MAIANI AND BENHAR, CHAPTER 15
APPLICATIONS : WEAK INTERACTIONS

OUTLINE of Chapter 15

Mandl and Shaw,
Chapter 11, "Weak Interactions"
11.6 Decay rates
11.7 The IVB theory

WEAK INTERACTIONS OF LEPTONS

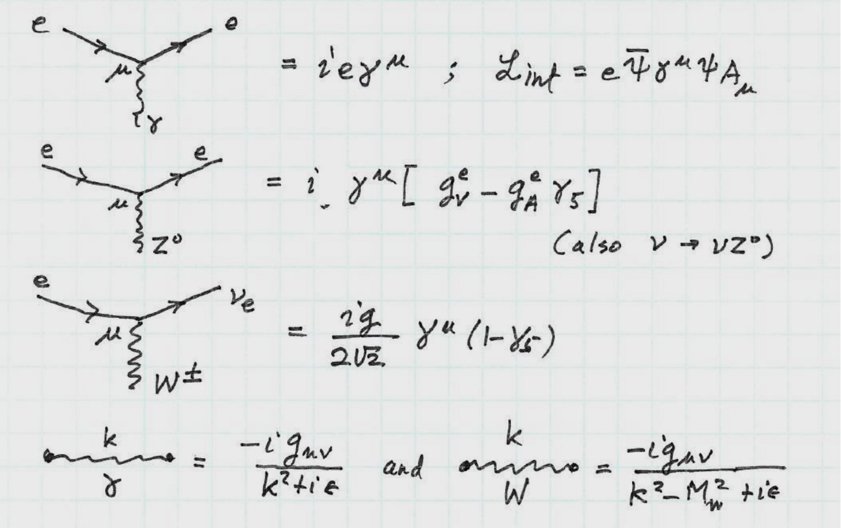
LEPTONS

CHARGED LEPTONS	SPIN	CHARGE	MASS c^2
e	1/2	$-e$	0.511 Mev
μ	1/2	$-e$	106 MeV
τ	1/2	$-e$	1777 MeV
NEUTRINOS			
ν_1	1/2	0	? ; $\lesssim$ 0.01 eV
ν_2	1/2	0	? ; $\lesssim$ 0.01 eV
ν_3	1/2	0	? ; $\lesssim$ 0.01 eV

VECTOR BOSONS

NEUTRAL	SPIN	CHARGE	MASS c^2 *(DECAY WIDTH c^2)*
γ	1	0	0
Z^0	1	0	91.2 GeV (2.50 GeV)
CHARGED			
W^+	1	$+e$	80.4 GeV (2.08 GeV)
W^-	1	$-e$	***ditto***

INTERACTION VERTICES AND PROPAGATORS



Section 15.2. Decay of the muon

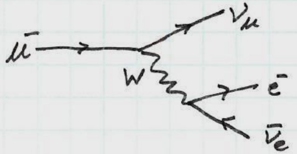
Muon decay modes

$\nu_\mu + e + \bar{\nu}_e \quad \approx 99\ \%$

$\nu_\mu + e + \bar{\nu}_e + \gamma \quad \approx 1\ \%$

$\nu_\mu + e + \bar{\nu}_e + e^+e^- \approx 1 \times 10^{-5}$

IVB



$$= \left[\bar{u}_2 \gamma^\mu (1-\gamma_5) u_1\right] \left[\bar{u}_3 \gamma_\mu (1-\gamma_5) v_4\right] \left(g/2\sqrt{2}\right)^2 \frac{1}{(p_1-p_2)^2 - M_W^2}$$

$$\approx \left[\bar{u}_2 \gamma^\mu (1-\gamma_5) u_1\right]\left[\bar{u}_3 \gamma_\mu (1-\gamma_5) v_4\right] G/\sqrt{2}$$

where $G/\sqrt{2} = \dfrac{g^2}{8 M_W^2}$

Fermi

$=$ 

with coupling $G_F/\sqrt{2}$

Quantum field theory for the muon decay calculation

$$\mu^-(P) \rightarrow \nu_\mu(p') + e^-(q) + \bar{\nu}_e(q')$$

$$S^{(1)}_{FI} = \int \langle \nu_\mu e\, \nu_e{}^b \mid \pounds_{int}(x) \mid \mu \rangle\, d^4x$$

$$\pounds_{int}(x) = -\, G_F/\sqrt{2}$$

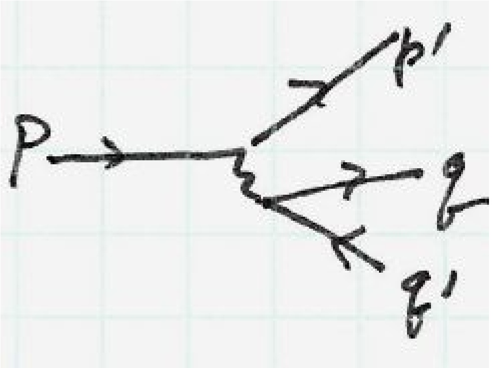
$$[\, \bar{\psi}_{nm} \gamma^\lambda (1-\gamma_5)\, \psi_m \,]_{\text{muon fields}}$$

$$[\, \bar{\psi}_e \gamma_\lambda (1-\gamma_5)\, \psi_{ne} \,]_{\text{electron fields}}$$

$+$ *hermitian conjugate*

Or sometimes we write

$$\pounds_{\text{int}}(x) = -\,G_F/\sqrt{2}$$
$$[\,\bar{\nu}_\mu(x)\,\gamma^\lambda\,(1-\gamma_5)\,\mu(x)\,]_{\text{muon fields}}$$
$$[\,\bar{e}(x)\,\gamma_\lambda\,(1-\gamma_5)\,\nu_e(x)\,]_{\text{electron fields}}$$
$$+\ \text{h.c.}$$



So the first-order S matrix element is

$$S^{(1)}_{FI} = i\int \langle \nu_\mu\, e\, \bar{\nu}_e \,|\, \frac{-G}{\sqrt{2}} \left[\bar{\nu}_\mu \gamma^\alpha (1-\gamma_5)\mu\right] \left[\bar{e}\,\gamma_\alpha (1-\gamma_5)\nu_e\right] \,|\mu\rangle\, d^4x$$

$$= \frac{-iG}{\sqrt{2}} \prod \frac{1}{\sqrt{2EV}} \prod \sqrt{2m} \left[\bar{u}(p')\gamma^\alpha(1-\gamma_5)u(P)\right] \left[\bar{u}(q)\gamma_\alpha(1-\gamma_5)v(q')\right] (2\pi)^4 \delta^4(p'+q+q'-P)$$

$P \to$

How to calculate the decay rate

- Quantum theory: The decay probability involves the square of the S-matrix element.
- *Recall the associated problems and tricks.*
- The result is Maiani&Benhar equation (11.41).
- The decay width (= inverse of the lifetime) is $d\Gamma$.

General equations for 3-body decay

$$A \rightarrow b + c + d$$

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta^4(P_f - P) \times \prod (2VE)^{-½} \prod (2m)^{½} \mathfrak{M}$$

$$d\Gamma = (2\pi)^4 \delta^4(P_f - P) (2E_A)^{-1} \times \prod \frac{d^3p}{(2\pi)^3 2E} \prod (2m) |\mathfrak{M}|^2$$

($E_A = M_A$)

The transition amplitude

$$\mathfrak{M} = -i\, G/\sqrt{2}\, [\, \bar{u}(q)\, \gamma^\alpha (1 - \gamma_5)\, v(q')\,]\; [\, \bar{u}(p')\, \gamma_\alpha (1 - \gamma_5)\, u(P)\,]$$

There are no propagators, just spinors for the initial and final Dirac particles.

Calculate $\mathfrak{M}\,\mathfrak{M}^*$.

Average over the initial spin and sum over the final spins ⇒ convert to traces of gamma matrices.

$$\mathfrak{X} = \tfrac{1}{2} \sum_{\text{spins}} |\mathfrak{M}|^2$$

$$= G^2/4 \ \left[\prod (2m)^{-1}\right]$$

$$\mathrm{Tr}\,\{ (\not{q} + m_e)\, \gamma^\alpha (1-\gamma_5)\, (\not{q}' - m_n)\, \gamma^\beta (1-\gamma_5) \}$$

$$\mathrm{Tr}\,\{ (\not{p}' + m_n)\, \gamma_\alpha (1-\gamma_5)\, (\not{P} + M)\, \gamma_\beta (1-\gamma_5) \}$$

$$= G^2/4 \ \left[\prod (2m)^{-1}\right] \ El^{\alpha\beta}\, Mu_{\alpha\beta}$$

Some approximations are reasonable,

$$El^{\alpha\beta} = \mathrm{Tr}\,\{ \not{q}\, \gamma^\alpha (1-\gamma_5)\, \not{q}'\, \gamma^\beta (1-\gamma_5) \}$$

$$Mu^{\alpha\beta} = \mathrm{Tr}\,\{ \not{p}'\, \gamma^\alpha (1-\gamma_5) \\ (\not{P} + M)\, \gamma^\beta (1-\gamma_5) \}$$

The term proportional to M is 0, because it is the trace of an odd number of gamma matrices; so

$$Mu^{\alpha\beta} = \mathrm{Tr}\,\{ p'\, \not{\gamma}^\alpha (1-\gamma_5) \\ \not{P}\, \gamma^\beta (1-\gamma_5) \}$$

After some algebra,

$$El^{\alpha\beta}\, Mu_{\alpha\beta} = 4\, G^2\, (P.q')\, (p'.q)$$

$\mu \ \bar{\nu}_e \ ; \ \nu_\mu \ e$

Combining the results,

$$d\Gamma = \frac{4G_F^{\,2}}{(2\pi)^5\, E}\ \delta^4(p' + q + q' - P)\ (P.q')\,(q.p')\ \frac{d^3p'\quad d^3q\quad d^3q'}{|\mathbf{p'}|\quad E_e\quad |\mathbf{q'}|}$$

Phase space integration

<u>Theorem</u>

$$J^{\rho\sigma} = \int \delta^4(q + p' + q' - P)\ \frac{d^3p'\, d^3q'}{4\,|\vec{p}'|\,|\vec{q}'|}\ p'^{\rho} q'^{\sigma}$$

$$= \frac{\pi}{24}\left(g^{\rho\sigma} Q^2 + 2Q^{\rho}Q^{\sigma}\right) \text{ when } Q = P - q$$

<u>Proof</u> By Lorentz invariance,

$$J^{\rho\sigma} = A(Q^2)\, g^{\rho\sigma} + B(Q^2)\, Q^{\rho}Q^{\sigma}$$

$$\Rightarrow J^{\rho}{}_{\rho} = 4A + Q^2 B = \int \delta^4(p' + q' - Q)\,\frac{d^3p'\, d^3q'}{4p'q'}\,(p'\cdot q')$$

$$p'\cdot q' = \tfrac{1}{2}Q^2 \text{ b/c } p'^2 = q'^2 = 0$$

$$\therefore 4A + Q^2 B = I\ Q^2/2$$

$$\Rightarrow Q_{\rho}Q_{\sigma}J^{\rho\sigma} = Q^2 A + (Q^2)^2 B = \int \delta^4(p' + q' - Q)\,\frac{d^3p'\, d^3q'}{4p'q'}\,(Q\cdot p')(Q\cdot q')$$

$$(Q\cdot p')(Q\cdot q') = (q'\cdot p')^2 = \tfrac{1}{4}(Q^2)^2$$

$$\therefore A + Q^2 B = I\ Q^2/4$$

Thus $A = \frac{Q^2}{12} I$ and $B = \frac{1}{6} I$

<u>Exercise</u>: $I = \int \delta^4(p' + q' - Q)\,\frac{d^3p'\, d^3q'}{4p'q'} = \pi/2$

Hint: let $Q = (Q^0, 0, 0, 0)$; WLOG.

Phase space integration, continued

Now we have, for the *total rate* Γ,

$$\Gamma = \frac{4G_F^{\,2}}{(2\pi)^5 E_\mu} \int \frac{d^3q}{E_e} \; P_\rho \, q_\sigma \; J^{\rho\sigma}$$

where E_μ = M (rest frame of the muon). The rest of the calculation is pretty easy

$$4 P_\rho q_\sigma J^{\rho\sigma} = \frac{\pi}{6} P\cdot q\,(P-q)^2 + \frac{\pi}{3} P\cdot(P-q)\,q\cdot(P-q)$$

$$= \frac{\pi}{6} M E_e \big(M^2 - 2ME_e + \underset{\substack{\uparrow \\ \text{neglect}}}{m^2}\big) + \frac{\pi}{3}\big(M^2 - ME_e\big)\big(ME_e - \underset{\substack{\uparrow \\ \text{neglect}}}{m^2}\big)$$

$$= \frac{\pi}{6}\left\{ M^3E - 2M^2E^2 + 2M^3E - 2M^2E^2 \right\}$$

$$= \frac{\pi}{6}\left\{ 3M^3E - 4M^2E^2 \right\}$$

$$\int \frac{d^3q}{E} \cdot 4(P_q \cdot J) = \int_{(m_e = 0)} \frac{4\pi E^2 dE}{E} \frac{\pi}{6}\left(3M^3E - 4M^2E^2\right)$$

$\boxed{\text{Integration range for } m_e = 0,\ 0 \le E \le M/2}$

$$= \frac{2\pi^2}{3}\left(3M^3 \tfrac{1}{3}\left(\tfrac{M}{2}\right)^3 - 4M^2 \tfrac{1}{4}\left(\tfrac{M}{2}\right)^4\right)$$

$$= \frac{1}{24}\pi^2 M^6$$

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The decay width Γ

$$\boxed{\Gamma = \frac{1}{\tau} = \frac{G_F^{\,2} M^5}{192\,\pi^3}}$$

Muon data

M = 105.7 MeV/c^2

τ = 2.197 × 10 $^{-6}$ sec

Famous trick question: Can a muon *cosmic ray secondary* reach the surface of the earth before it decays?

$\Rightarrow G_F$ = **SQRT** [192 π^3 M^{-5} τ^{-1})]

But what are the units!?

(Revert to ħ and c in real units.)

G_F = 1.16 × 10 $^{-5}$ GeV^2

But we only calculated the *total decay rate.*

There is a lot more to say about muon decay. These are observed:

- the electron energy spectrum (problem 28)
- the electron angular distribution, for polarized muons
- the electron helicity for unpolarized muons
- parity violation

Homework Problems

due Friday April 21

25. Maiani and Benhar problem 15.4.1
(the massive vector field)

26. Maiani and Benhar problem 15.4.2
(IVB propagator)

27. Maiani and Benhar problem 15.4.3
(Fermi 4-point coupling)

28. Mandl and Shaw problem 11.1
(muon decay electron energy)

29. Mandl and Shaw problem 11.2. Compare the result to the total decay width. Explain the difference. *(width of the W)*