MAIANI AND BENHAR, CHAPTER 15
APPLICATIONS : WEAK INTERACTIONS

OUTLINE of Chapter 15
15.1 ▸ Neutron decay
15.2 ▸ Muon decay
15.3 ▸ The current current theory
15.4 ▸ The intermediate vector boson theory
15.5 ▸ Problems for Chapter 15

Mandl and Shaw,
Chapter 11, "Weak Interactions"
11.6 Decay rates
11.7 The IVB theory

History of the science of weak interactions

- Nuclear beta decays
- Beta particles (electrons) have a continuous energy spectrum.
- Pauli proposed the existence of an unobserved particle.
- Fermi named it the neutrino.
- Fermi's 4-fermion coupling
- Discovery of the muon
- Parity violation
- Two neutrinos
- The electroweak gauge theory
- The standard model
- Neutrino oscillations, which imply neutrino masses

Section 15.1 ▸ Neutron decay

This is the most basic beta decay ;
decay of a free neutron:

$$n(P) \rightarrow p(p') + e^-(q) + \nu_e(q')$$

$$P^\mu = p'^\mu + q^\mu + q'^\mu$$

In the neutron rest frame, $P^\mu = (M, 0, 0, 0)$.

The mean lifetime is 881.5(15) sec.

<u>Masses</u>

n : 939.565 MeV/c^2
p : 938.272 MeV/c^2
e : 0.511 MeV/c^2
$\nu_{1,2,3}$: ? but very small; < 0.01 eV?

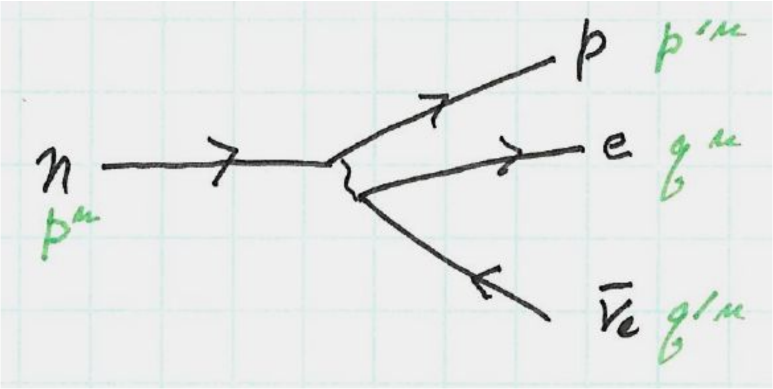
The Fermi theory (= low-energy limit of the Standard Model)

$$\mathcal{L}_{int}(x) = -G/\sqrt{2}$$

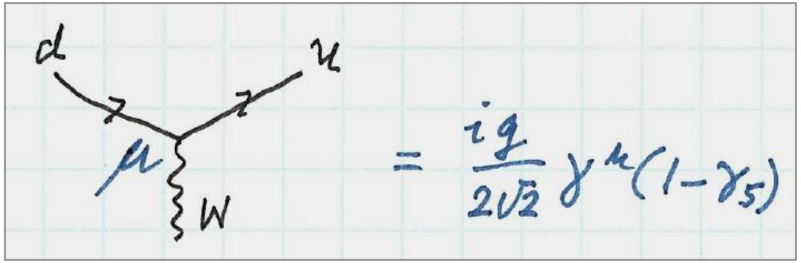
$$[\, \bar{p}(x)\, \gamma^\mu(1 + \lambda\gamma_5)\, n(x) \,]$$

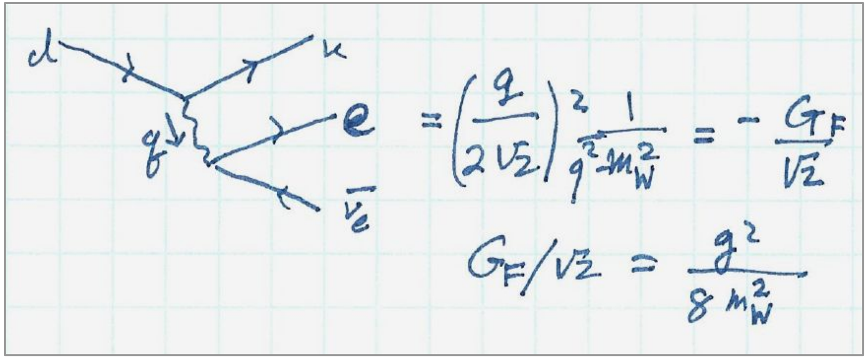
$$[\, \bar{e}(x)\, \gamma_\mu(1 - \gamma_5)\, \nu_e(x) \,]$$

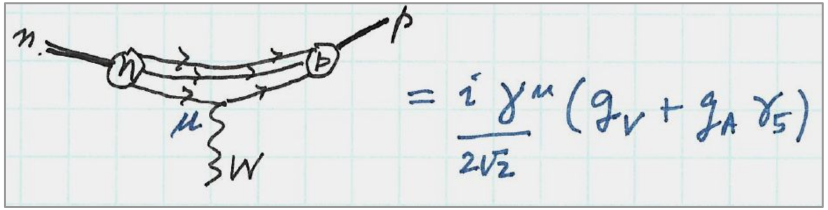
where $\lambda = g_A/g_V$.



Quarks and the weak interaction







The S-matrix for the Fermi theory in first order pert. theory

$$S_{FI} = i \int \langle p,e,\nu | \, \mathcal{L}_{int}(x) \, | n \rangle \, d^4x$$

$$= -i \, G/\sqrt{2} \, (2\pi)^4 \, \delta^4(p' + q + q' - P)$$

$$[\prod\sqrt{(m/EV)}] \, (\bar{u}(p') \, \gamma^\mu(1 + \lambda\gamma_5) \, u(P))$$

$$(\bar{u}(q) \, \gamma_\mu(1 - \gamma_5) \, v(q'))$$

⇒ The decay rate

$$d\Gamma = G^2/2 \, (2\pi)^4 \, \delta^4(p' + q + q' - P)$$

$$[\prod(m/E)][\prod d^3p/(2\pi)^3] \, H_{\mu\nu} \, L^{\mu\nu}$$

Sum over e and ν spins ⇒

$$L^{\mu\nu} = (2m_e)^{-1}(2m_\nu)^{-1} \, \mathrm{Tr}\{ (\not{q}+m_e)\gamma^\mu(1-\gamma_5) (\not{q}' - m_{\nu e})\gamma^\nu(1-\gamma_5) \}$$

Neglect $m_{\nu e}$; then the term $\propto m_e$ is zero.

$$L^{\mu\nu} = (2m_e)^{-1}(2m_{\nu e})^{-1}\, l^{\mu\nu}$$

where $l^{\mu\nu} = \mathrm{Tr}\{\, \not{q}\, \gamma^\mu (1-\gamma_5)\, \not{q}'\, \gamma^\nu (1-\gamma_5) \}$

$$= \mathrm{Tr}\{\, \not{q}\, \gamma^\mu\, \not{q}'\, \gamma^\nu\, \mathbf{2}\, (1-\gamma_5) \}$$

Similarly,

$$H^{\mu\nu} = (2M_p)^{-1}(2M_n)^{-1}\, h^{\mu\nu}$$

where $h^{\mu\nu} = \mathrm{Tr}\{\, (\not{p}' + M_p)\ \gamma^\mu\ (1+\lambda\gamma_5)$

$$(\not{P} + M_n)\ \gamma^\nu\ (1+\lambda\gamma_5) \}$$

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In[303]:= Ltensor = 2 * Tr[GS[q].GA[μ].GS[q2].GA[ν].(1 - DiracMatrix[5])]
           Htensor = Tr[(GS[p2] + Mp).GA[μ].(1 + λ * DiracMatrix[5]).
              (GS[P] + Mn).GA[ν].(1 + λ * DiracMatrix[5])]
           Asq = Calc[Ltensor.Htensor]
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Out[303]= $8\left(\frac{1}{2}\mathrm{me}^2 g^{\mu\nu} - \frac{s\, g^{\mu\nu}}{2} + q^\nu\, \mathrm{q2}^\mu + q^\mu\, \mathrm{q2}^\nu - i\, \epsilon^{\mu\nu q\, \mathrm{q2}}\right)$

Out[304]= $4\left(-\frac{1}{2}\lambda^2 \mathrm{Mn}^2 g^{\mu\nu} - \frac{1}{2}\mathrm{Mn}^2 g^{\mu\nu} - \lambda^2 \mathrm{Mn\, Mp}\, g^{\mu\nu} + \mathrm{Mn\, Mp}\, g^{\mu\nu} - \frac{1}{2}\lambda^2 \mathrm{Mp}^2 g^{\mu\nu} - \frac{1}{2}\mathrm{Mp}^2 g^{\mu\nu} + \frac{1}{2}\lambda^2 s\, g^{\mu\nu} + \frac{s\, g^{\mu\nu}}{2} + \lambda^2 P^\nu \mathrm{p2}^\mu + \lambda^2 P^\mu \mathrm{p2}^\nu + P^\nu \mathrm{p2}^\mu + P^\mu \mathrm{p2}^\nu - 2\, i\, \lambda\, \epsilon^{\mu\nu P\, \mathrm{p2}}\right)$

Out[305]= $-16\lambda^2 \mathrm{me}^2 \mathrm{Mn}^2 + 32\lambda\, \mathrm{me}^2 \mathrm{Mn}^2 - 16\, \mathrm{me}^2 \mathrm{Mn}^2 - 32\lambda^2 \mathrm{me}^2 \mathrm{Mn\, Mp} + 32\, \mathrm{me}^2 \mathrm{Mn\, Mp} - 16\lambda^2 \mathrm{me}^2 \mathrm{Mp}^2 - 32\lambda\, \mathrm{me}^2 \mathrm{Mp}^2 - 16\, \mathrm{me}^2 \mathrm{Mp}^2 + 16\lambda^2 \mathrm{me}^2 t + 32\lambda\, \mathrm{me}^2 t + 16\, \mathrm{me}^2 t + 16\lambda^2 \mathrm{me}^2 u - 32\lambda\, \mathrm{me}^2 u + 16\, \mathrm{me}^2 u - 32\lambda^2 \mathrm{Mn}^2 \mathrm{Mp}^2 - 32\, \mathrm{Mn}^2 \mathrm{Mp}^2 + 16\lambda^2 \mathrm{Mn}^2 t + 32\lambda\, \mathrm{Mn}^2 t + 16\, \mathrm{Mn}^2 t + 16\lambda^2 \mathrm{Mn}^2 u - 32\lambda\, \mathrm{Mn}^2 u + 16\, \mathrm{Mn}^2 u + 32\lambda^2 \mathrm{Mn\, Mp}\, s - 32\, \mathrm{Mn\, Mp}\, s + 16\lambda^2 \mathrm{Mp}^2 t + 32\lambda\, \mathrm{Mp}^2 t + 16\, \mathrm{Mp}^2 t + 16\lambda^2 \mathrm{Mp}^2 u - 32\lambda\, \mathrm{Mp}^2 u + 16\, \mathrm{Mp}^2 u - 16\lambda^2 t^2 - 32\lambda\, t^2 - 16\, t^2 - 16\lambda^2 u^2 + 32\lambda\, u^2 - 16\, u^2$

We can calculate $l^{\mu\nu}\, h_{\mu\nu}$ with FeynCalc.

The result is ugly.

[308]= $-128\, \mathrm{Ee}^2 \lambda^2 \mathrm{Mn}^2 - 128\, \mathrm{Ee}^2 \mathrm{Mn}^2 - 128\, \mathrm{Ee\, Ep}\, \lambda^2 \mathrm{Mn}^2 + 256\, \mathrm{Ee\, Ep}\, \lambda\, \mathrm{Mn}^2 - 128\, \mathrm{Ee\, Ep\, Mn}^2 + 64\, \mathrm{Ee}\, \lambda^2 \mathrm{me}^2 \mathrm{Mn} + 64\, \mathrm{Ee\, me}^2 \mathrm{Mn} + 128\, \mathrm{Ee}\, \lambda^2 \mathrm{Mn}^3 - 128\, \mathrm{Ee}\, \lambda\, \mathrm{Mn}^3 + 128\, \mathrm{Ee\, Mn}^3 - 128\, \mathrm{Ee}\, \lambda\, \mathrm{Mn\, Mp}^2 - 64\, \mathrm{Ep}^2 \lambda^2 \mathrm{Mn}^2 + 128\, \mathrm{Ep}^2 \lambda\, \mathrm{Mn}^2 - 64\, \mathrm{Ep}^2 \mathrm{Mn}^2 + 32\, \mathrm{Ep}\, \lambda^2 \mathrm{me}^2 \mathrm{Mn} - 64\, \mathrm{Ep}\, \lambda\, \mathrm{me}^2 \mathrm{Mn} + 32\, \mathrm{Ep\, me}^2 \mathrm{Mn} + 96\, \mathrm{Ep}\, \lambda^2 \mathrm{Mn}^3 - 192\, \mathrm{Ep}\, \lambda\, \mathrm{Mn}^3 + 96\, \mathrm{Ep\, Mn}^3 - 64\, \mathrm{Ep}\, \lambda^2 \mathrm{Mn}^2 \mathrm{Mp} + 64\, \mathrm{Ep\, Mn}^2 \mathrm{Mp} + 32\, \mathrm{Ep}\, \lambda^2 \mathrm{Mn\, Mp}^2 - 64\, \mathrm{Ep}\, \lambda\, \mathrm{Mn\, Mp}^2 + 32\, \mathrm{Ep\, Mn\, Mp}^2 - 32\lambda^2 \mathrm{me}^2 \mathrm{Mn}^2 + 64\lambda\, \mathrm{me}^2 \mathrm{Mn}^2 - 32\, \mathrm{me}^2 \mathrm{Mn}^2 - 32\lambda^2 \mathrm{me}^2 \mathrm{Mn\, Mp} + 32\, \mathrm{me}^2 \mathrm{Mn\, Mp} - 32\lambda^2 \mathrm{Mn}^4 + 64\lambda\, \mathrm{Mn}^4 - 32\, \mathrm{Mn}^4 + 32\lambda^2 \mathrm{Mn}^3 \mathrm{Mp} - 32\, \mathrm{Mn}^3 \mathrm{Mp} - 32\lambda^2 \mathrm{Mn}^2 \mathrm{Mp}^2 + 64\lambda\, \mathrm{Mn}^2 \mathrm{Mp}^2 - 32\, \mathrm{Mn}^2 \mathrm{Mp}^2 + 32\lambda^2 \mathrm{Mn\, Mp}^3 - 32\, \mathrm{Mn\, Mp}^3$

However, some approximations are justified. *Note that*

M_n = 939.565 MeV/c^2 ~ 1000 MeV

M_p = 938.272 MeV/c^2 ~ 1000 MeV

$\delta M = M_n - M_p$ = 1.293 MeV/c^2 ~ 1 MeV

m_e = 0.511 MeV/c^2 ~ 1 MeV

$|\mathbf{p}'|$, $|\mathbf{q}|$, $|\mathbf{q}'|$ ~ 1 MeV

∴ *We can neglect* m_e^2 , δM^2 , $|\mathbf{p}'|^2$, $|\mathbf{q}|^2$, $|\mathbf{q}'|^2$, when they are combined with M_n or M_p .

For example,

$$E_p = \sqrt{(M_p^2 + q^2)} \approx M_p + q^2/2M_p$$

$$\sim 1000 + 1/2000 = 1000.0005 \approx M_p .$$

Another example,

Exact: $M_n = E_p + E_e + |q'|$

Approximate:

$$\delta M = \sqrt{(m_e^2 + q^2)} + |q'|$$

in which all terms are order 1 MeV.

Here is the point:

$l^{\mu\nu}\, h_{\mu\nu}$ has units of Mass^4 .

We can write $l \bullet h$

$$= c_4 M_n^4 + c_3 M_n^3 + c_2 M_n^2 + c_1 M_n + c_0 ;$$

$c_4 = 0$ and $c_3 = 0$;

c_2 we want to keep;

c_1 and c_0 we can neglect.

We obtain this approximate result:

$$l^{\mu\nu} h_{\mu\nu} =$$

$$32M_n^2 \times \{ \delta M^2 (\lambda^2 - 1)$$

$$+ 4\, \delta M\, E_e\, (\lambda^2+1) - 4\, q^2\, (\lambda^2+1)$$

$$- (5\, \lambda^2 + 3)\, m_e^2 \}$$

(may not be quite right;
didn't have time to check it;
but this is the right idea)

The phase space integral

This is another 3-body final state (like muon decay) but with different relative magnitudes.

For neutron decay we have

$$d\Gamma = \frac{G^2/2}{(2\pi)^5\, 16M_n} \int \frac{d^3q\, d^3q'}{E_p\, E_e\, |\mathbf{q'}|}$$

$$\delta^4 (E_p + E_e + |\mathbf{q'}| - M_n)\ [l.h]$$

where $E_p = \sqrt{M_p^2 + (\mathbf{q}+\mathbf{q'})^2} \approx \mathbf{M_p}$

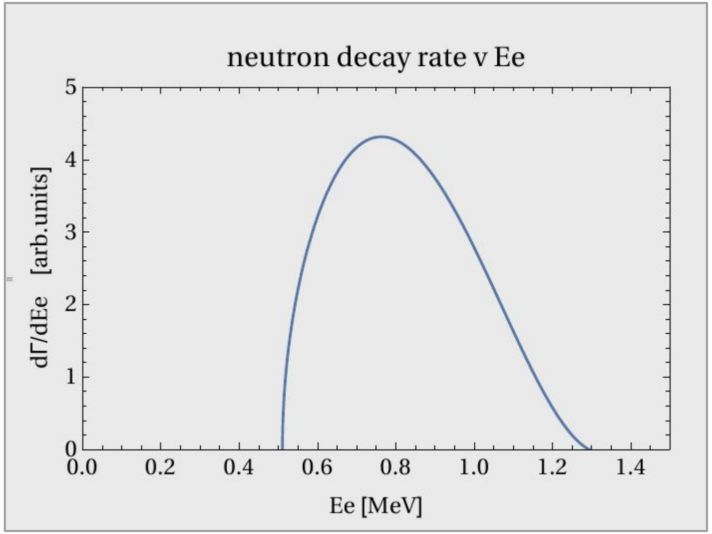
With this approximation, the calculation simplifies.

$$|\mathbf{q'}| = M_n - M_p - E_e$$

The electron energy spectrum

$$d\Gamma = \frac{G^2/2\ (4\pi)^2}{(2\pi)^5\ 16 M_n M_p} \quad (\ q^2\, dq\ /E_e\ )\ (\ M_n - M_p - E_e\ ) \quad [l.h]$$

$$\frac{d\Gamma}{dE_e} = \textbf{const.} \quad (E_e - m_e{}^2)^{1/2}\,(\ M_n - M_p - E_e\ ) \quad [l.h]$$



The total decay rate

Maiani and Benhar

$$\Gamma = \frac{G_F^2 (\delta M)^5}{60 \pi^3} (1 + 3\lambda^2) \, I$$

$G_F = 1.16 \times 10^{-5}$ $(\mathrm{GeV})^{-2}$ and $\lambda = -1.27$ $(= g_A/g_V)$

$\delta M = M_p - M_n = 1.293$ MeV

$$I = 30 \int_x^1 dt \; t (1-t)^2 \sqrt{t^2 - x^2} \quad \text{where } x = \frac{m_e}{\delta M}$$

(electron energy spectrum, integrated)

$t = E_e/\delta M$ goes from $\frac{m_e}{\delta M}$ to $\frac{\delta M}{\delta M}$

$I = 0{,}472$

$$\tau = \frac{\hbar}{\Gamma} = 912 \text{ seconds}$$

[P.D.G. value is 881.5 (15) sec]

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***Colloquium*: The neutron lifetime**

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TABLE I. A summary of neutron lifetime measurements. When applicable, statistical and systematic errors have been added in quadrature. Asterisks indicate the 8 experiments included in our global averages.

Reference	Neutron lifetime (s)	Uncertainty (s)
Beam Experiments		
Robson, 1951	1110	220
Spivak *et al.*, 1956	1040	130
D'Angelo, 1959	1100	160
Sosnovsky *et al.*, 1959	1013	26
Christensen *et al.*, 1972	918	14
Last *et al.*, 1988	876	21
Spivak, 1988*	891	9
Kossakowski *et al.*, 1989	878	30
Byrne *et al.*, 1996*	889.2	4.8
Nico *et al.*, 2005*	886.3	3.4
Bottle Experiments		
Kosvintsev *et al.*, 1980	875	95
Kosvintsev, Morozov, and Terekhov, 1986	903	13
Morozov, 1989	893	20
Mampe *et al.*, 1989*	887.6	3.0
Alfimenkov *et al.*, 1992	888.4	3.3
Mampe *et al.*, 1993*	882.6	2.7
Arzumanov *et al.*, 2000	885.4	0.98
Serebrov *et al.*, 2005*	878.5	0.76
Pichlmaier *et al.*, 2010*	880.7	1.8
Magnetic Trap Experiments		
Paul *et al.*, 1989*	877	10
Ezhov *et al.*, 2009	878.2	1.9

Homework Problems

due Friday April 21

25. Maiani and Benhar problem 15.4.1

26. Maiani and Benhar problem 15.4.2

27. Maiani and Benhar problem 15.4.3

28. Mandl and Shaw problem 11.1

29. Mandl and Shaw problem 11.2

30. Use equation (15.24) in Maiani and Benhar. Plot two graphs: the differential decay rate as a function of (i) electron energy and (ii) electron angle, assuming the muons are polarized with spin $\mathbf{S}_z = +\frac{1}{2}$.