MAIANI AND BENHAR, CHAPTER 15
APPLICATIONS : WEAK INTERACTIONS

OUTLINE of Chapter 15

history

Mandl and Shaw,
Chapter 11, "Weak Interactions"

NEUTRINO SCATTERING

- Create a neutrino (or antineutrino) beam. *For high energy neutrinos, first produce hadrons; the pions and kaons will decay to muons and neutrinos.*
- Create a target. *Because the cross section is very small, the target must have high density, e.g., iron or other heavy nuclei.*
- Create a detector. *Because the cross section is very small, the target and the detector are one and the same.*

CHARGED CURRENT (CC) INTERACTIONS

Typical *quark interactions* are

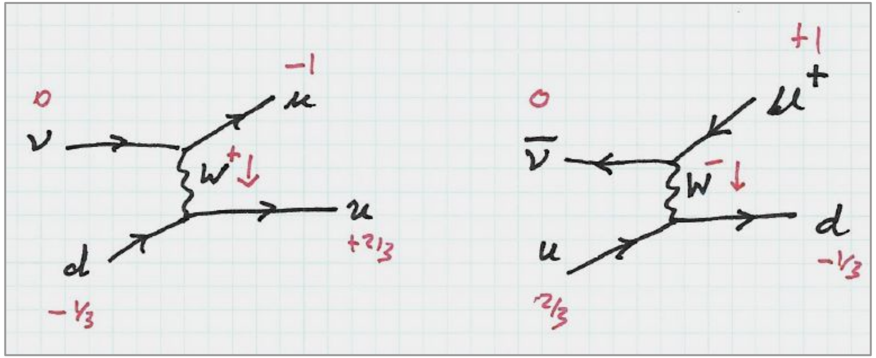
$$\nu_\mu + d \longrightarrow \mu^- + u$$

(check conservation laws!)

and

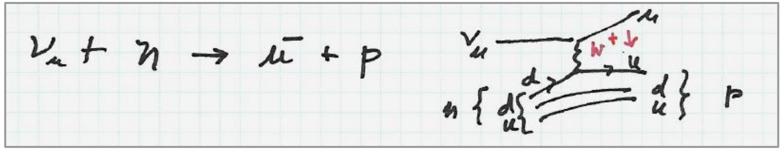
$$\bar{\nu}_\mu + u \longrightarrow \mu^+ + d$$

(check conservation laws!)

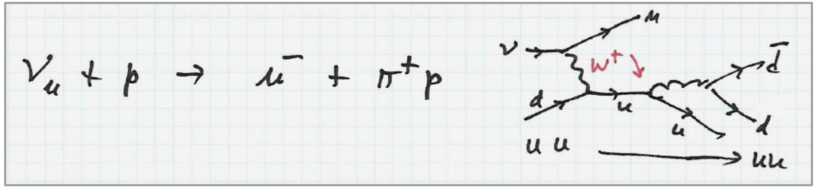


There are different kinds of interactions with hadrons (all hadrons are color singlets): e.g.,

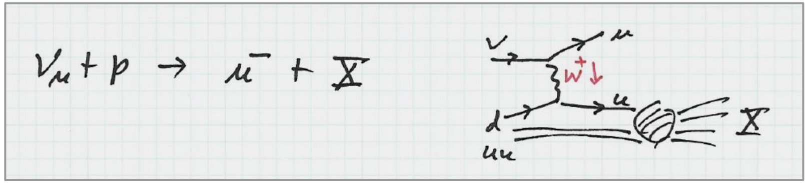
- elastic or quasi elastic scattering;



- exclusive hadron production;



- deep-inelastic scattering;



A compendium of data on ν_μ and $\bar{\nu}_\mu$ interactions

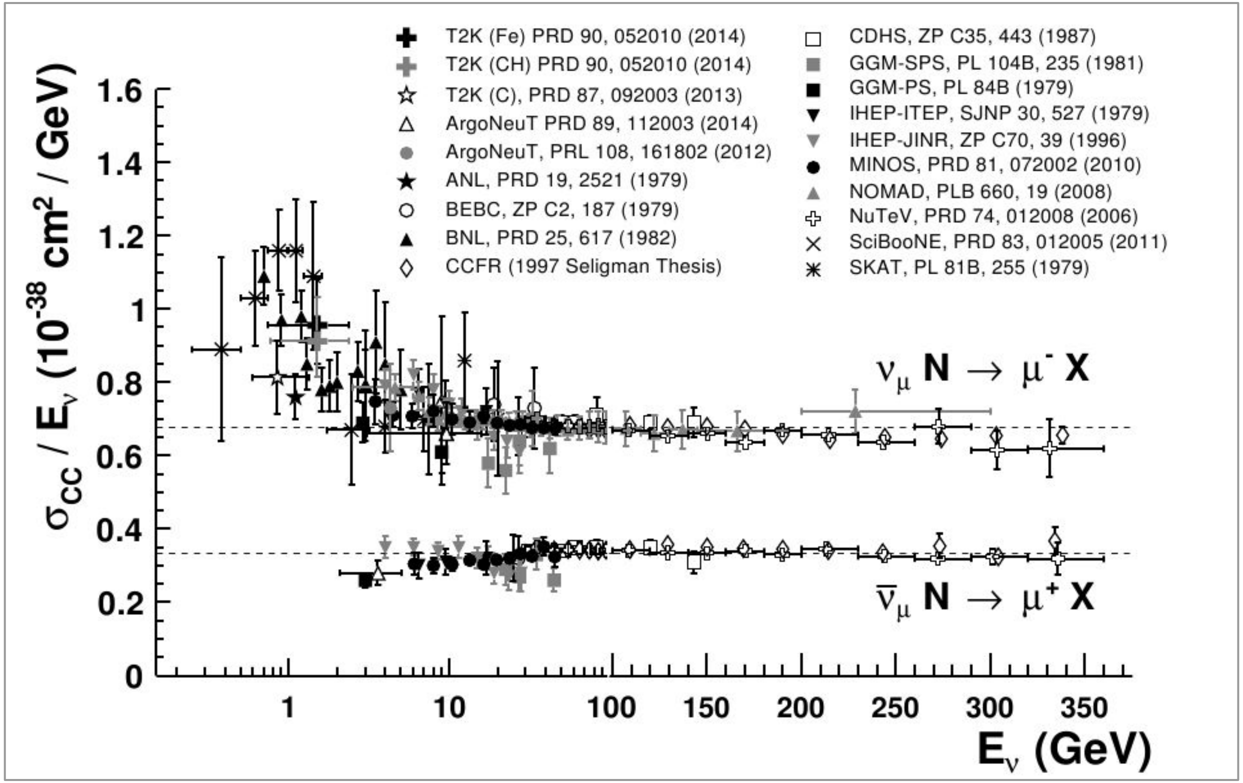
(Particle Data Group)

We observe two things:

$\sigma \propto E_\nu$;

$\sigma(\nu) / \sigma(\bar{\nu}) \approx 2$.

Can we explain why?



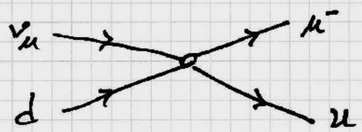
Neutrino and antineutrino interactions with quarks

Case 1: $\nu_\mu + d \rightarrow \mu^- + u$

$$\sim \left(\frac{g}{\sqrt{2}}\right)^2 \frac{1}{q^2 - M_W^2} \gamma^\alpha \frac{(1-\gamma_5)}{2} \gamma^\beta \frac{(1-\gamma_5)}{2}$$

$$= \frac{-g^2}{8 M_W^2} \gamma^\alpha (1-\gamma_5) \gamma^\beta (1-\gamma_5)$$

Make the 4-fermion coupling approximation



$$\sim -\frac{G_F}{\sqrt{2}} \gamma^\alpha (1-\gamma_5) \gamma^\beta (1-\gamma_5)$$

$$\frac{G_F}{\sqrt{2}} = g^2 / 8 M_W^2$$

Also make the relativistic approximation, E >> m ; i.e., neglect the fermion masses.

<u>Theorem</u>. Because of the V – A coupling, there is only one nonzero helicity combination in the relativistic approximation.

<u>Proof.</u> The helicity eigenstates of the Dirac equation (using Dirac-Pauli gamma matrices)

$$u_{(+)}(\vec{p}) = N \begin{pmatrix} \cos\theta/2 \\ e^{i\phi} \sin\theta/2 \\ \kappa \cos\theta/2 \\ \kappa\, e^{i\phi} \sin\theta/2 \end{pmatrix} \qquad u_{(-)}(\vec{p}) = N \begin{pmatrix} -\sin\theta/2 \\ e^{i\phi} \cos\theta/2 \\ \kappa \sin\theta/2 \\ -\kappa\, e^{i\phi} \cos\theta/2 \end{pmatrix}$$

$$N = (1-\kappa^2)^{-1/2} \quad \text{and} \quad \kappa = \frac{p}{E+m}$$

κ = SQRT [(E − m) / (E + m)]

Exercise.

Verify, for $u_{(+)}(p)$ and $u_{(-)}(p)$, that

$$(\boldsymbol{\alpha}.\mathbf{p} + \beta m)\, u(p) = E\, u(p) \,;$$

$$E = \mathbf{sqrt}(m^2+\boldsymbol{p}^2)$$

and

$$(\hat{\mathbf{p}}.\boldsymbol{\Sigma})\, u_{(\pm)}(\mathbf{p}) = \pm\, u_{(\pm)}(\mathbf{p})$$

helicity

where $\boldsymbol{\Sigma} = \{ \{ \boldsymbol{\sigma} , 0 \} , \{ 0 , \boldsymbol{\sigma} \} \}$

spin

Now consider γ_5 .
Using Dirac-Pauli gamma matrices,
$\gamma_5 = \{\{ 0 , 1 \} , \{ 1 , 0\}\}$

$$\gamma_5\, u_{(-)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} N \begin{bmatrix} -\sin\theta/2 \\ e^{i\phi}\cos\theta/2 \\ K\sin\theta/2 \\ -K e^{i\phi}\cos\theta/2 \end{bmatrix}$$

$$= N \begin{bmatrix} K\sin\theta/2 \\ -K e^{i\phi}\cos\theta/2 \\ -\sin\theta/2 \\ e^{i\phi}\cos\theta/2 \end{bmatrix}$$

Thus $\gamma_5\, u_{(-)} = -\, u_{(-)}$ if $K=1$

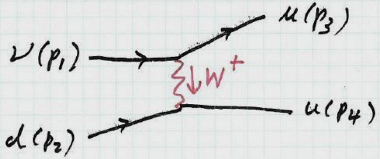
also $\gamma_5\, u_{(+)} = +\, u_{(+)}$ if $K=1$

$$K = \frac{p}{E+m} = \sqrt{\frac{E-m}{E+m}} = 1 \quad \text{if } m=0$$

For massless fermions, helicity eigenstates are γ_5 eigenstates ; eigenvalue = helicity.

The cross section for ν_μ-d scattering

Again we'll average and sum over helicities; *but there is only one nonzero helicity combination (LLLL)* .



$p_1^\mu = (E, 0\,0\,E)$

$p_2^\mu = (E, 0\,0\,-E)$

$p_3^\mu = (E, E\sin\theta, 0, E\cos\theta)$

$p_4^\mu = (E, -E\sin\theta, 0, -E\cos\theta)$

neglecting masses

$$\mathcal{M} = \frac{G}{\sqrt{2}}\,\bar{u}(p_3)\,\gamma^\mu(1-\gamma_5)\,u(p_1)\;\bar{u}(p_4)\,\gamma_\mu(1-\gamma_5)\,u(p_2)$$

High energy ($m << E$) $\Rightarrow$ only $\mathcal{M}_{LLL}$ is nonzero.

$$\mathcal{M}_{LLLL} = \frac{G}{\sqrt{2}}\,2\cdot 2\cdot\ \bar{u}_{(-)}(p_3)\,\gamma^\mu\,u_{(-)}(p_1)\;\bar{u}_{(-)}(p_4)\,\gamma_\mu\,u_{(-)}(p_2)$$

The spinor with helicity $= -1$ is

$u_{(-)}(\vec{p}) = N\begin{bmatrix} A \\ -kA \end{bmatrix}$ where $A = \begin{pmatrix} -\sin\theta/2 \\ e^{i\phi}\cos\theta/2 \end{pmatrix}$.

$u_{(-)}(p_1) = N_1\begin{bmatrix} \binom{0}{1} \\ \binom{0}{1} \end{bmatrix}$ $(\theta=0)$ and $u_{(-)}(p_3) = N_3\begin{bmatrix} A \\ -kA \end{bmatrix}$ with $\phi = 0$.

Exercise.

$$\begin{array}{lcccc} & \mu=0 & \mu=1 & \mu=2 & \mu=3 \\ \bar{u}_{(-)}(p_3)\,\gamma^\mu\,u_{(-)}(p_1) = 2N_3N_1\{ & \cos\frac{\theta}{2}, & \sin\frac{\theta}{2}, & -i\sin\frac{\theta}{2}, & \cos\frac{\theta}{2}\} \end{array}$$

Also

$$\bar{u}_{(-)}(p_4)\,\gamma^\mu\,u_{(-)}[p_2] = 2N_4N_2\left\{\cos\frac{\theta}{2}, -\sin\frac{\theta}{2}, -i\sin\frac{\theta}{2}, -\cos\frac{\theta}{2}\right\}$$

$$\mathcal{M}_{LLLL} = \frac{G}{\sqrt{2}}\cdot 2\cdot 2\cdot 2N_3N_1\cdot 2N_4N_2\left(\cos^2+\sin^2+\sin^2+\cos^2\right)$$

$$\mathcal{M}_{LLLL} = \frac{G}{\sqrt{2}}\,32\,N_1N_2N_3N_4$$

The matrix element is independent of θ.

"Spherical symmetry"

$S_z = 0$.

$$\overset{s_1 \leftarrow}{\longrightarrow} \quad \overset{\rightarrow s_2}{\longleftarrow}$$

$\nu(p_1)$ L (helicity $= -1$) $\qquad d(p_2)$ L

$$\mathcal{M}_{LLLL} = \sqrt{2}\, G \prod_{i=1}^{4} (2N_i) \quad \text{where} \quad N_i = \sqrt{\frac{E+m_i}{2m_i}} \approx \sqrt{\frac{E}{2m_i}}$$

Unpolarized scattering:

$$\overline{|\mathcal{M}|^2} = \frac{1}{2} |\mathcal{M}_{LLLL}|^2 = G_F^2 \frac{E^4 \times 16^2}{[\prod_i (2m_i)]}$$

The center of mass cross section is

$$\frac{d\sigma}{d\Omega_3} = \frac{[\prod_i (2m_i)]}{64\pi^2 s} \overline{|\mathcal{M}|^2} = \frac{4G_F^2 E^4}{\pi^2 s};$$

and $E = \sqrt{s}/2 \Rightarrow \dfrac{d\sigma}{d\Omega_3} = \dfrac{G_F^2 s}{4\pi^2}$

$$\therefore \boxed{\sigma_{total}(\nu_\mu d \to \mu^- u) = \frac{G_F^2 s}{\pi}}$$

Similarly, $\left(\dfrac{d\sigma}{d\Omega_3}\right)_{\bar{\nu}_\mu u \to \mu^+ d} = \dfrac{G_F^2 s}{16\pi^2}(1+\cos\theta)^2$

$$\therefore \boxed{\sigma_{total}(\bar{\nu}_\mu u \to \mu^+ d) = \frac{G_F^2 s}{3\pi}}$$

(violation of unitarity as $s \to \infty$)

Results

We found $\sigma(\nu+d)$ and $\sigma(\mathbf{a}\nu+u)$ $\propto$ s ; and $\sigma(\nu+d)/\sigma(\mathbf{a}\nu+u) = 3$.

How do these results compare to neutrinos scattering from nucleons?

- In the lab frame (i.e., the nucleon rest frame) $s = (p_1 + P)^2$
$$= m_p^2 + 2\, m_\mathbf{p}\, E_\mathbf{\nu} \approx 2\, m_\mathbf{p}\, E_\mathbf{\nu} \;.$$
Thus, $\sigma \propto E_\nu$. ✔
- Fe-56 has Z = 26 and N = 30; i.e., 82 u quarks and 86 d quarks; we might expect $\sigma(\nu+Fe)/\sigma(a\nu+Fe) > 3$.

(expt'l ratio ~ 2)

better theory = the parton model

Homework Problems

due Friday April 21

25. Maiani and Benhar problem 15.4.1

26. Maiani and Benhar problem 15.4.2

27. Maiani and Benhar problem 15.4.3

28. Mandl and Shaw problem 11.1

29. Mandl and Shaw problem 11.2

30. Use equation (15.24) in Maiani and Benhar. Plot two graphs: the differential decay rate as a function of (i) electron energy and (ii) electron angle, assuming the muons are polarized with spin $S_z = +\frac{1}{2}$.