta$
  $= m^2 - 2\ k'\ (E + p\ \cos\theta)$

Check: ***$s + t + u = 2m^2$;***

therefore $0 = k(E+p) - kk'(1\text{-}\cos) - k'(E+p\cos)$;

solve for $k' = k\ (E+p)\ /\ [E+p\cos + k(1\text{-}\cos)]$.

THE FINAL PHOTON ENERGY

$$E' = \sqrt{m^2 + k'^2 + (k-p)^2 - 2(k-p)k'\cos\theta}$$

$$E' = \omega + E - \omega' = k - k' + E$$

$$E'^2 = m^2 + k'^2 + k^2 + p^2 - 2kp - 2(k-p)k'\cos\theta$$

$$E'^2 = m^2 + p^2 + k^2 + k'^2 - 2kk' + 2E(k-k')$$

cancellations →

$$2k'[E + k - (k-p)\cos\theta] = 2k(E+p)$$

$$\frac{k'}{k} = \frac{E+p}{E + p\cos\theta + k(1-\cos\theta)}$$

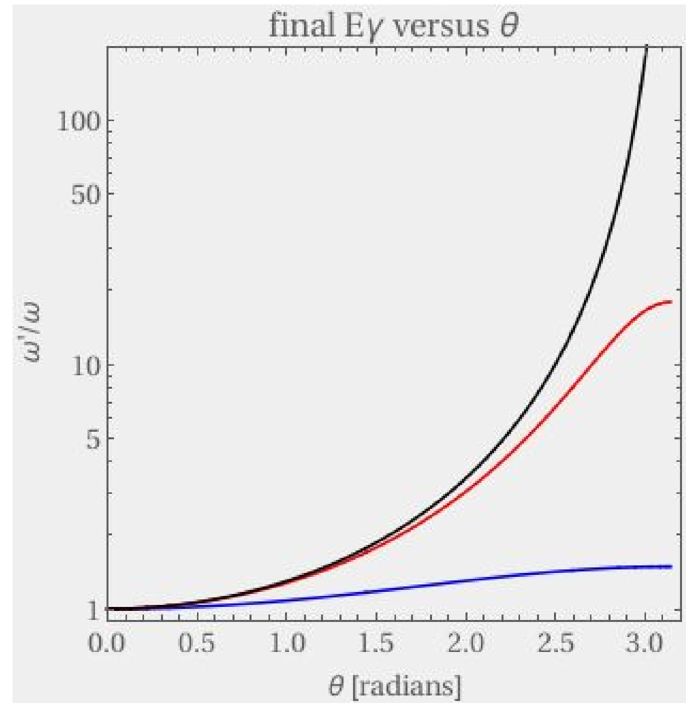
Note: If $p=0$ then

$$\frac{k'}{k} = \frac{m}{m + k(1-\cos\theta)}$$

= the Compton effect in e, rest frame

THE FINAL PHOTON ENERGY

ω = 10 eV;
p = 0.1 MeV , 1.0 MeV , 10.0 MeV



So here's the result :
To make a high energy photon beam, use inverse Compton scattering with photons scattered in the direction $\theta \approx \pi$.

Max $\omega' = (E+p) / (E-p+2\omega) \approx E/\omega$ if E is large.

(1 GeV) / (10 eV) = 100 MeV

THE CROSS SECTION

We need to go through the calculation again, for this frame of reference.

$$d\sigma = \frac{(2m)^2}{4E_1E_2 v_{rel}} (2\pi)^4 \delta^4(P_f - P_i) |\mathcal{M}|^2 \frac{d^3p_3}{(2\pi)^3 2E_3} \frac{d^3p_4}{(2\pi)^3 2E_4}$$

$$d\sigma = \frac{m^2}{\omega E \left| \frac{k}{\omega} + \frac{p}{E} \right|} \frac{1}{(2\pi)^2} \int \frac{d^3k'}{4\omega' E'} \delta(\omega' + E' - \omega - E) |\mathcal{M}|^2$$

$\omega = k$ and $\omega' = k'$

$$d\sigma = \frac{m^2}{Ek + \omega p} \frac{1}{16\pi^2} \int \frac{k'^2 dk' d\Omega'}{k'E'} \delta[f(k')] |\mathcal{M}|^2$$

$$f(k') = k' + \sqrt{m^2 + k'^2 + (k-p)^2 - 2(k-p)k'\cos\theta} - \omega - E$$

$$\frac{df}{dk'} = 1 + \frac{1}{2}\frac{1}{\sqrt{\cdots}}\left(2k' - 2(k-p)\cos\theta\right)$$

$$= \frac{1}{E'}\left(E' + k' - (k-p)\cos\theta\right)$$

$$= \frac{1}{E'}\left(E + k - (k-p)\cos\theta\right)$$

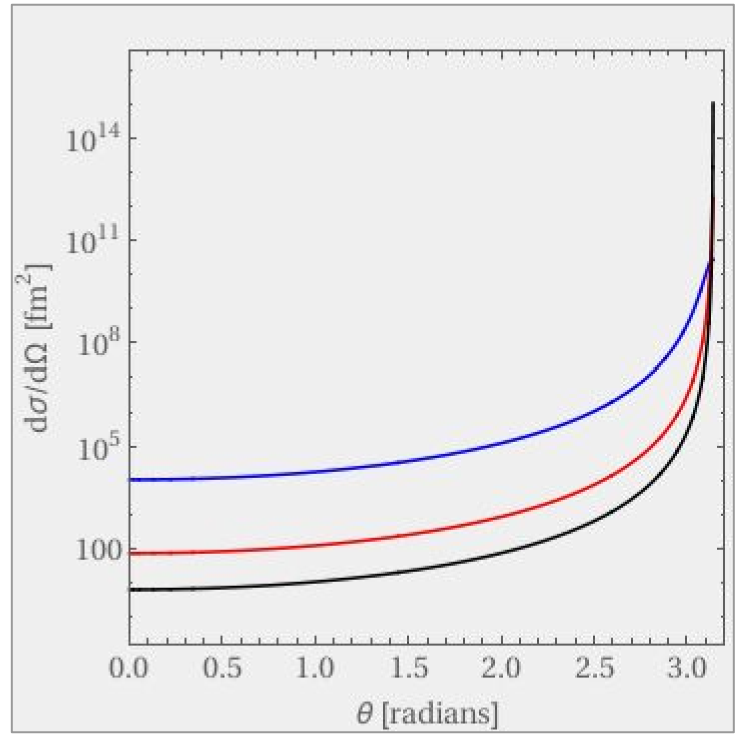
$$= \frac{1}{E'}\left(E + p\cos\theta + k(1-\cos\theta)\right) = \frac{k(E+p)}{E'k'}$$

$$\frac{d\sigma}{d\Omega'} = \frac{m^2}{(E+p)k} \frac{1}{16\pi^2} \frac{k'}{E'} \frac{E'k'}{k(E+p)} |\mathcal{M}|^2$$

$$\frac{d\sigma}{d\Omega'} = \frac{m^2}{16\pi^2} \left(\frac{k'}{k}\right)^2 \frac{|\mathcal{M}|^2}{(E+p)^2}$$

THE CROSS SECTION

To make this plot we need Ma'tica!



So here's the result :
Use photons scattered in the direction $\theta \approx \pi$. The cross section for scattering in that direction is large.

THE EXAMPLE IN MAIANI AND BENHAR

- Frascati Laboratory, Rome
- Adone storage ring
  - E_e = 1.5 GeV
- Laser photons
  - ω_γ = 2.45 eV
- high-energy photon beam
  - $\omega' \approx 80$ MeV
- The high energy photons can be polarized, by creating them with polarized electrons.

ANOTHER APPLICATION OF INVERSE COMPTON SCATTERING

GAMMA RAY ASTRONOMY

- ❏ Astronomical gamma rays are observed in gamma ray telescopes.
- ❏ E.g., Fermi LAT (Large Area Telescope) – a NASA satellite.
- ❏ E.g., HAWC , the High Altitude Water Cerenkov observatory in Mexico.
- ❏ How are astronomical gamma rays produced? There are many theories, and some of them rely on *inverse compton scattering*.

Homework Problems

due April 14

18. Some calculations from Chapter 14.
If you use FeynCalc (the easy way) then hand in a printout of the program including the result. If you do the calculation by hand, hand in the step by step details.
(a) Given equation 68 derive equation 71.
(b) Given equations 97 – 100, derive equations 107 and 112.
(c) Derive the result in equation 118.

19. Derive the center of mass cross section for Bhabha scattering.
If you use FeynCalc (the easy way) then hand in a printout of the program including the result. If you do the calculation by hand, hand in the step by step details. Compare your result, in the form of a plot of $d\sigma/d\Omega$ versus θ, to measurements from the PETRA e+e- collider.

20. We sometimes hear the statement "The electron is a point particle." What does that mean? Ref: G. Kopp, D. Schaile, M. Spira, P.M. Zerwas, Z. Phys. C 65 , 545 (1995). According to this reference, what is the experimental bound on the radius of the electron?