Evaluations

```
In[91]:= Remove["Global`*"];
```

```
In[92]:=
(* ignore *)
eq24[expr_] = Style[expr, FontFamily → "Kalam", 32];
tx24[expr_] = Style[expr, FontFamily → "Kalam", 32];
line0 = {tx24["charge and current"], tx24["symbol"], tx24["equation"]};
line1 = {tx24["charge density"], eq24["ρ"], eq24["ρ δV = δQ (inside δV)"]};
line2 = {tx24["current density"], eq24["J⃗"], eq24["J⃗ · δA⃗ δt = δQ (thru δA⃗)"]};
line3 = {tx24["continuity equation"], " ", eq24["∇·J⃗ = - ∂ρ /∂t"]};
a1 = Grid[{line0, line1, line2, line3}, Frame → All];
```

```
In[99]:=
(* ignore *)
n1 = tx24["Gauss"] ; FGL = eq24["∇ · E⃗ = 4 π ρ"] ; FGLB = eq24["∇ · B⃗  = 0"];
n2 = tx24["Ampere"]; FAL = eq24["∇×B⃗-(1/c)(∂E⃗/∂t)=(4π/c)J⃗" ];
n3 = tx24["Faraday"]; FFL = eq24["∇×E⃗+(1/c)(∂B⃗/∂t) = 0 "];
a2 = Grid[{{tx24["Maxwell's equations"], tx24["fundamental fields"], "#"},
   {n1, FGL, "(1)"}, {n2, FAL, "(2)"}, {n3, FFL, "(3)"}, {n1, FGLB, "(4)"}}, Frame → All];
```

```
In[103]:=
(* ignore *)
eq1 = eq24["∇ · D⃗  = 4 π ρ_free"];
eq4 = eq24["∇ · B⃗  = 0"];
eq2 = eq24["∇ × H⃗  - (1/c) (∂D⃗/∂t)  = (4π/c) J⃗_free "];
eq3 = eq24["∇ × E⃗  + (1/c) (∂B⃗/∂t)  = 0 "];
tt = Join[{{tx24["Maxwell Equations"], tx24["phenomelogical fields"], "#"}},
  {{n1, eq1, "(1)"}}, {{n2, eq2, "(2)"}}, {{n3, eq3, "(3)"}}, {{n1, eq4, "(4)"}}];
a3 = Grid[tt, Frame → All];
tc = Join[{{tx24["Constitutive equations"], tx24["Comment"]}},
  {{eq24["D⃗ = ϵ E⃗"], tx24["but it's not that simple for time-dependent fields!"]}},
  {{eq24["H⃗ = B⃗ /μ"], tx24[" '' "]}},
{{eq24["J⃗_free = σ E⃗"], tx24[" '' "]}}];
a4 = Grid[tc, Frame -> All];
```

In[111]:=

```
(* ignore *)
n1 = tx24["Gauss"]; F0GL = eq24["∇ · e⃗ = 4 π ρ"]; F0GLB = eq24["∇ · b⃗ = 0"];
n2 = tx24["Ampere"]; F0AL = eq24["∇×b⃗ - (1/c) ∂e⃗/∂t = (4 π/c) J⃗ "];
n3 = tx24["Faraday"]; F0FL = eq24["∇×e⃗ + (1/c) ∂b⃗/∂t = 0 "];
a5 = Grid[{{tx24["Maxwell's equations"], tx24["fundamental fields"], "#"}, {n1, F0GL, "(1)"},
    {n2, F0AL, "(2)"}, {n3, F0FL, "(3)"}, {n1, F0GLB, "(4)"}}, Frame → All];
```

In[115]:=

```
(* 8 ignore *)
defP = {
   {tx24["Polarization = P⃗( x⃗ , t) = electric dipole moment density"]},
{tx24[Style["Bound charge density = ρ_b = - ∇ · P⃗   " , Red , Bold ]]},
{tx24["Charge density = ρ_tot = ρ_free + ρ_b = ρ_free - ∇ · P⃗ " ]},
{tx24["Displacement field = D⃗ = e⃗ + 4 π P⃗ " ]},
{tx24["Susceptibility χ is defined by P⃗ = χ e⃗ " ]},
{tx24["Permittivity ϵ is defined by D⃗ = ϵ e⃗ = (1 + 4π χ) e⃗ " ]}};
a7 = Grid[defP, Alignment → Left, Frame → All];
```

In[117]:=

```
(* ignore *)
l1 = StringJoin[ "Magnetization = " , "M⃗(x⃗,t)" , " = magnetic dipole moment density"] ;
l2 = Style[StringJoin["Bound current density = ", "J⃗_b", " = c ∇ × M⃗ + ∂P⃗ /∂t"], Red, Bold] ;
l3 = "Current density = J⃗_tot = J⃗_Free + J⃗_bound  \n = J⃗_Free + c ∇ × M⃗ + ∂P⃗/∂t ";
l4 = "Magnetic field H⃗ = b⃗ - 4 π M⃗ ";
l5 = "Susceptibility  χ_M  is defined by M⃗ = χ_M b⃗ ";
l6 = "Permeability μ is defined by H⃗ = b⃗ /μ  ";
defs = {{tx24[l1]}, {tx24[l2]}, {tx24[l3]}, {tx24[l4]}, {tx24[l5]}, {tx24[l6]}};
a8 = Grid[defs, Alignment → Left, Frame → All];
```

lecture 2 — fri aug 31

1–

PHY 842 - Lecture 2

Outline of today’s lecture

1/ The microscopic equations of classical electromagnetism

2/ The macroscopic equations; linear media

−1

“The Equations of Classical Electrodynamics”

■ The Microscopic equations of electromagnetism

First, these are the fundamental equations, appropriate for fields with only free charges and currents in empty space; i.e., no macroscopic materials are present.

● Charge and current

In[125]:= a1

Out[125]=

charge and current	symbol	equation
charge density	ρ	$\rho\,\delta V = \delta Q$ (inside δV)
current density	$\vec{J}$	$\vec{J}\cdot\delta\vec{A}\,\delta t = \delta Q$ (thru $\delta\vec{A}$)
continuity equation		$\nabla\cdot\vec{J} = -\,\partial\rho/\partial t$

The continuity equation states that electric charge is *locally* conserved.
(This is related to gauge invariance).

Do you know why?

2–

● Fields and Forces

$$\vec{F}(\vec{x}, \vec{v}, t) = q\,\vec{E}(\vec{x}, t)$$
$$+\ q\ (\vec{v}/c) \times \vec{B}(\vec{x}, t)$$

Lorentz force in Gaussian units!

−2

● Maxwell's equations - the microscopic equations

In[126]:= a2

Out[126]=

Maxwell's equations	fundamental fields	⌗
Gauss	$\nabla \cdot \vec{E} = 4\pi\rho$	(1)
Ampere	$\nabla \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{J}$	(2)
Faraday	$\nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$	(3)
Gauss	$\nabla \cdot \vec{B} = 0$	(4)

3−

■ The Macroscopic Equations

Electromagnetism with *Linear Continuous Media* (Dielectrics and Magnetic Materials)

These are the "phenomenological" equations, for a system with

- fields { $\vec{E}$, $\vec{B}$, $\vec{D}$, $\vec{H}$ }
- materials (parameters ϵ , μ , σ)
- free charge and free current (ρ_{free} and $\vec{J}_{\text{free}}$)

-3

● Maxwell's equations – the macroscopic equations

In[127]:= a3

Out[127]=

Maxwell Equations	phenomelogical fields	⌗
Gauss	$\nabla \cdot \vec{D} = 4\pi\,\rho_{free}$	(1)
Ampere	$\nabla \times \vec{H} - \frac{1}{c}\frac{\partial \vec{D}}{\partial t} = \frac{4\pi}{c}\vec{J}_{free}$	(2)
Faraday	$\nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$	(3)
Gauss	$\nabla \cdot \vec{B} = 0$	(4)

In[128]:= a4

Out[128]=

Constitutive equations	Comment
$\vec{D} = \epsilon\,\vec{E}$	but it's not that simple for time–dependent fields!
$\vec{H} = \vec{B}/\mu$	''
$\vec{J}_{free} = \sigma\,\vec{E}$	''

4−

Derivations: Microscopic equations ⟹ Macroscopic equations

Start with the microscopic equations;
call the fundamental fields $\vec{e}(\vec{x}, t)$ and $\vec{b}(\vec{x}, t)$

In[129]:= a5

Out[129]=

Maxwell's equations	fundamental fields	⌗
Gauss	$\nabla \cdot \vec{e} = 4\,\pi\,\rho$	(1)
Ampere	$\nabla\times\vec{b} - \frac{1}{c}\frac{\partial\vec{e}}{\partial t} = \frac{4\pi}{c}\vec{J}$	(2)
Faraday	$\nabla\times\vec{e} + \frac{1}{c}\frac{\partial\vec{b}}{\partial t} = 0$	(3)
Gauss	$\nabla \cdot \vec{b} = 0$	(4)

5−

■■ DIELECTRICS: Polarization, Displacement, and Permittivity

WT Section 5.4
Jackson chapter 4

What is a dielectric material? (You should know ...)

The Oxford Dictionary of Physics

dielectric --
-- A nonconductor of electric charge; an applied electric field causes displacement of electric charge but not flow of charge; the displacement of negative charge relative to positive charge produces the polarization $\vec{P}(\vec{x},t)$.

−5

In[130]:= a7

Out[130]=

Polarization = $\vec{P}(\vec{x}, t)$ = electric dipole moment density
Bound charge density = $\rho_b = -\nabla \cdot \vec{P}$
Charge density = $\rho_{tot} = \rho_{free} + \rho_b = \rho_{free} - \nabla \cdot \vec{P}$
Displacement field = $\vec{D} = \vec{e} + 4\pi \vec{P}$
Susceptibility χ is defined by $\vec{P} = \chi \vec{e}$
Permittivity ϵ is defined by $\vec{D} = \epsilon \vec{e} = (1 + 4\pi \chi) \vec{e}$

Now the field equation (1) becomes

$$\nabla \cdot \vec{D} = 4\pi \rho_{\text{total}} + 4\pi (-\rho_b) = 4\pi \rho_{\text{free}} .$$

So, instead of keeping the charge due to polarization in the equations and calculations, we introduce a new field, $\vec{D}$.

We need a constitutive equation, $\vec{D} = \epsilon \vec{e}$.

Electrostatics $D(x) = (1+4\pi \chi) E(x)$;
but $D(x,t)$ is not $\propto E(x,t)$.

6–

■■ MAGNETISM: MAGNETIZATION, H-FIELD, AND PERMEABILITY

WT Section 6.10.

What is “magnetism”?
The Oxford Dictionary of Physics

magnetism --
-- A group of phenomena associated with the interaction between magnetic fields and materials.

MAGNETISM ⊃ {diamagnetism, paramagnetism, ferromagnetism, antiferromagnetism, ferrimagnetism}

−6

In[131]:= a8

Out[131]=

Magnetization = $\vec{M}(\vec{x},t)$ = magnetic dipole moment density
Bound current density = $\vec{J}_b = c\,\nabla\times\vec{M} + \partial\vec{P}/\partial t$
Current density = $\vec{J}_{tot} = \vec{J}_{Free} + \vec{J}_{bound} = \vec{J}_{Free} + c\,\nabla\times\vec{M} + \partial\vec{P}/\partial t$
Magnetic field $\vec{H} = \vec{b} - 4\pi\vec{M}$
Susceptibility χ_M is defined by $\vec{M} = \chi_M\,\vec{b}$
Permeability μ is defined by $\vec{H} = \vec{b}/\mu$

Now the field equation (2) becomes

$$\nabla\times\vec{H} = \frac{4\pi}{c}\vec{J}_{\text{free}} + \frac{1}{c}\frac{\partial\vec{D}}{\partial t}$$

So, instead of keeping the currents due to magnetization and polarization in the equations, we introduce a new field, $\vec{H}$.

We need another constitutive equation, $\vec{H} = \vec{b}/\mu$.

In[132]:=

7–

In[133]:= a3

Out[133]=

Maxwell Equations	phenomelogical fields	⌗
Gauss	$\nabla \cdot \vec{D} = 4\pi \rho_{free}$	(1)
Ampere	$\nabla \times \vec{H} - \frac{1}{c} \frac{\partial \vec{D}}{\partial t} = \frac{4\pi}{c} \vec{J}_{free}$	(2)
Faraday	$\nabla \times \vec{E} + \frac{1}{c} \frac{\partial \vec{B}}{\partial t} = 0$	(3)
Gauss	$\nabla \cdot \vec{B} = 0$	(4)

−7

Friday August 31

Hand in — Homework Assignment #1.

Pick up — Homework Assignment #2.