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Chapter 9.

PLANE E. M. WAVES AND
PROPAGATION IN MATTER

Electromagnetic waves in matter are similar to and different from electromagnetic waves in vacuum.

Before we go to Chapter 9, today's lecture is

"A Review of D and H"

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Sections 5.4 : Electrostatics with Dielectrics

● Classical microscopic electrostatics

$\nabla \cdot \vec{E} = 4\pi\rho$ and $\nabla \times \vec{E} = 0$

Quantum theory is different;
the classical theory is accurate for large $N\gamma$.

● Classical macroscopic electrostatics

Matter is molecular, and molecules contain electric charges. A molecule is electrically neutral, but it can be polarized (polar or induced). Then it has an electric dipole moment.

Molecular electric dipole moments

$\vec{p} = \int d^3x \; \vec{x}\, \rho(\vec{x})$;

$\Longrightarrow$ a macroscopic polarization, $\vec{P}(\vec{x})$

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Define polarization $\vec{P}(\vec{x})$ = electric dipole moment density

= moment per unit volume

= $\vec{p}(\vec{x})$ n($\vec{x}$)

⇒ What are the dimensions?

QL L^{-3} = Q L^{-2} = same as electric field

Polarization does not produce a net charge density.

However, *divergence of polarization* does produce a net charge density.

The crucial equation is

$\rho_{\text{bound}}(\vec{x}) = -\nabla \bullet \vec{P}(\vec{x})$.

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Out[•]=

Schwinger's Method –
analyze the force on a sample

$\vec{F}_{atom} = (\vec{p}\cdot\nabla)\vec{E}(\vec{x})\,\Big|_{\vec{x}=\text{position}}$ ....

$\vec{F}_{bulk} = \int d^3x\; n(\vec{x})\,\vec{F}_{atom}(\vec{x})$

$= \int d^3x\;(\vec{P}(\vec{x})\cdot\nabla)\,\vec{E}(\vec{x})$

polarization $\vec{P}(\vec{x}) = n(\vec{x})\vec{p}(\vec{x})$

$= \int_V d^3x\,(-\nabla\cdot\vec{P})\vec{E} + \int_S da(\hat{n}\cdot\vec{P})\,\vec{E}$

$\therefore$

$\rho_{bound} = -\nabla\cdot\vec{P}\;;\quad \sigma_{bound} = \hat{n}\cdot\vec{P}$

$(\vec{F} = q\vec{E})$

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Electric dipoles ten to shield free charges.
Figures 5.8 and 5.9.

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f59

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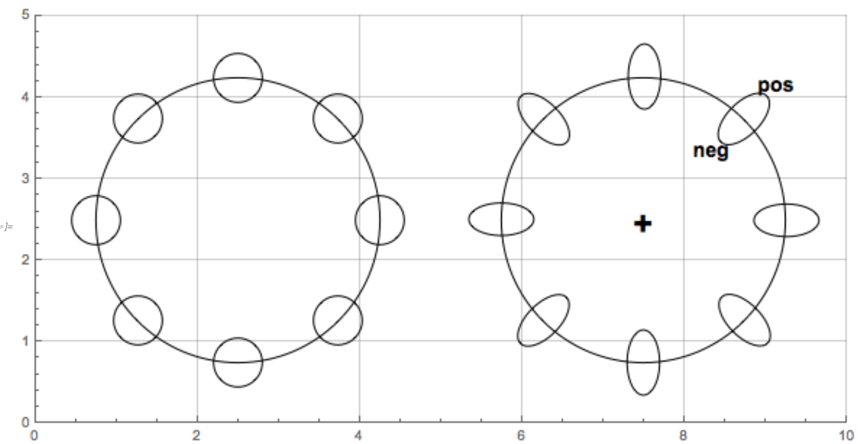

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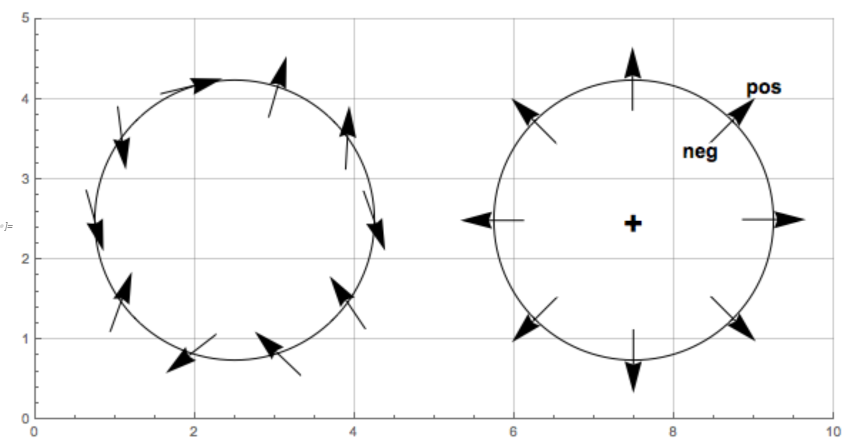

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● The displacement field

We can write

$\rho_{\text{total}}(\vec{x}) = \rho_{\text{free}}(\vec{x}) + \rho_{\text{bound}}(\vec{x}) = \rho_{\text{free}}(\vec{x}) - \nabla \bullet P(\vec{x})$

Then Gauss's law becomes

$$\nabla \bullet \vec{E} = 4\,\pi\,\rho_{\text{free}} - 4\,\pi\ \nabla \bullet \vec{P}$$

or

$$\boxed{\nabla \bullet \vec{D} = 4\,\pi\,\rho_{\text{free}} \quad \text{where} \quad \vec{D} = \vec{E} + 4\,\pi\,\vec{P}}$$

● Susceptibility χ and Permittivity ϵ

For a simple macroscopic medium — "linear isotropic dielectric" — we can write

$$\vec{P}(\vec{x}) = \chi(\vec{x})\,\vec{E}(\vec{x})\,.$$

Then,

$$\vec{D}(\vec{x}) = \epsilon(\vec{x})\,\vec{E}(\vec{x}) \quad \text{where} \quad \epsilon(\vec{x}) = 1 + 4\pi\,\chi(\vec{x})$$

For a uniform isotropic medium, χ and ϵ are independent of $\vec{x}$.

● Electric dipoles tend to shield charges

Therefore, $\chi > 0$ and $\epsilon > 1$.

The range of values of ϵ is large, from 1 to ~10^4.

In[◦]:=

$$\boxed{\begin{array}{l}\text{Electrostatics}\\ \nabla \times \vec{E} = 0\,; \quad \nabla \bullet \vec{D} = 4\pi\,\rho_{\text{free}}\,; \quad \vec{D} = \epsilon\,\vec{E}\end{array}}$$

What about time-dependent systems?
For time-dependent fields ...
– apply Fourier analysis ; $F(t) \longrightarrow \tilde{F}(\omega)$
– ϵ depends on frequency ; $\epsilon = \epsilon(\omega)$
– $\epsilon(\omega)$ is complex!
$\vec{P}(x,t)$ is not $\propto \vec{E}(x,t)$ because of time delay.

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Sections 6.9 — 6.11 : Magnetostatics with continuous media

What is magnetism?
The Oxford dictionary of physics

magnetism --
-- A group of phenomena associated with the interaction between magnetic fields and materials.

MAGNETISM ⊃ {diamagnetism, paramagnetism, ferromagnetism, antiferromagnetism, ferrimagnetism}

● Classical microscopic magnetostatics

$$\nabla \cdot \vec{B} = 0 \quad \text{and} \quad \nabla \times \vec{B} = \frac{4\pi}{c} \vec{J}$$

Quantum theory is different, but the classical theory is accurate for $N\gamma \to \infty$.

● Classical macroscopic magnetostatics

Matter is molecular, and molecules contain electric currents from electron orbital motion and electron spin. Electric currents make magnetic dipoles. The dipole moment of an isolated molecule may be zero (⇒ diamagnetism) or nonzero (⇒ paramagnetism or ferromagnetism)

Molecular magnetic dipole moments

$$\vec{m} = \frac{1}{2c} \oint \vec{x} \times d\vec{I}$$

produce a macroscopic magnetization.

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Define magnetization $\vec{M}(\vec{x})$ = magnetic dipole moment density

= moment per unit volume

= $\vec{m}(\vec{x})$ n($\vec{x}$)

⇒ What are the dimensions?

$I\,T/L\;L^2\,L^{-3}$= Q L^{-2} , same as $\vec{B}$ and $\vec{E}$

Magnetization does not produce a net current density.

However, curl of magnetization does produce a net current density.

The crucial equation is $\vec{J}_{\text{bound}}(\vec{x})$ = c ∇ × $\vec{M}(\vec{x})$.

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In[•]:= DHSc2

analyze The force on a sample

$\vec{F}_{atom} = \nabla(\vec{m}\cdot\vec{B}) = \sum_i m_i \nabla B_i$

$\vec{F}_{bulk} = \int d^3x \; n(\vec{x}) \; \vec{F}_{atom}(\vec{x})$

$= \int d^3x \; n \, m_i \, \nabla B_i$

$= \int d^3x \; M_i \, \nabla B_i$

magnetization $\vec{M}(\vec{x}) = n(\vec{x}) m(\vec{x})$

Out[•]=

= ... see page 280 ...

$= \int_V d^3x \, (\nabla\times\vec{M})\times\vec{B} + \int_S da \, (\vec{M}\times\hat{n})\times\vec{B}$

$\vec{F} = \frac{1}{c}\int d^3x \; \vec{J}\times\vec{B}$

$\therefore \vec{J}_{bound} = c\,\nabla\times\vec{M}$; $\vec{K}_{bound} = c\,\hat{n}\times\vec{B}$

$\left(\vec{F} = q\,\frac{\vec{v}}{c}\times\vec{B}\right)$

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● The magnetic fields $\vec{B}$ and $\vec{H}$

Some people call $\vec{B}$ the magnetic field and they call $\vec{H}$ the magnetic field. The nomenclature in the Oxford dictionary of physics ...

$\vec{B}$ = magnetic flux density, or magnetic induction;

$\vec{H}$ = magnetic field strength, or magnetizing force, or magnetic intensity;

We can write

$$\vec{J}_{\text{total}}(\vec{x}) = \vec{J}_{\text{free}}(\vec{x}) + \vec{J}_{\text{bound}}(\vec{x})$$
$$= \vec{J}_{\text{free}}(\vec{x}) + c\, \nabla \times \vec{M}(\vec{x})$$

Then Ampere's law becomes

$$\nabla \times \vec{B} = \frac{4\pi}{c} \vec{J}\text{free} + 4\pi\, \nabla \times \vec{M}$$

or

$$\nabla \times \vec{H} = \frac{4\pi}{c} \vec{J}_{\text{free}} \quad \text{where} \quad \vec{H} = \vec{B} - 4\pi \vec{M}$$

● Magnetic Permeability μ

For a simple continuum medium — an isotropic diamagnetic or paramagnetic material,

$$\vec{B}(\vec{x}) = \mu(\vec{x})\, \vec{H}(\vec{x}) \;\; ;$$

or, for a uniform material, $\vec{B} = \mu\, \vec{H}$.

Paramagnetism $\mu > 1$; polar molecules align with $\vec{B}$

Diamagnetism $\mu < 1$; induced molecular moments anti-align with $\vec{B}$

Typical size of the magnetic effect is $(\mu - 1) \sim + 10^{-4}$ or $- 10^{-5}$.

(much larger for ferromagnetism)

Magnetostatics

$$\nabla \cdot \vec{B} = 0 \; ; \quad \nabla \times \vec{H} = \frac{4\pi}{c} \vec{J}_{\text{free}} \; ; \quad \vec{B} = \mu \vec{H}$$

What about time-dependent systems?

$\mu = \mu(\omega)$; but this is usually not as important as $\epsilon = \epsilon(\omega)$.

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Section 7.1 — Time-Varying Fields
"Plausibility argument for the time-dependent Maxwell equations"

QUOTE (page ?)
In summary, we have given plausible justifications for the four Maxwell equations,

"Coulomb" $\nabla \cdot \vec{E} = 4\pi\ \rho$

"Faraday" $\nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$

"Ampere/Maxwell" $\nabla \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{J}$

"No-monopole" $\nabla \cdot \vec{B} = 0$

UNQUOTE

Obviously those are the equations for *microscopic electrodynamics*; i.e., fields and isolated charges and isolated currents in otherwise empty space.

Now, *what are Maxwell's equations for macroscopic electrodynamics?*
I.e., fields $(\vec{E}, \vec{D}, \vec{B}, \vec{H})$, free charges (ρ_{free}) and free currents ($\vec{J}_{\text{free}}$) in the presence of macroscopic media (i.e., materials).

Review Section 7.3
7.3.1 = Schwinger's method (forces)
7.3.2 = Jackson's method (currents)

$\Longrightarrow\ \rho_{bound} = -\nabla \cdot \vec{P}$

and $\vec{J}_{bound} = c\nabla \times \vec{M} + \frac{\partial \vec{P}}{\partial t}$

The Results

$\rho = \rho_{\text{free}} + \rho_{\text{bound}} = \rho_{\text{free}} - \nabla \bullet \vec{P}$

$\vec{J} = \vec{J}_{\text{free}} + \vec{J}_{\text{bound}} = \vec{J}_{\text{free}} + \text{c}\, \nabla \times \vec{M} + \frac{\partial \vec{P}}{\partial t}$

Homogeneous field equations ...

$\nabla \times \vec{E} + \frac{1}{c} \frac{\partial \vec{B}}{\partial t} = 0$ and $\nabla \bullet \vec{B} = 0$;

and inhomogeneous equations ...

$\nabla \bullet \vec{D} = 4\pi\, \rho_{\text{free}}$ and $\nabla \times \vec{H} - \frac{1}{c} \frac{\partial \vec{D}}{\partial t} = \frac{4\pi}{c}$ $\vec{J}_{\text{free}}$;

here

$\vec{D} \equiv \vec{E} + 4\pi\, \vec{P}$ and $\vec{H} \equiv \vec{B} - 4\pi\, \vec{M}$;

$\vec{P}(\vec{x},\text{t})$ and $\vec{M}(\vec{x}, t)$ are the elec. and mag. dipole densities.

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Summary

(1) The electrodynamics of materials may be summarized by ...

$\rho_{\text{bound}} = -\nabla \bullet \vec{P}$

$\vec{J}_{\text{bound}} = \text{c}\, \nabla \times \vec{M} + \frac{\partial \vec{P}}{\partial t}$

(2) For simple materials, we describe the effects of molecular electric and magnetic dipoles by introducing the fields

$\vec{D} \equiv \vec{E} + 4\pi\, \vec{P}$ and $\vec{H} \equiv \vec{B} - 4\pi\, \vec{M}$.

(3) Then we need “constitutive equations”; i.e., relations $\vec{D} = \epsilon\, \vec{E}$ and $\vec{B} = \mu\, \vec{H}$.