CHAPTER 14 - APPLICATIONS : QED

OUTLINE of the chapter

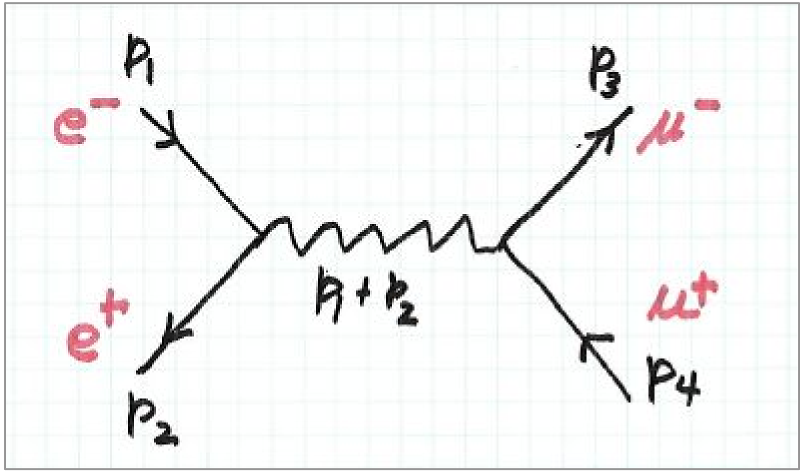
14.1 ▸ Scattering in a Coulomb field ✓
14.2 ▸ Form factors ✓
14.3 ▸ The Rosenbluth formula ✓
14.4 ▸ Compton scattering ✓
14.5 ▸ Inverse Compton scattering ✓
14.6 ▸ Processes $\gamma\gamma \to e^+e^-$ and $e^+e^- \to \gamma\gamma$ ✓
14.7 ▸ $e^+e^- \to \mu^+\mu^-$ annihilation
14.8 ▸ Problems for Chapter 14

Section 14.7 ▸

e+ e– annihilation to μ+ μ–

and related processes!

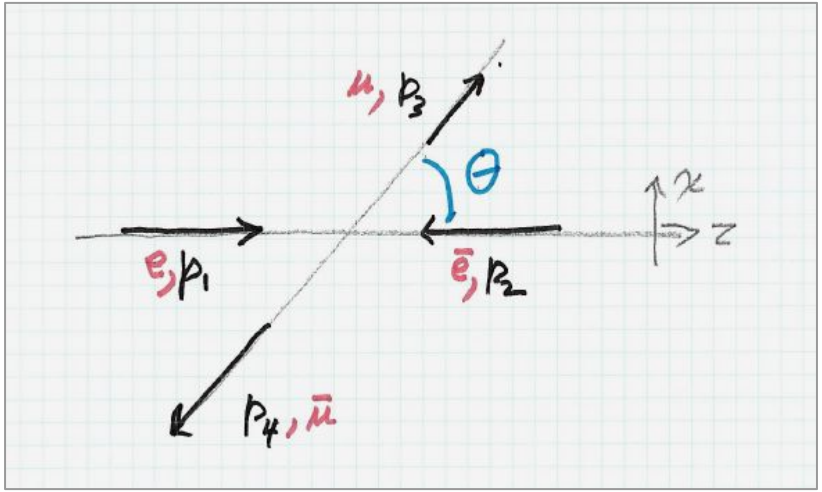
There is one Feynman diagram for this process.



Kinematics

It is most natural to use the center of mass frame of reference.

This is also the *lab frame of reference* for an electron-positron collider (e.g., DESY-PETRA , SLAC-PEP , SLAC-SLC , CERN-LEP).



In the center of mass frame of reference, let the z-axis be the electron beam,

$p_1^\mu = (E_e , 0, 0, p);$
$p_2^\mu = (E_e , 0, 0, -p)$

$E_e = \mathrm{SQRT}(p^2 + m^2)$

$p_3^\mu = (E_\mu , p_3 \sin\theta, 0, p_3 \cos\theta)$
$p_4^\mu = (E_\mu , -p_3 \sin\theta, 0, -p_3 \cos\theta)$

$E_\mu = \mathrm{SQRT}(p_3^{\,2} + M^2) = E_e$

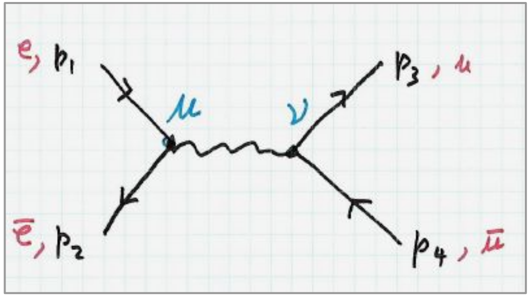
$s = (p_1+p_2)^2 = 4 E_e^{\,2}$

$t = (p_1 - p_3)^2 = (E_e - E_\mu)^2 - (\mathbf{p_1} - \mathbf{p_3})^2$
$= - (p_3 \sin\theta)^2 - (p - p_3 \cos\theta)^2$
$= - p_3^{\,2} - p^2 + 2 p p_3 \cos\theta$

$u = (p_1 - p_4)^2 = - p_3^{\,2} - p^2 - 2 p p_3 \cos\theta$

Check: $s + t + u = 2m^2 + 2M^2$

The matrix element



$M = [\, \bar{v}_2 \,(ie\gamma^\mu)\, u_1 \,]\, [\, \bar{u}_3 (ie\gamma^\nu)\, v_4 \,]\, (-ig_{\mu\nu}/\, q^2\,)$

where $q^\mu = p_1{}^\mu + p_2{}^\mu = p_3{}^\mu + p_4{}^\mu$.

Now calculate the amplitude squared,

$$|M|^2 = e^4 / s^2 \; [\, \bar{v}_2\, \gamma^\mu\, u_1 \,]\, [\, \bar{u}_3\, \gamma^\nu\, v_4 \,] \; g_{\mu\nu}$$
$$[\, \bar{u}_1\, \gamma^\alpha\, v_2 \,]\, [\, \bar{v}_4\, \gamma^\beta\, u_3 \,] \; g_{\alpha\beta}$$

$$|M|^2 = e^4 / s^2 \; [\, \bar{v}_2\, \gamma^\mu\, u_1 \,]\, [\, \bar{u}_1\, \gamma^\alpha\, v_2 \,]$$
$$[\, \bar{u}_3\, \gamma^\nu\, v_4 \,]\, [\, \bar{v}_4\, \gamma^\beta\, u_3 \,] \; g_{\mu\nu}\, g_{\alpha\beta}$$

$$\overline{|M|^2} = e^4 / s^2 \; /(4m^2M^2)$$
$$\mathrm{Tr}\, \{\, \gamma^\mu\, (\not{p}_1 + m)\, \gamma^\alpha\, \{\not{p}_2 - m)\, \}$$
$$\mathrm{Tr}\, \{\, \gamma_\mu\, (\not{p}_4 - M)\, \gamma_\alpha\, \{\not{p}_3 + M)\, \}$$
$$= e^4 / s^2 / (4m^2M^2) \;\; E^{\mu\alpha}\, M_{\mu\alpha}$$

I'll neglect the mass of the electron; that is,

- ❑ $m = 0$
- ❑ $E_e = p$
- ❑ $E_{muon} = E_e$ implies $p_3{}^2 + M^2 = p^2$
- ❑ $s = 4\, p^2$
- ❑ $t = M^2 - 2p^2 + 2\, p\, p_3 \cos\theta$

The matrix element squared

$|M|^2 = e^4 / s^2 / (4m^2M^2) \quad E^{\mu\alpha} M_{\mu\alpha}$

Note 4 m^2 M^2 = **[Πn^2]** .

Calculate $E^{\mu\alpha} M_{\mu\alpha}$ using FeynCalc, summing and averaging over spins for the unpolarized process. Also, we can approximate m = 0 because m << M.

For m = 0,

$$E^{\mu\alpha} M_{\mu\alpha} = 16 M^4 - 32 M^2 t + 8 s^2 + 16 s t + 16 t^2 .$$

In the center of mass frame of reference,

$p = E_e$; $E_3 = E_e$; p_3 = **SQRT**$(E_e{}^2 - M^2)$

$s = 4E_e{}^2$;

$t = M^2 - 2p^2 + 2 p p_3 \cos\theta$.

```
In[14]:= tenE = Tr[GA[μ].(GS[p1] + m).GA[α].(GS[p2] - m)]
         tenM = Tr[GA[μ].(GS[p4] - M).GA[α].(GS[p3] + M)]
         ξ = Calc[tenE.tenM]
         ξ = Simplify[Expand[ξ /. {u → 2 m^2 + 2 M^2 - s - t}]]
         (* set m = 0 *)
         ξ = Expand[ξ /. {m → 0}]
```

Out[14]= $4\left(-\frac{1}{2} m^2 g^{\alpha\mu} - \frac{s g^{\alpha\mu}}{2} + p1^\mu p2^\alpha + p1^\alpha p2^\mu\right)$

Out[15]= $4\left(-\frac{1}{2} s g^{\alpha\mu} + p3^\mu p4^\alpha + p3^\alpha p4^\mu\right)$

Out[16]= $16 m^4 + 48 m^2 M^2 + 16 m^2 s - 16 m^2 t - 16 m^2 u + 16 M^4 + 16 M^2 s - 16 M^2$

Out[17]= $8\left(2 m^4 + m^2 (6 M^2 - 4 t) + 2 M^4 - 4 M^2 t + s^2 + 2 s t + 2 t^2\right)$

Out[18]= $16 M^4 - 32 M^2 t + 8 s^2 + 16 s t + 16 t^2$

The cross section in the C.o.M.

$$\frac{d\sigma}{d\Omega} = \frac{[\Pi n^2]}{64 \pi^2 s} \frac{p_3}{p} |M|^2 \; [\Pi n^2]$$

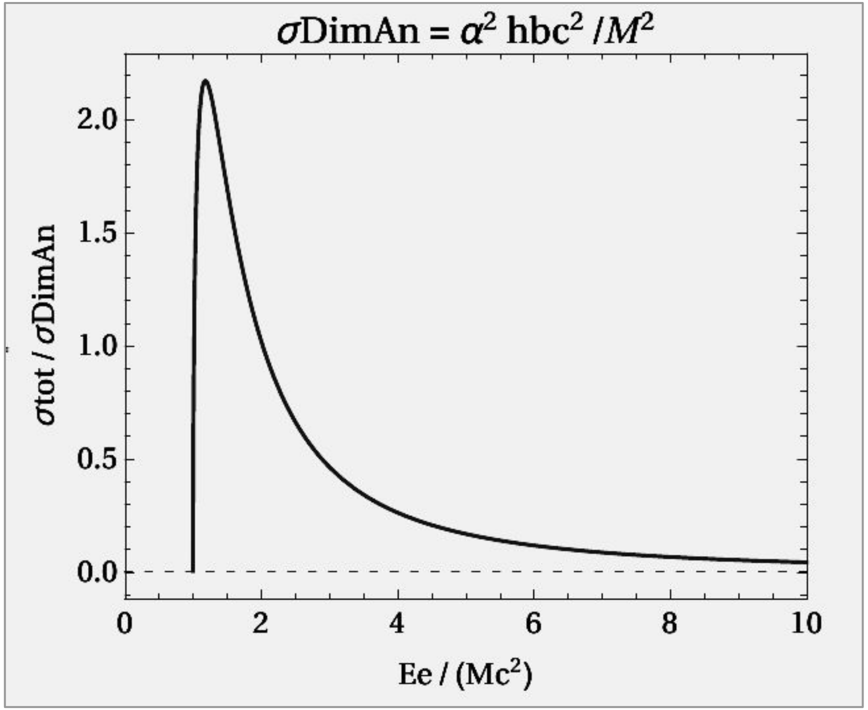
```
Δσ = Πnsq / (64 * Pi^2 * s) * (p3 / p) * Msq
```

$$\frac{\alpha^2 \sqrt{\mathrm{Ee}^2 - M^2}\,\left(\mathrm{Ee}^2\left(1+\cos^2\right) - M^2\left(\cos^2 - 1\right)\right)}{4\,\mathrm{Ee}^5}$$

⇒ the total cross section

$$\frac{2\,\pi\,\alpha^2 \sqrt{\mathrm{Ee}^2 - M^2}\,\left(2\,\mathrm{Ee}^2 + M^2\right)}{3\,\mathrm{Ee}^5}$$

Graphical analysis of the cross section



★ The process e+ + e− → hadrons

- This was very important in the history of high energy physics; it had a big influence on the acceptance of QCD.
- QCD is asymptotically free. Therefore the cross section can be approximated by

$$\sigma(e\bar{e}\to\text{hadrons}) \approx \sigma(e\bar{e}\to\text{quarks})$$

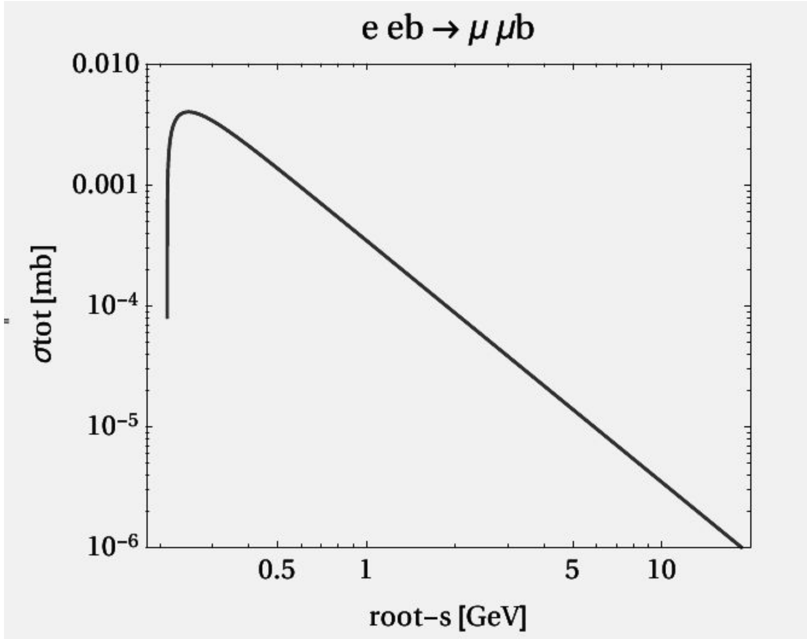
- $e\bar{e} \to q\bar{q}$ is just like $e\bar{e} \to \mu\bar{\mu}$, except for the difference in electric charges.
- So we might guess

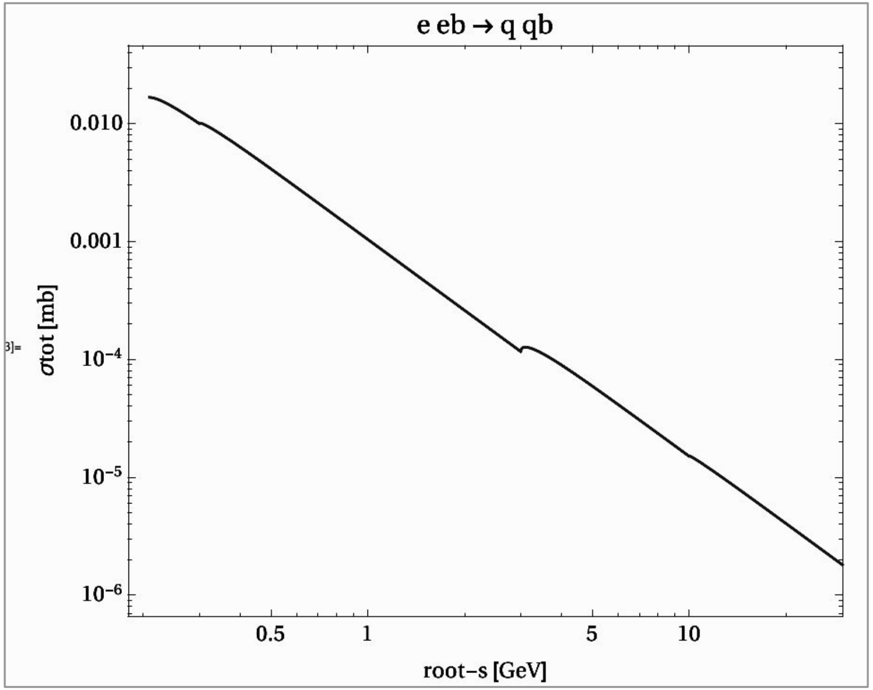
$$R = \sigma_{\text{hadrons}} / \sigma_{\mu\mu} = \sum e_q^{\,2}/e^2$$

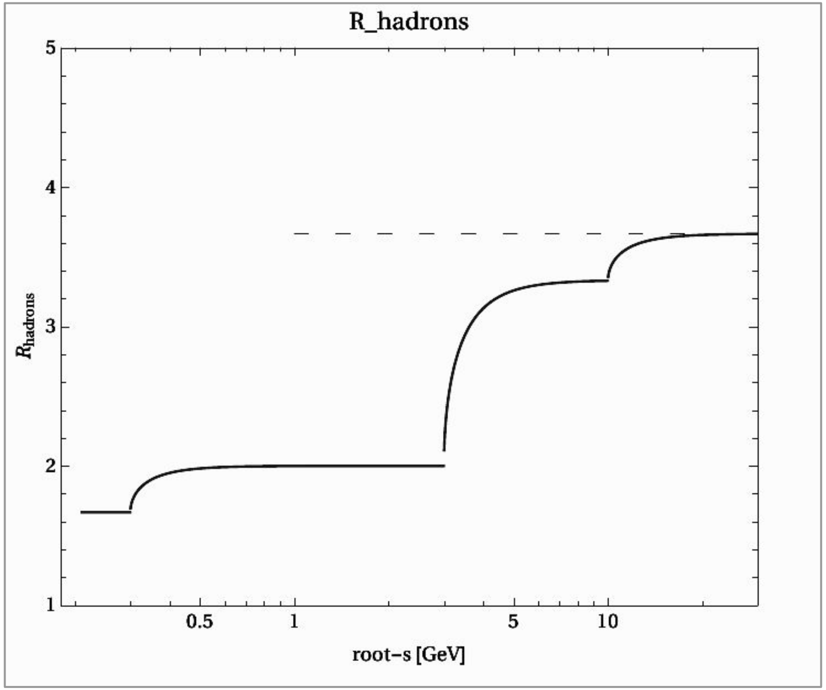
At $\sqrt{s} > 6$ GeV we expect

$$R = 3\,(⅓)^2 + 3\,(⅔)^2 + 3\,(⅓)^2 + 3\,(⅔)^2 + 3\,(⅓)^2$$

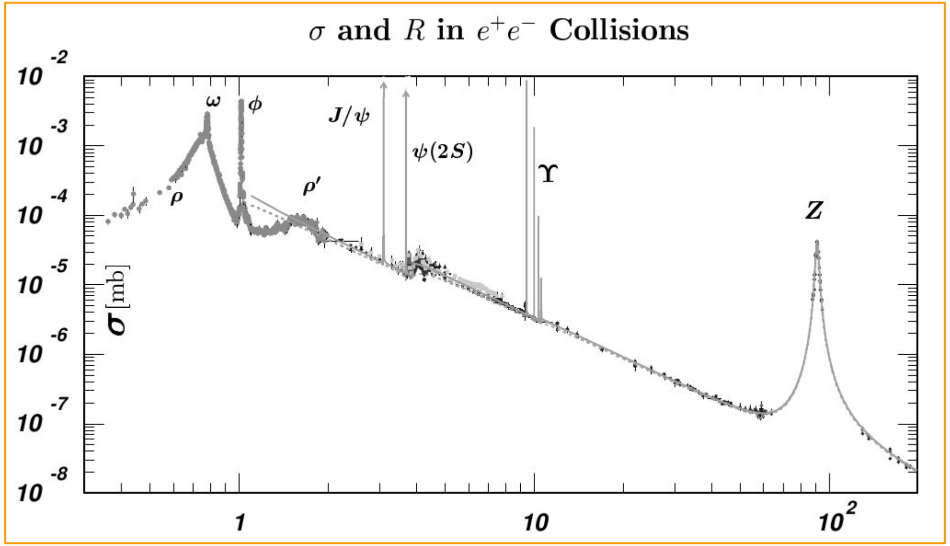
$$= 11/3 = 3.667$$

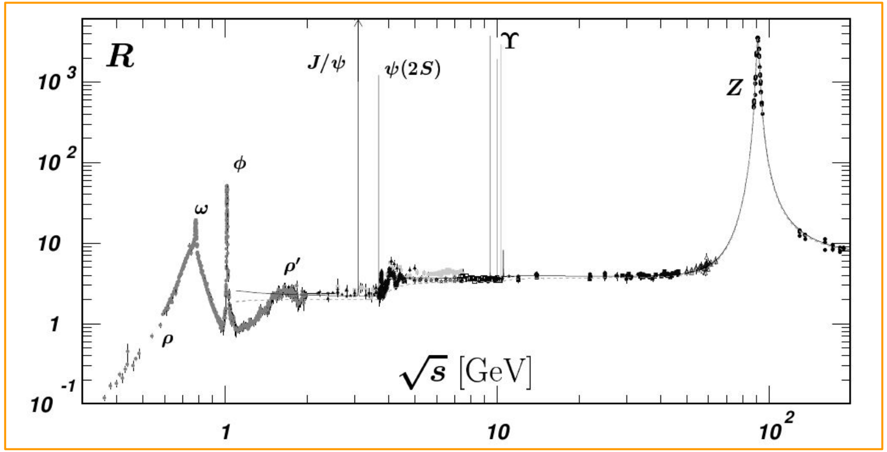


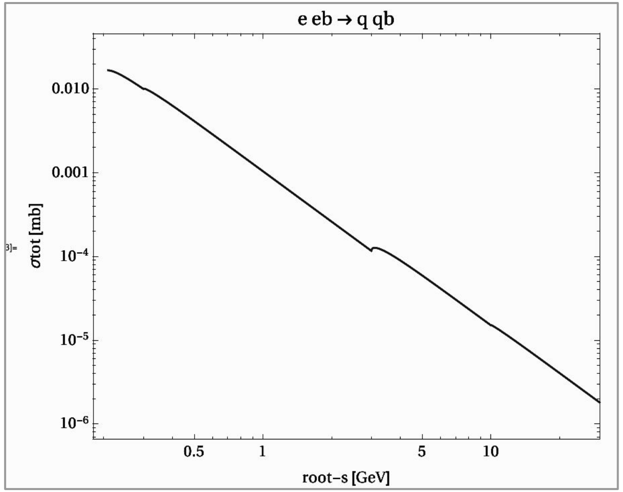


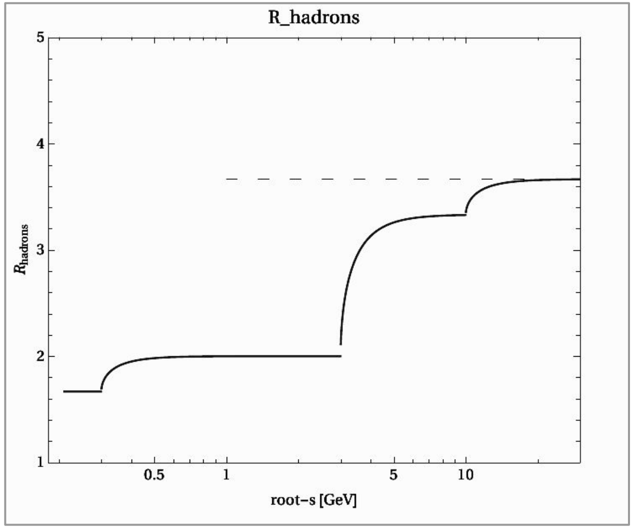


Thresholds occur at $\sqrt{s} = 2\, m_q$.


σ and R in e+e− Collisions
σ [mb]
ρ
ω
φ
ρ′
J/ψ
ψ(2S)
ϒ
Z
1
10
10^2


R
J/ψ
ψ(2S)
ϒ
Z
ω
φ
ρ′
ρ
√s [GeV]
1
10
10^2


e eb → q qb
σtot [mb]
0.010
0.001
root-s [GeV]
0.5
1
5
10


R_hadrons
R_hadrons
root-s [GeV]
0.5
1
5
10

HOMEWORK

NOTHING NEW TO ASSIGN

Homework Problems due April 14

18. Some calculations from Chapter 14.
If you use FeynCalc (the easy way) then hand in a printout of the program including the result. If you do the calculation by hand, hand in the step by step details.
(a) Given equation 68 derive equation 71.
(b) Given equations 97 − 100, derive equations 107 and 112.
(c) Derive the result in equation 118.

19. Derive the center of mass cross section for Bhabha scattering.
If you use FeynCalc (the easy way) then hand in a printout of the program including the result. If you do the calculation by hand, hand in the step by step details. Compare your result, in the form of a plot of $d\sigma/d\Omega$ versus θ, to measurements from the PETRA e+e- collider.

20. We sometimes hear the statement "The electron is a point particle." What does that mean? Ref: G. Kopp, D. Schaile, M. Spira, P.M. Zerwas, Z. Phys. C 65 , 545 (1995). According to this reference, what is the experimental bound on the radius of the electron?

21. Is it possible to observe $\gamma\gamma \rightarrow e^- + e^+$.
If so, how. If not, why not?

22. Calculate the mean lifetime of para-positronium. (You can use the results in the book, but fill in the gaps.)

23. Show why the annihilation process $e^- + e^+ \rightarrow \gamma$ *cannot* occur.