1 ✓

Chapter 9.
PLANE E. M. WAVES
AND PROPAGATION IN MATTER

★ Lecture #1 on waves in matter

★ Section 9.1
Plane waves in dielectric media

■1

Plane Waves in Dielectric Media

Recall plane waves in vacuum ...

$$\vec{E}(\vec{x},t) = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$$

and $\vec{B}(\vec{x},t) = \vec{B}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$

- The real parts are the physical fields.
- The vectors $\{\vec{k}, \vec{E}_0, \vec{B}_0\}$ form a right-handed orthogonal triad.
- Dispersion relation $\omega = c\,k$
- $|\vec{B}_0| = |\vec{E}_0|$

Now calculate plane waves in a linear medium, with permittivity ϵ and permeability μ.

Today we' ll take ϵ and μ to be constant real numbers.
Later we’ll find that this assumption is not generally valid.

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Macroscopic electrodynamics with no free sources

The equations

i.e., Maxwell' s equations with $\rho_{\text{free}} = 0$ and $\vec{J}_{\text{free}} = 0$,

$$\nabla \times \vec{E} + \frac{\partial \vec{B}}{c\,\partial t} = 0 \quad \text{and} \quad \nabla \bullet \vec{B} = 0$$

$$\nabla \bullet \vec{D} = 0 \quad \text{and} \quad \nabla \times \vec{H} - \frac{\partial \vec{D}}{c\,\partial t} = 0.$$

The solutions

The plane wave solutions are complete, so we seek ...

$$\vec{E}(\vec{x},t) = \vec{E}_0 \; e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$$

and $\vec{B}(\vec{x},t) = \vec{B}_0 \; e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$

Substituting into Maxwell's equations,

∇ gets replaced by i $\vec{k}$ and

∂/∂t gets replaced by –i ω $\Longrightarrow$

■2

$$i\,\vec{k} \times \vec{E}_0 - i\,(\omega/c)\,\vec{B}_0 = 0$$

$$i\,\vec{k} \bullet \vec{B}_0 = 0$$

$$i\,\vec{k} \bullet \vec{E}_0\,\epsilon = 0$$

$$i\,\vec{k} \times \vec{B}_0\,/\mu \;+ i\,(\omega/c)\,\epsilon\,\vec{E}_0 = 0$$

(you can cancel all the factors of i)

3✓

$$\vec{k}\times\vec{E}_0 - (\omega/c)\,\vec{B}_0 = 0 \quad \& \quad \vec{k}\bullet\vec{B}_0 = 0$$
$$\vec{k}\bullet\vec{E}_0\,\epsilon = 0 \quad \& \quad \vec{k}\times\vec{B}_0/\mu + (\omega/c)\,\epsilon\,\vec{E}_0 = 0$$

RESULTS

● Note that $\vec{k}\cdot\vec{E}_0 = 0$ and $\vec{k}\cdot\vec{B}_0 = 0$ and $\vec{k}\times\vec{E}_0$ is parallel to $\vec{B}_0$;
$\therefore \{\vec{k}, \vec{E}_0, \vec{B}_0\}$ form a right-handed orthogonal triad of vectors.

● $\vec{k}\times\vec{E}_0 = (\omega/c)\ \vec{B}_0 \implies$
$$k\,E_0 = (\omega/c)\,B_0$$
$\vec{k}\times\vec{B}_0 = -\mu\epsilon\,(\omega/c)\ \vec{E}_0 \implies$
$$k\,B_0 = \mu\epsilon\,(\omega/c)\ E_0$$

So $\therefore$

▪ $v_{\text{phase}} = \dfrac{\omega}{k} = \dfrac{c}{\sqrt{\epsilon\mu}}$

▪ the *index of refraction*
$$n \equiv c\,/\,v_{\text{phase}} = \sqrt{\epsilon\mu} > 1$$

▪ $B_0 = \sqrt{\epsilon\mu}\ E_0$ (i.e., $B_0 > E_0$)

Is there anything more to say?

Light travels slower in a material medium than in vacuum.

The phase velocity is $v_{phase} = c/n$, where $n = \sqrt{\epsilon\mu}$ is the index of refraction; this leads us to GEOMETRICAL OPTICS.

The group velocity is $v_{group} = \frac{d\omega}{dk}$;
$v_{group} \neq v_{phase}$ because of "dispersion" : ϵ and μ vary with ω.

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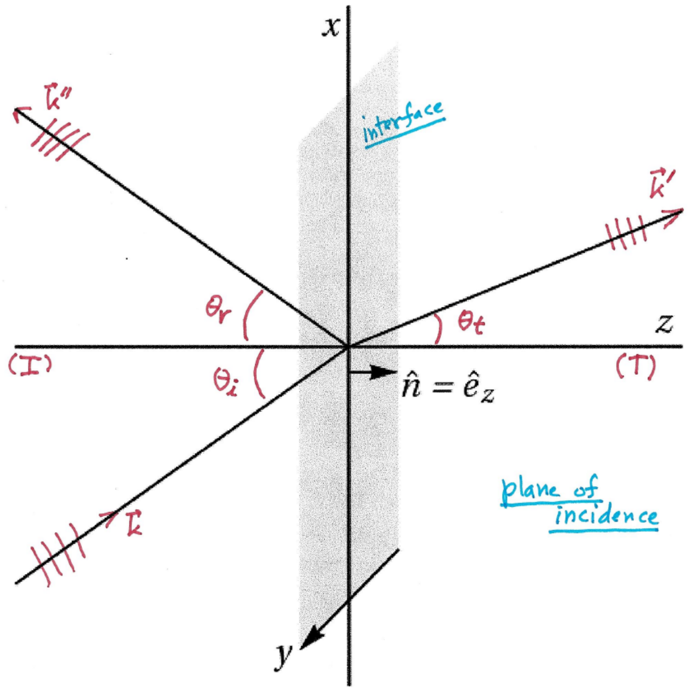
Section 9.2: Reflection and Refraction at a Dielectric Interface

In[•]:= W1F1

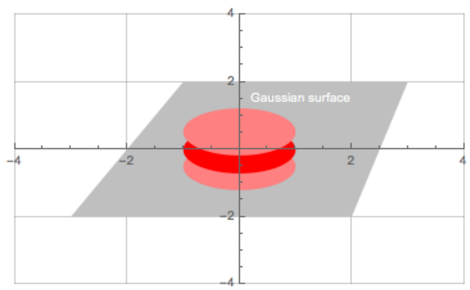
Out[•]=


■4

But first we need to review something — Boundary Conditions of the Fields at Surfaces

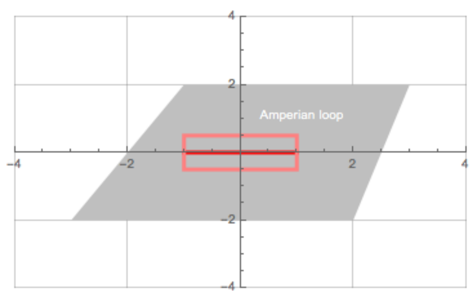
/1/ B_{normal} is continuous across any surface. (Eq. 1.21)

In[•]:= W1F2

Out[•]=


/2/ $E_{\text{tangential}}$ is continuous across any surface. (Eq. 1.26)

In[•]:= W1F3

Out[•]=


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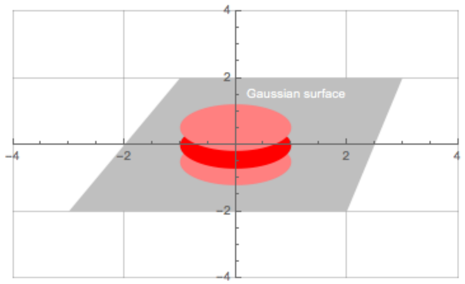
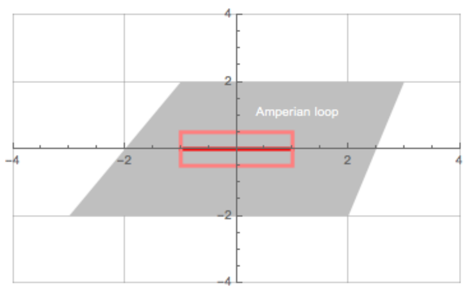
/3/ D_{normal} is continuous across any electrically neutral surface ($\sigma_{\text{free}} = 0$). (Eq. 1.20)

In[=]:= W1F2

Out[=]=


/4/ $H_{\text{tangential}}$ is continuous across any current-free surface ($\vec{K}_{\text{free}} = 0$). (Eq. 1.25)

In[=]:= W1F3

Out[=]=


■5

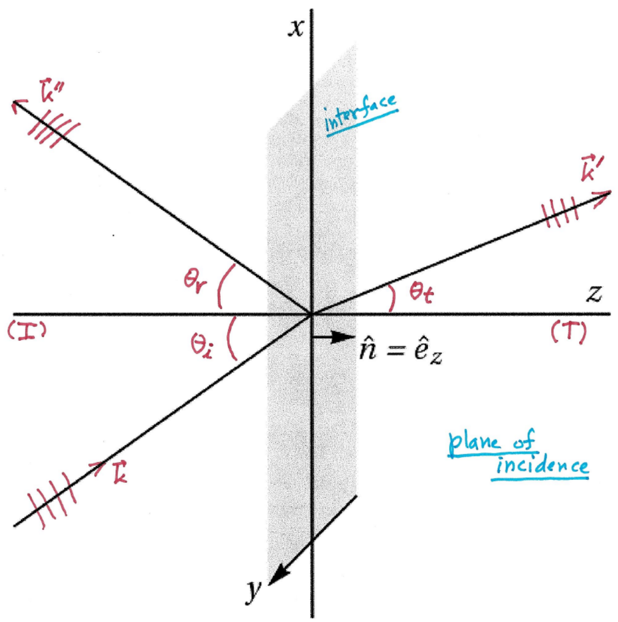
To calculate the reflection and refraction at a dielectric interface, all we need to do is to apply these boundary conditions on the fields: E_{tang}, D_{norm}, B_{norm}, H_{tang} are continuous across the interface.

6 ✓

Start again; Section 9.2:
Reflection and Refraction at a Dielectric Interface

In[ ]:= W1F1

Out[ ]=



Study the figure.

■ The *interface* is the xy-plane; i.e., z = 0; normal vector $\hat{n} = \hat{e}_z$.

■ The *plane of incidence* is the xz-plane (≡ the plane spanned by the normal vector $\hat{n} = \hat{e}_z$ and the incident vector $\vec{k} = k_x\,\hat{e}_x + k_z\,\hat{e}_z$.)

■ We'll calculate reflection and refraction for incident plane waves. (The reflected and transmitted waves will also be plane waves.)

the incident wave:

$\vec{E}$ and $\vec{B}$; $\vec{k}$ and ω ; $\hat{e}$ and $\hat{k} \times \hat{e}$

the transmitted wave ;

$\vec{E'}$ and $\vec{B'}$; $\vec{k'}$ and ω'; $\hat{e}'$ and $\hat{k}' \times \hat{e}'$

the reflected wave ;

$\vec{E}''$ and $\vec{B}''$; $\vec{k''}$ and ω'' ; $\hat{e}''$ and $\hat{k}'' \times \hat{e}''$

■ There are two polarization cases:

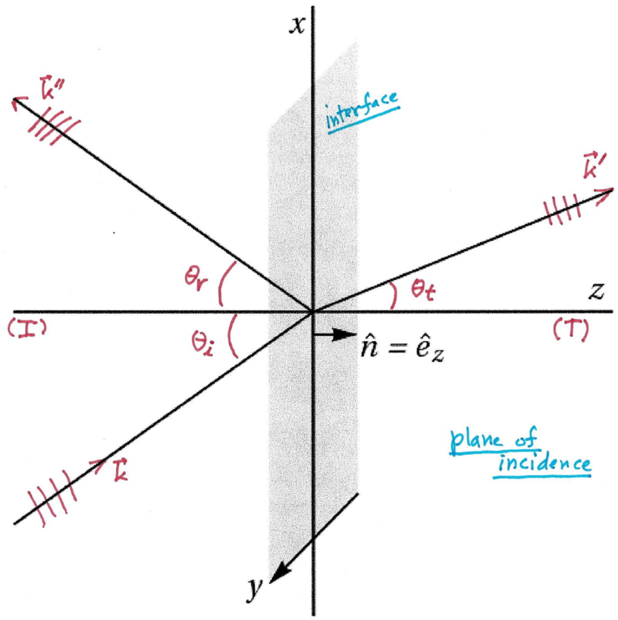
TE : Transverse Electric polarization; i.e., $\hat{e}$ is perpendicular to the plane of incidence

TM : Transverse Magnetic polarization; i.e.,

$\hat{k} \times \hat{e}$ is perpendicular to the plane of incidence

In[•]:= W1F1

Out[•]=



We have 4 boundary conditions to apply.
I'll work out one example.
Consider TE incident polarization.
Then we have $\hat{n} = \hat{e}_z$; $\vec{k} = k_x\,\hat{e}_x + k_z\,\hat{e}_z$; and $\hat{e}_{\text{inc}} = \hat{e}_y$.
$\vec{E}_{\text{tangential}}$ is continuous at z = 0.

In[•]:= W1Sc1

Out[•]=

TE incident wave:
(E⃗ᵢ transverse to plane of incidence)

$$\hat{e} = \hat{e}_y \quad \text{and} \quad \vec{k} = k_x \hat{e}_x + k_z \hat{e}_z$$

$$\vec{k} = n_I \frac{\omega}{c}\left[\hat{e}_x \sin\theta_i + \hat{e}_z \cos\theta_i\right]$$

$$\hat{e}' = \hat{e}'' = \hat{e}_y$$

$$\vec{k}' = n_T \frac{\omega'}{c}\left[\hat{e}_x \sin\theta_t + \hat{e}_z \cos\theta_t\right]$$

$$\vec{k}'' = n_I \frac{\omega''}{c}\left[\hat{e}_x \sin\theta_r - \hat{e}_z \cos\theta_r\right]$$

$$\vec{E}_{tang.}(z=0) = \vec{E}_0\, e^{i(k_x x - \omega t)} + \vec{E}_0''\, e^{i(k_x'' x - \omega'' t)} = \vec{E}_0'\, e^{i(k_x' x - \omega' t)}$$

The final equation must hold for all t;
$\therefore \quad \omega = \omega' = \omega''$.

And the equation must hold for all x;
$\therefore \quad k_x = k'_x = k''_x$.

8 ✓

Angles

Write $\theta_{\text{incident}} = \theta_i$;
$\theta_{\text{transmitted}} = \theta_t$; $\theta_{\text{reflected}} = \theta_r$.

In[•]:=

$k_x = k''_x \implies$
$(n_I\,\omega/c) \sin\theta_i = (n_I\,\omega''/c) \sin\theta_r$
$\therefore\ \theta_i = \theta_r$
(the law of equal angles for reflection)

$k_x = k'_x \implies$
$(n_I\,\omega/c) \sin\theta_i = (n_T\,\omega'/c) \sin\theta_t$
$\therefore\ n_I \sin\theta_i = n_T \sin\theta_t$
(Snell's law for refraction)

▪8

These familiar laws of geometrical optics are required by the electromagnetic field theory, in order to satisfy one of the boundary conditions ($E_{\text{tangential}}$ is continuous). The other three boundary conditions require exactly the same laws of geometrical optics. (It is easy to see why!)

So now we understand the directions of reflection and refraction. Next, what are the intensities of reflection and refraction? For that we need to derive Fresnel's equations (Sections 9.2 and 9.3).